A Comparative Study of the Influence of Shift Factor Equations on Master Curve Construction

Nur Izzi Md. Yusoff†, Emmanuel Chailleux2, and Gordon D. Airey3

Abstract: This paper evaluates the applicability of different shifting techniques for constructing complex modulus master curves using the time-temperature superposition principle (TTSP). A database of complex modulus results of unmodified bitumens, polymer modified bitumens (PMBs), bitumen-filler mastics, unaged and aged bitumens was used together with a Generalized Logistic Sigmoidal Model to assess the validity of seven different shifting approaches. Except for the Laboratoire Central des Ponts et Chaussées (LCPC) method, the construction of master curves was done using the Generalised Logistic Sigmoidal Model and non-linear least squares regression optimization with the aid of the Microsoft Excel Spreadsheet Solver function. The goodness-of-fit for the various shifting techniques and functions was assessed through graphical and statistical methods. From the study, it was found that a numerical shift approach using non-linear least squares produced the best fit between experimental and predicted data in terms of both graphical and goodness-of-fit statistics. The ranking of the other shifting techniques consisted of the LCPC approach, followed by the Williams, Landel and Ferry (WLF), Modified Kaelble, Viscosity Temperature Susceptibility (VTS), Arrhenius and Log-Linear methods. However, most of the equations are basically empirical and they are not expected to be strictly obeyed by any materials. Discrepancies were still evident, even for the better functions, for those materials that demonstrated a deviation from the thermorheological simplicity of the rheological behaviour as found for highly structured bitumens following high degrees of polymer modification and/or oxidative ageing.

Key words: Arrhenius equation, Generalised logistic sigmoidal model, Master curves, Shift factors, WLF equation.

Introduction

The rheological properties of bitumens are usually determined by means of dynamic mechanical analysis (DMA) using oscillatory type, dynamic shear rheometer (DSR) tests, generally conducted within the linear viscoelastic (LVE) region [1]. The principal viscoelastic coefficients that are obtained from the DSR are the magnitude of the complex modulus (|G*|) and the phase angle (δ). |G*| can be defined as the ratio of maximum shear stress to maximum strain and provides a measure of the total resistance to deformation when bitumen is subjected to shear loading. It consists of two components, namely the storage (G') and loss (G'') moduli. The phase angle (δ or φ) is the phase, or time difference between stress and strain in harmonic oscillation and is an indication of the viscoelastic balance of the material behaviour.

Studies into the viscoelastic behaviour of bitumen have received increased interest from various researchers since the early 1990s, following the activities of the Strategic Highway Research Program (SHRP) [2-3]. The rheological properties of bitumens are normally presented in terms of |G*| and δ master curves together with the determination of shift factors associated with temperature shifting of the rheological parameters. The temperature dependency of the viscoelastic behaviour of bitumens is indicated using shift factors and expressed as:

\[ a_T = \frac{f_r}{f} \]  

where \( a_T \) is the shift factor, \( f \) is the tested frequency and \( f_r \) is the reduced frequency at a reference temperature. Fig. 1 shows an example of how these shift factors at different temperatures are used to construct the \(|G^*|\) and δ master curve at a particular reference temperature. Temperature dependency should not be confused with temperature susceptibility. Temperature dependency can be defined as a fundamental concept that indicates how the relaxation process within bitumen changes with temperature. Meanwhile temperature susceptibility is an empirical concept based on the change of consistency or hardness of bitumen with temperature.

The construction of master curves can be done using an arbitrarily selected reference temperature to which all rheological data are shifted. At the reference temperature, \( T_{ref} \), the value of \( a_T \) is equal to one (\( \log a_T = 0 \)). In general, several different \( a_T \) functions can be used to model the time-temperature equivalency relationship of bitumens and asphalt mixtures. Details of these functions will be discussed in full in section 2, where all the functions only involve horizontal line movements and do not take vertical shifts into account. The vertical shift, \( b_T \), represents temperature induced density changes and involves shifts along the modulus axis. According to Wada and Hirose [4], since the temperature range of the experimental data for bitumen is usually relatively narrow, vertical shifts would only slightly modify results and moreover, the physical meaning of vertical shift is not a priori indent for bitumen, whose molecular structure is extremely complex. Furthermore, vertical shifts are highly dependent on the thermal history of the sample [5] and have been found to give a poor...
prediction of density change upon heating for most bitumens [6].

A considerable number of studies have been conducted using various shift factor equations in order to construct smooth and continuous $[G^*]$ and δ master curves [7-13]. However, the different shift factor equations are usually used in isolation and no comparative study of the different shift factor functions or their applicability to different types of bituminous binders has been undertaken. This study was therefore conducted to assess the validity of several different shifting functions for constructing $[G^*]$ master curves of different bituminous binders by applying the time-temperature superposition principle (TTSP) [14].

Seven different shifting techniques or methods; namely a numerical, non-functional form shift approach, the Williams, Landel and Ferry (WLF) equation, a Laboratoire Central des Ponts et Chaussées (LCPC) approach, the modified Kaelble equation, a Viscosity Temperature Susceptibility (VTS) equation, an Arrhenius equation, and a Log-Linear approach have been used together with a large LVE rheological database held by the Nottingham Transportation Engineering Centre (NTEC). The NTEC database includes different combinations of unaged and aged samples of unmodified bitumens, polymer modified bitumens (PMBs) and bitumen-filler mastics [15-18]. Correlations between the numerical, non-functional form (non-linear least squares) shift approach and other predicted functions were assessed using both graphical and goodness-of-fit statistical analysis methods.

