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Effective Modulus Variation and Field Distributions with Depth --Using a Multilayered Pavement Program

Wael Alkasawneh[1], Ernie Pan[2+], Feng Han[3], Ronghua Zhu[4], and Roger Green[5]

Abstract: Elasticity solutions have been used for many years in design and analysis of flexible pavements. Current available techniques do not consider the temperature induced modulus variation with depth. This variation can be performed by subdividing the elastic layer into several sublayers of known thicknesses and elasticity parameters that vary with depth. However, most available programs can handle only a total number of 20 layers/sublayers which have been shown to be insufficient for modulus variation with depth due to temperature variation. An innovative multilayered elastic solution has been developed and implemented into a new software product ***(MultiSmart3D)*** to model any number of sublayers/layers with any thickness. Results show that this new method and program can be used successfully to model the quadratic