**Shift Factor Laws**

**Numerical, Non-linear Least Squares Shift**

In the numerical, non-functional form shift approach, all the shift factors are solved simultaneously with the coefficients of the sigmoidal fitting function (described in section 3) using non-linear least squares fitting. This is achieved with the aid of the Microsoft Excel Spreadsheet Solver function without assuming any functional form for the relationship of $\alpha_T$ versus temperature as described by Pellinen et al. [9].

**Williams, Landel and Ferry (WLF) Equation**

The WLF equation, after it’s discovers Williams, Landel and Ferry, has been widely used to describe the relationship between the $\alpha_T$ and temperature dependency and thereby determine the $\alpha_T$ of bitumens [19]:

$$\log a_T = \frac{-C_1(T - T_{ref})}{C_2 + (T - T_{ref})}$$

(2)

where $T$ is temperature, $T_{ref}$ is the reference temperature, $C_1$ and $C_2$ are taken as constants. The other parameters are as previously defined. This method is found to be applicable for both bitumens [20-22] and asphalt mixtures [23].

**Modified Kaelble Equation**

The Modified Kaelble equation can be thought of as a modification of the WLF equation and shown in the following form [3]:

$$\log a_T = \frac{-C_1(T - T_{ref})}{C_2 + (T - T_{ref})}$$

(3)

where the parameters are as defined in Eq. (2). Like the WLF equation, the $C_1$ and $C_2$ coefficients are used to minimise the error between measured and predicted $[G^*]$ data. The glass transition temperature ($T_g$) in the original equation has been replaced by $T_{ref}$ since this study did not involve very low temperatures.

**Arrhenius Equation**

The Arrhenius equation can be described as the following:

$$\log a_T = C \left(1 - \frac{1}{T} \right) = \frac{0.4347}{R} \left(1 - \frac{1}{T} \right)$$

where $C$ is a constant, $E_a$ is the activation energy (J/mol) and $R$ is the ideal gas constant (8.314 J/mol.K). The other parameters are as previously defined. In the literature, different values, such as 10920,
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13060 and 7680 K were reported for the constant $C$ [24]. The Arrhenius expression requires only one constant to be determined, $E_m$ which describes the minimum energy needed before any intermolecular movement can occur.

**Log-Linear Equation**

Eq. (5) shows the form of the equation with the concept of reference temperature:

$$\log a \left( \frac{T}{T_{ref}} \right) = \beta(T - T_{ref})$$  \hspace{1cm} (5)

where $\beta$ is the slope of the straight line relationship between $\log a_T$ and temperature $T$. As discussed by Garcia and Thompson [22], the Log-Linear equation is normally only used for asphalt mixtures. However, this equation has been used in this study as a comparison to the other methods.

**Viscosity Temperature Susceptibility (VTS) Equation**

In the Mechanistic-Empirical Pavement Design Guide (MEPDG), $a_T$ was expressed as a function of binder viscosity to allow ageing over the life of the pavement to be considered using a global ageing model, developed by Mirza and Witzczak [25]. Mirza and Witzczak later devised the shift factor equation used in the MEPDG termed the viscosity temperature susceptibility (VTS) equation [26]:

$$\log a_T = c(0.4 + VTS \log T_R - 0.4 VTS \log A)$$  \hspace{1cm} (6)

where $T_R$ is temperature (Rankine), $(T_R)_0$ is the reference temperature (Rankine), $A$ is the regression line intercept, $VTS$ is regression line slope (called VTS coefficient) and $c$ is a constant.

**Laboratoire Central des Ponts et Chaussées (LCPC) Approach**

Chailleux et al. [21] from the LCPC, France established a mathematical basis procedure in order to construct master curves from dynamic measurements. Using the Kramers-Kronig relations and considering two close frequencies ($f_i$ and $f_j$), they showed that:

$$\delta^{(f_i,f_j)}_{a_T} \cdot \frac{2}{\pi} = \frac{\log \left( G^* (T, f_i) \right) - \log \left( G^* (T, f_j) \right)}{\log(f_i) - \log(f_j)}$$  \hspace{1cm} (7)

where $\delta^{(f_i,f_j)}_{a_T}$ is the average of the two phase angles measured at $f_i$ and $f_j$ for temperature $T$ and $G^*(T, f)$ represents the complex modulus. Note that the Kramers-Kronig relations are integral transform relationships between the real and imaginary parts of a complex function including the relation: $\delta(\alpha) = \pi \frac{\log(G^* (\alpha))}{2 \log(\alpha)}$. A shift factor, $a_{T_1,T_2} = f_2f_1$, exists as the TTSP is presumably valid. For two close temperatures, Eq. (7) can be rewritten as:

$$\delta^{(T_1,T_2)}_{a_T} \cdot \frac{2}{\pi} = \frac{\log \left( G^* (T_1, f) \right) - \log \left( G^* (T_2, f) \right)}{\log(a_{T_1,T_2})}$$  \hspace{1cm} (8)

where $\delta^{(T_1,T_2)}_{a_T}$ is the average of two phase angles measured at $T_1$ and $T_2$ for $f_2$. Shift factors can be calculated using Eq. (8) for close isotherms, at only one frequency. If measurements are carried out at temperatures $T_{1}$, $T_{2}$, $T_{3}$, $T_{4}$, ..., $T_{n}$ master curve construction related to $T_{ref}$ (with reference between 1 to $n$) can be performed using the cumulative sum of $\log \left( a_{T_1,T_{ref}} \right)$. Hence, the shift factor needed for an isotherm $T_i$ according to $T_{ref}$ will be:

$$\log(a_{T_i,T_{ref}}) = \sum_{j=i}^{j=n} \log \left( a_{T_j,T_{ref}} \right)$$  \hspace{1cm} (9)

Combining Eqs. (8) and (9), allows the shift factors for the rheological data to be calculated as:

$$\log(a_{T_i,T_{ref}}) = \sum_{j=i}^{j=n} \frac{\log \left( G^* (T_j, f) \right) - \log \left( G^* (T_{ref}, f) \right)}{\delta^{(f_j,f_{ref})}_{a_T}} \times \frac{\pi}{2}$$  \hspace{1cm} (10)

The calculation of $a_T$ using the LCPC approach is based on linear viscoelasticity theory and derived solely from measurements of $|G^*(\mu, \nu)|$ and $\delta (\mu, T)$ without the need for any adjustable coefficients.

**Constructing Master Curves using the Generalized Logistic Sigmoidal Model**

Master curves have been used by various researchers to describe and represent the LVE characteristics of bitumens and asphalt mixtures over a wide range of temperatures and frequencies. A large number of these studies have been devoted to the development of models to predict these rheological master curves. Among the various predictive models, the Sigmoidal Model used in the MEPDG can be considered to be one of the most popular. This model can be represented as follows [9, 24, 26]:

$$\log |G^* (f, T)| = \delta + \frac{\alpha}{1 + e^{B \gamma (\log f_i)}}$$  \hspace{1cm} (11)

where $|G^*(f, T)|$ is complex modulus as a function of frequency and temperature, $\log f_i$ is the log reduced frequency, $\delta$ is the lower asymptote, $\alpha$ is the difference between the values of the upper and lower asymptotes, $\beta$ and $\gamma$ are defined as shape coefficients. Fig. 2 shows a graphical definition of the sigmoidal function. Recently, the use of a Generalized Logistic Sigmoidal Model (or Richards Model) was recommended by Rowe et al. [3] to obtain a better fit of the non-symmetric curve of the master curve. This equation can be shown as:

$$\log |G^* (F, T)| = \delta + \frac{\alpha}{\left( 1 + \lambda e^{B \gamma (\log f_i)} \right)^{1/\nu}}$$  \hspace{1cm} (12)
where the coefficients are as previously defined. The $\lambda$ coefficient allows the curve to take a non-symmetrical shape (Fig. 3). When $\lambda$ reduces to one, the above equation shrinks to the standard sigmoidal function as represented by Eq. (11) [3].

### Details of the Fitting Procedure

The construction of the $[G^*]$ master curve was done with the aid of the Microsoft Excel Solver function, used for performing optimisation of data with non-linear least squares regression techniques. The procedure consisted of minimising the sum of square error (SSE) between numerical shift (hereafter called measured) and equation-based shift factor (hereafter called predicted) data as shown in Eq. (13):

$$
SSE = \sum \left( \frac{\left( \log |G^*_{\text{exp}}(f, T)| - \log |G^*_{\text{pre}}(\alpha_1(T, T_{\text{ref}}) \cdot f, T_{\text{ref}})| \right)^2}{\log |G^*_{\text{exp}}(f, T)|} \right)
$$

(13)

where $|G^*_{\text{exp}}(f, T)|$ is measured complex modulus, $|G^*_{\text{pre}}(f, T)|$ is predicted complex modulus, $T$ is temperature (°C), $T_{\text{ref}}$ is the reference temperature, $f$ is frequency (Hz) and $\alpha_1(T, T_{\text{ref}})$ is shift factor. In this study, the $T_{\text{ref}}$ was arbitrarily taken as $25^\circ$C. By combining Eqs. (12) and (13), the following equation is obtained:

$$
SSE = \sum \left( \frac{\left( \frac{\alpha}{\left(1 + \lambda e^{(\beta(T + \gamma) - \delta)}\right)^{1/\beta}} \right)^2 - \log |G^*_{\text{exp}}(f, T)|}{\log |G^*_{\text{exp}}(f, T)|} \right)
$$

(14)

The coefficients $\alpha$, $\delta$, $\beta$, $\lambda$, and $\alpha_1(T, T_{\text{ref}})$ are fitted in the minimization procedure between measured and predicted data. For the shift factor $\alpha_1(T, T_{\text{ref}})$ coefficient, the various expressions described in section 2 were used. For the numerical shift and LCPC methods, the only parameters used to minimise the error between measured and predicted $|G^*|$ data are those from the Generalised Logistic Sigmoidal Model. In contrast, for the other functions, such as the WLF, Modified Kaelble, Arrhenius, VTS and Log-Linear equations, their coefficients were used together with the master curve model. For instance, when the WLF equation is used to represent the shift factor function, Eq. (14) becomes:

$$
SSE = \sum \left( \frac{\left( \frac{\alpha}{\left(1 + \lambda e^{(\beta(T + \gamma) - \delta)}\right)^{1/\beta}} \right)^2 - \log |G^*_{\text{exp}}(f, T)|}{\log |G^*_{\text{exp}}(f, T)|} \right)
$$

(15)

The coefficients that need to be determined are now $\alpha$, $\delta$, $\beta$, $\lambda$, $C_1$ and $C_2$. The Solver function, together with initial seed values for the coefficients, is used to obtain the optimum values of the coefficients using a number of minimisation runs [27]. When no further changes are observed, the iteration process is terminated and the final values quoted for the coefficients.

However, it should be noted that this study only focuses on the comparison between measured and predicted $\alpha_1$ values and does not consider anomalies that can be seen in the construction of some of the $|G^*|$ master curves. This refers particularly to anomalies associated with the presence of highly crystalline bitumens (wax content > 7%), structured bitumens with high asphaltene content and highly PMBs (> 5% polymer content). Fig. 4 shows an example of the WLF shift factor function for one of the unmodified bitumen samples used in this study. Fig. 5 subsequently depicts $|G^*|$ and $\delta$ master curves constructed using the above shift factors.

### Goodness-of-Fit Statistics

Several statistical methods have been used in this study to indicate the goodness-of-fit between measured and predicted data [28-30].

#### Standard Error Ratio ($S_s$/$S_n$)

The standard error of estimation, $S_n$, and standard error of deviation, $S_s$, can be defined as follows:

$$
S_s = \sqrt{\frac{n}{(n-k)}} \left( \frac{\sum (Y - \bar{Y})^2}{(n-k)} \right)
$$

(16)
Table 1. Criteria of the Goodness-of-fit Statistics [28].

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$R^2$</th>
<th>$S_e/S_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
<td>$\geq 0.90$</td>
<td>$\leq 0.35$</td>
</tr>
<tr>
<td>Good</td>
<td>$0.70 – 0.89$</td>
<td>$0.36 – 0.55$</td>
</tr>
<tr>
<td>Fair</td>
<td>$0.40 – 0.69$</td>
<td>$0.56 – 0.75$</td>
</tr>
<tr>
<td>Poor</td>
<td>$0.20 – 0.39$</td>
<td>$0.76 – 0.89$</td>
</tr>
<tr>
<td>Very Poor</td>
<td>$\leq 0.19$</td>
<td>$\geq 0.90$</td>
</tr>
</tbody>
</table>

where $n$ is sample size, $k$ is number of independent variables in the model, $Y$ is measured $a_f$, $\bar{Y}$ is predicted $a_f$ and $\bar{Y}$ is mean value of measured $a_f$.

The Coefficient of Determination, $R^2$

The coefficient of determination, $R^2$ is determined as follows:

$$R^2 = 1 - \left(\frac{n-k}{n-1}\right) \left(\frac{S_e}{S_y}\right)^2$$  \hspace{1cm} (18)

where the coefficients are as previously defined. For the perfect fit, $R^2 = 1$. The criteria for the $S_e/S_y$ and $R^2$ goodness-of-fit statistics are shown in Table 1 [28].

The Discrepancy Ratio, $r_i$

The discrepancy ratio, $r_i$ is calculated as follows:

$$r_i = \frac{a_{f_i}}{a_{m_i}}$$  \hspace{1cm} (19)

where $a_{f_i}$ and $a_{m_i}$ are the predicted and measured $a_f$, respectively. The subscript $i$ denotes the data set number. For a perfect fit, $r_i$ is equal to 1.

The Mean Normalized Error (MNE)

The mean normalized error (MNE) is determined as follows:

$$MNE = \frac{100}{J} \sum_{i=1}^{J} \left|\frac{a_{m_i} - a_{f_i}}{a_{m_i}}\right|$$  \hspace{1cm} (20)

where $J$ is the total number of the data points and for a perfect fit, $MNE$ is equal to 0.

The Average Geometric Deviation (AGD)

The average geometric deviation (AGD) is defined as:

$$AGD = \left\{ \frac{1}{J} \prod_{i=1}^{J} \tilde{R}_i \right\}, \tilde{R}_i = \begin{cases} a_{f_i}/a_{m_i} & \text{for } a_{f_i} \geq a_{m_i} \\ a_{m_i}/a_{f_i} & \text{for } a_{f_i} < a_{m_i} \end{cases}$$  \hspace{1cm} (21)

For a perfect fit, $AGD$ is equal to 1.
Experimental Data Set

A collection of DSR tests, conducted in the LVE response region, of unaged and aged unmodified bitumens, polymer modified bitumens (PMBs) and bitumen-filler mastics has been used in this study to verify the validity of the shift factor equations. Details of the physical properties some of the tested binders and mastics can be found in the following references [15-17]. For example, the penetration and softening point values of unaged PMBs decrease and increase with increasing polymer content and subsequent polymer modification. The increase of binder hardness can be directly attributed to the stiffening effect caused by the addition of polymer (ethylene vinyl acetate (EVA) and styrene butadiene styrene (SBS) polymers). A similar observation was made for the aged PMBs samples.

All the binders and mastics were subjected to both amplitude (strain) as well as frequency sweeps to; firstly, establish the linearity region of response of the material and subsequently to determine their LVE rheological properties. The frequency sweep tests were performed under controlled strain loading conditions using frequencies between 0.1 to 10 Hz at 5°C temperature intervals between 5 and 75°C. Tests at lower temperatures (generally between 5 and 35°C) were undertaken with a 8 mm diameter and 2 mm gap testing geometry and at higher temperatures (generally between 25 to 75°C) with a 25 mm diameter and 1 mm gap testing geometry.

At low frequencies and/or high temperatures, the PMBs and bitumen-filler mastics show higher \( G' \) values compared to the unmodified bitumens. On the other hand, at high frequencies and/or low temperatures, the curves approach a limiting value, known as the glassy modulus \( G_g \), at \( 1 \times 10^9 \) Pa for unaged and aged unmodified bitumens and PMBs. However, for the unaged and aged bitumen-filler mastics, the \( G_g \) values vary, depending on the percentage and a type of mineral fillers used. This phenomenon occurs due to the existence of physical interaction in a mixture. In addition, the presence of polymer and mastics increase the elastic response (reduced \( \delta \) as temperature increased. Details of the LVE rheological properties of the binders and mastics can be found in the following references [15-17].

Results and Discussion

Graphical Comparison

Comparisons between predicted (shift factor functions) and measured (numerical) shift factors are shown graphically in Figs. 6-7. These plots are intended to visually and qualitatively show the agreement between measured and predicted values and to display any errors associated with the predictive equations and/or material combinations [31]. The predictive equations consist of the WLF, Modified Kaelble, Arrhenius, Log-Linear, VTS and LCPC functions/procedures. The measured shift factor data consists of the numerical, non-functional form shift approach. This approach used non-linear least squares fitting with the aid of the Microsoft Excel spreadsheet Solver function to simultaneously determine the coefficients associated with Eq. (15). As discussed by Pellinen et al. [9], the numerical shift approach produces the best results in terms of data shifting flexibility due to the fact that this method has the highest degree of freedom. This method, however, has no physical meaning (or functional form) and has simply been used as a comparison to the other shift factor methods.

The numerical shifts are always plotted on the \( x \)-axis (Figs. 6-7). A combination of comparisons between measured and predicted \( a_T \) for the unaged unmodified bitumens, PMBs and bitumen-filler mastics are shown in Fig. 6. Fig. 7 represents the combinations of these samples that have undergone various ageing processes. From Fig. 6, it can be seen that the WLF and Arrhenius equations show the best results with the predicted \( a_T \) values being close to the equality line. Reasonably good correlations can also be seen for the VTS and Modified Kaelble methods. The WLF equation, originally derived from the empirical Doolittle equation relating fractional free volume theory to temperature, is clearly applicable for all bituminous materials. As mentioned by Dealy and Larson [32], the WLF equation generally provides a better fit of the data at temperatures closer to the glass transition temperature, \( T_g \). However, the results in Figs. 6-7 also show that the WLF equation is also applicable at higher temperatures for bituminous binders.

Rowe et al. [3] found that the Modified Kaelble method was applicable for asphalt mixtures particularly at low temperatures. This method shows a sigmoidal type behaviour which does not result in excessively high values of \( a_T \) as the temperature reduces. However, it is observed that the Modified Kaelble method underestimates the unmodified bitumens at low temperatures compared to the original WLF equation. The Log-Linear equation, as expected, showed the lowest correlation between measured and predicted \( a_T \). This expected result could relate to the fact that the equation is only suitable for the construction of asphalt mixture master curves. The VTS equation appears to be unsuitable for predicting \( a_T \) of the unmodified bitumens at low frequencies and/or high temperatures. Finally, the LCPC method showed a dispersion of predicted \( a_T \) data particularly for the unaged bitumen-filler mastics. Similar findings were observed by Chailleux et al. [21] for asphalt mixture where anomalies had been seen particularly at high frequencies. From their study, they found that the dispersion of \( a_T \) data normally occurs at the transition between the highest tested frequency for a particular temperature and the lowest tested frequency for the next (higher) temperature. The low data quality can be partly attributed to compliance errors associated with the measurement of \( \delta \) and the subsequent use of the \( \delta \) function [1, 33].

Fig. 7 shows a comparison between measured and predicted \( a_T \) of the different \( a_T \) equations for the aged unmodified bitumens, PMBs and bitumen-filler mastic samples. The results show a larger discrepancy between the predicted and measured \( a_T \) values, particularly at both lower and higher temperatures. The WLF, Modified Kaelble, VTS, LCPC and Arrhenius methods produced almost identical results when comparing the measured and predicted \( a_T \). The Log-Linear equation slightly overestimates the measured \( a_T \). In general, it is observed that all the models suffer from a similar drawback where they are unable to accurately predict the \( a_T \) data. This lack of agreement between measured and predicted \( a_T \) particularly for the aged mastics and PMBs can be attributed to the increased complexity of the rheological response of the materials.
The following oxidation and increased structuring. Moreover, most of the $a_T$ equations (functions) are empirical and are therefore unable to account for changes in the physicochemical properties of the materials after ageing.
Fig. 7. Comparison between $a_T$ (Numerical Shift) and $a_T$ (Predicted) of Different Shift Factor Equations for the Aged Samples ($T_{ref} = 25^\circ C$).

Goodness-of-Fit Statistics

Tables 2-3 show the $S/S_y$ and $R^2$ goodness-of-fit statistics associated with the different $a_T$ equations on unaged and aged samples. The unaged PMBs and bitumen-filler mastics shifted by means of the LCPC method show good correlations between measured and
Table 2. Summary of the $S_i/S_y$, $R^2$, $r_i$, MNE and AGD goodness-of-fit for Unaged Samples.

<table>
<thead>
<tr>
<th>Method</th>
<th>Binders</th>
<th>$n$</th>
<th>$S_i/S_y$</th>
<th>$R^2$</th>
<th>Data in Range of Discrepancy Ratio, $r_i$ (%)</th>
<th>MNE</th>
<th>AGD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.98 - 1.02</td>
<td>0.96 - 1.04</td>
<td>0.94 - 1.06</td>
</tr>
<tr>
<td>WLF</td>
<td></td>
<td>71</td>
<td>0.086</td>
<td>0.993</td>
<td>42.25 60.56</td>
<td>73.24 81.69</td>
<td>87.32</td>
</tr>
<tr>
<td>Modified Kaelble</td>
<td>Unmodified</td>
<td>71</td>
<td>0.177</td>
<td>0.969</td>
<td>12.86 20.00</td>
<td>30.00 35.71</td>
<td>47.14</td>
</tr>
<tr>
<td>Arrhenius</td>
<td>Unmodified Bitumens</td>
<td>71</td>
<td>0.110</td>
<td>0.988</td>
<td>23.94 38.03</td>
<td>50.70 60.56</td>
<td>69.01</td>
</tr>
<tr>
<td>Log-Linear</td>
<td></td>
<td>71</td>
<td>0.307</td>
<td>0.906</td>
<td>7.04 15.49</td>
<td>26.76 38.03</td>
<td>46.48</td>
</tr>
<tr>
<td>VTS</td>
<td></td>
<td>71</td>
<td>0.147</td>
<td>0.979</td>
<td>32.39 52.11</td>
<td>56.34 60.56</td>
<td>64.79</td>
</tr>
<tr>
<td>LCPC</td>
<td></td>
<td>71</td>
<td>0.057</td>
<td>0.997</td>
<td>40.85 54.93</td>
<td>78.87 84.51</td>
<td>90.14</td>
</tr>
<tr>
<td>WLF</td>
<td></td>
<td>106</td>
<td>0.050</td>
<td>0.998</td>
<td>40.95 71.43</td>
<td>82.86 90.48</td>
<td>93.33</td>
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<td>Modified Kaelble</td>
<td>PMBs</td>
<td>106</td>
<td>0.085</td>
<td>0.993</td>
<td>5.71 32.38</td>
<td>50.48 58.10</td>
<td>68.57</td>
</tr>
<tr>
<td>Arrhenius</td>
<td>Bitumen-filler Mastics</td>
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<td>0.106</td>
<td>0.989</td>
<td>11.43 28.57</td>
<td>38.10 52.38</td>
<td>58.10</td>
</tr>
<tr>
<td>Log-Linear</td>
<td></td>
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<td>0.173</td>
<td>0.970</td>
<td>3.81 11.43</td>
<td>14.29 16.19</td>
<td>21.91</td>
</tr>
<tr>
<td>VTS</td>
<td></td>
<td>106</td>
<td>0.050</td>
<td>0.998</td>
<td>45.71 69.52</td>
<td>84.76 92.38</td>
<td>93.33</td>
</tr>
<tr>
<td>LCPC</td>
<td></td>
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<td>0.410</td>
<td>0.831</td>
<td>13.33 31.43</td>
<td>49.52 58.10</td>
<td>69.52</td>
</tr>
</tbody>
</table>

predicted $a_T$ data. The $S_i/S_y$ and $R^2$ goodness-of-fit parameters tend to indicate excellent correlation between measured and predicted $a_T$ data for most all tested material and shift factor combinations. However, according to Tran and Hall [28], the correlation coefficient, $R^2$, is not always a reliable coefficient to measure the goodness-of-fit for non-linear regression analysis. In addition, it is questionable whether $S_i/S_y$ is a good tool to perform a comparison between measured and predicted data. For example, in the case of $|G^*|$ measurements over a large temperature range, the standard deviation ($S_i$) has no meaning. $S_i$ is only scientifically founded for multiple $|G^*|$ measurements under the same experimental condition (i.e. at one temperature and one frequency). In this case, $S_i$ corresponds to the average distance between the mean value and all the experimental data.

Therefore, a more microscopic statistical analysis is needed and subsequently the discrepancy ratio ($r_i$), mean normalized error (MNE) and average geometric deviation (AGD) goodness-of-fit statistics are introduced, as shown in Tables 2-3. As discussed by Wu et al. [30], it is not straightforward to determine which method performs best since different statistical methods lead to different
rankings. The discrepancy ratio, \( r_i \) (%) is used to observe the predicted data’s tabulation from the equality line with a perfect value being one. When the \( r_i \) is larger or smaller than one, it measures how much wider the prediction interval has to be to cover the observed number of cases [30]. A smaller range means a closer range to the perfect agreement. In this study, an interval of \( 1 \pm (0.02, 0.04, 0.06, 0.08 \text{and } 0.10) \) is used. An example is shown for the WLF equation of unmodified bitumens, with an understanding that the commentary applies to other equations, both on the unaged and aged samples. A value of 0.98–1.02 represents an area where the ratio between predicted and measured \( a_T \) data is taken 0.02 to each left and right side from the equality line. It is observed that the \( r_i \) is equal to 42.25%. When the region widens with 0.02 more on each right and left sides (now the range between 0.96–1.04), another 18.31% of data is included. In this range, the data’s tabulation increased up to 60.56%. A similar process is repeated for the \( r_i \) in the range of 0.94–1.06, 0.92–1.08 and 0.90–1.10, resulting the data’s tabulation up to 73.24, 81.69 and 87.32%. In general, the improvement in \( r_i \) happens in all ranges. The \( MNE \) is related to the overall discrepancy between measured and predicted data. Meanwhile, the \( AGD \) is a measure of the average ratio between measured and predicted \( a_T \) data. As defined in the equation, the \( \delta \) values are always greater or equal to unity. \( \delta \) is equal to one when the predicted and measured \( a_T \) are identical. Thus the lowest possible value for \( AGD \) is 1. For instance, if it is 2, it means that the predicted \( a_T \) will be 2 (or 0.5) times the measured \( a_T \).

Taking the range of \( r_i \) of 0.90–1.10 as an example, the LCPC method shows the best result, followed by the WLF, VTS, Arrhenius, Modified Kaelble and Log-Linear equations. A similar finding is observed with the use of \( MNE \) goodness-of-fit parameters. However, no different in terms of the \( AGD \) values could be observed on the unaged samples. The Arrhenius equation might produce a better fit of [\( G^* \)] (or \( \delta \)) master curves for low temperatures [34]. With one coefficient needing to be determined, the Arrhenius equation shows a low degree of freedom in its equation. At low temperatures, the activation energy, \( E_{aT} \), that associated with the Arrhenius equation varies. However, the \( E_{aT} \) values become more constant as the temperatures increased [9]. Since the \( E_{aT} \) values are relatively consistent, the Arrhenius equation is only reliant on temperature which explains why this equation becomes invalid at high temperatures. The use of one \( E_{aT} \) value is obviously unable to yield a complete behaviour of [\( G^* \)] and \( \delta \) master curves of bituminous binders.

In general, the LCPC equation shows the best correlation between measured and predicted \( a_T \) of the \( r_i \) distribution in the range of \( r_i \) of 0.90–1.10 for the aged unmodified bitumens, as shown in Table 3. It is followed by the Modified Kaelble, VTS, WLF, and Arrhenius equations. As expected, the Log-Linear shows the least correlation in term of goodness-of-fitting statistical analysis. The breakdown of the TTSP could be related to the ageing process that results in increased of asphaltenes content. It is also interesting to note that the Modified Kaelble and LCPC methods show comparable results in terms of \( r_i \) and \( MNE \), followed by the WLF, VTS, Arrhenius and Log-Linear equations for the aged PMBs. The WLF, Modified Kaelble, LCPC and VTS equations are well dispersed around the equality line for the aged bitumen-filler mastics. However, the

| Table 4. Shifting Coefficients of the WLF Equation. |
|-----------------|-----------------|-----------------|-----------------|
| Unaged          | Unmodified Bitumen | Bitumen-filler Mastics | PMBs |
|                 | C1  | C2  | C1  | C2  | C1  | C2  |
| Average         | 22  | 125 | 12  | 107 | 17  | 158 |
| Minimum         | 12  | 109 | 11  | 93  | 11  | 104 |
| Maximum         | 46  | 162 | 14  | 130 | 51  | 479 |

<table>
<thead>
<tr>
<th>Aged</th>
<th>Unmodified Bitumen</th>
<th>Bitumen-filler Mastics</th>
<th>PMBs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
<td>C1</td>
</tr>
<tr>
<td>Average</td>
<td>15</td>
<td>131</td>
<td>13</td>
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<tr>
<td>Minimum</td>
<td>14</td>
<td>124</td>
<td>11</td>
</tr>
<tr>
<td>Maximum</td>
<td>16</td>
<td>150</td>
<td>21</td>
</tr>
</tbody>
</table>

Arrhenius and Log-Linear equations show the largest scatter in predicted results with small \( a_T \) values of \( r_i \) in the range of 0.90–1.10 for aged PMBs and bitumen-filler mastics. There is a dispersion of [\( G^* \)] data for the aged bitumen-filler mastics at high temperatures, due to the influence of the material’s granular skeleton, which renders the TTSP invalid for these materials [35].

It can also be inferred that as all the above methods are essentially empirical, they cannot be expected to be valid for all the material combinations [32]. The \( AGD \) parameter shows comparable results for most of the aged samples. This finding concludes that method is not always reliable at detecting the goodness-of-fit between measured and predicted \( a_T \) of bituminous binders, unaged and aged samples. Finally, it is worth mentioning that there are many possible solutions that have been used to find the goodness-of-fit statistical parameters. The methods explained in this section are limited and only include those that have been used by researchers and practitioners in the bitumen industry.

### WLF Coefficients

Previous researchers have shown that the \( C_1 \) and \( C_2 \) values for unaged and aged bitumens can be taken as 19 and 92, respectively [24, 36-37]. However, the results from this study, as shown in Table 4, found that the values of \( C_1 \) and \( C_2 \) are inconsistent from one sample to the next. Similar results were obtained by Di Benedetto and co-workers [10-11] with the values for \( C_1 \) being more consistent than those for \( C_2 \). Generally, the WLF equation does not fit bitumen rheological data with the same set of constants above and below the softening point, where [\( G^* \)] is increasingly dominated by viscous and elastic effects, respectively. Based on this argument, one might expect poor superposition of [\( G^* \)] data taken over a frequency interval sufficiently wide as to include both glassy and viscous asymptotes for the master curve [20, 38]. Recently, Chailleux et al. [21] attempted to link the WLF coefficients to the physico-chemical state of bituminous binders. They found the \( C_2 \) parameter was linked to the thermal dependency of the materials with an increase in \( C_2 \) value with ageing.

### Activation Energy

The activation energy, \( E_{aT} \), can be defined as a minimum energy required before any particular molecular movement can occur. Table 5 shows the \( E_{aT} \) values calculated using Eq. (4) for a set of...
particularly of materials like bitumens. It can be inferred that the coefficients. Not all chemical reactions will give a straight line. A general form of equation that relates \( \ln \eta \) to \( T \) can be written as follows:

\[
\ln \eta = a \times \left( \frac{1}{T} \right)^2 - b \times \left( \frac{1}{T} \right) + c
\]

where \( \eta \) is viscosity, \( T \) is temperature (K), \( a \), \( b \), and \( c \) are unknown coefficients. Not all chemical reactions will give a straight line particularly of materials like bitumens. It can be inferred that the \( E_a \) values of unmodified bitumens are varies and dependent on temperatures. Similar observations might be expected from PMBs and bitumen-filler mastics as their behaviour are much more complicated compared to the unmodified bitumens.

### Table 5. The \( E_a \) Values of Different Polymer Contents.

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Unmodified</th>
<th>EVA</th>
<th>PMBs</th>
<th>SBS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>5%</td>
<td>7%</td>
</tr>
<tr>
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<td>199</td>
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<tr>
<td>348.15</td>
<td>141</td>
<td>150</td>
<td>166</td>
<td>185</td>
</tr>
</tbody>
</table>

**Fig. 8.** The \( \ln \eta \) Versus Temperatures Curve of Unmodified Bitume.

unmodified bitumens and PMBs. In general, the \( E_a \) values are relatively constant for this set of binders with only a slight decrease with increasing temperature. However, results may actually reflect some testing (compliance) errors and also the occurrence of plastic deformation during testing at high temperatures. In addition, it is important to note that changing certain parameters, such as the type of bitumen, type and content of polymer modification, bitumen-filler mastics and ageing process will potentially alter the \( E_a \) values [39, 40].

In theory, a curve between a natural logarithmic of viscosity (\( \ln \eta \)) and temperatures should produce a straight line to show that the \( E_a \) values are independent on temperatures. In this investigation, the viscosity of a 40/60 penetration grade bitumen was tested at a range of temperatures from 80 to 180°C. It is observed in Fig. 8 that the \( \ln \eta \) versus temperatures is more conveniently presented in the form of a second order of polynomial fitting function rather than a straight line. A general form of equation that relates \( \ln \eta \) and temperatures can be written as follows:

\[
\ln \eta = a \times \left( \frac{1}{T} \right)^2 - b \times \left( \frac{1}{T} \right) + c
\]

### Conclusions

For the range of temperatures from 5 to 75°C, the Generalised Logistic Sigmoidal Model is able to accurately model the complex modulus data obtained from LVE frequency sweep tests undertaken with a DSR. With the exception of the LCPC method, all the shift factor equations can be used together with this model to construct complex modulus master curves at an arbitrary selected reference temperature. The numerical, non-functional form shift function produces the most consistent set of results due to high degree of freedom and overall flexibility of this non-linear least squares fitting approach. The LCPC and WLF equations generally produced the best results compared to the numerical shift approach (predicted versus measured shift factor data) for all the material combinations studied, followed by the Modified Kaelble, VTS, Arrhenius and Log-Linear methods. As expected, the Log-Linear equation showed the lowest correlation with measured shift factor data, with this equation being more suitable for the construction of asphalt mixture complex modulus master curves.

The LCPC method, which is based on the Kramers-Kronig relationship, produced comparable \( a_T \) data with the numerical shift approach. However, this approach is highly reliant on consistent phase angle data and very sensitive to any compliance errors associated with the particular DSR machine. Results from this study showed that dispersions of \( a_T \) data normally occurred at changes between frequency sweep temperatures, particular the highest tested frequency at a particular temperature and the beginning of the frequency sweep (lowest tested frequency) at the next highest temperature. However, most of the equations are basically empirical and it is not expected that they will strictly obeyed by any materials.

In terms of comparing the different shift factor functions and methods, both a graphical and a number of goodness-of-fit statistics were used. The graphical plots were able to visually observe the agreement between predicted and measured data, although this method is unreliable at detecting small changes in data. It is found that the \( R^2 \) and \( S_y/S_x \) parameters are not suitable for describing mismatches between predicted and experimental data. The \( R^2 \) is more applicable for linear models, while \( R^2 \) and \( S_y/S_x \) cannot be considered to be independent of each other. The average geometric...
deviation (AGD) is also not always reliable at detecting goodness-of-fit even though it is meant for data scattered over a logarithmic scale. It is therefore recommended that the discrepancy ratio ($r$) and the mean normalized error (MNE) provide the best means of identifying the goodness-of-fit statistics for measured and predicted data associated with the shifting of rheological data over wide temperature and frequency domains.

References


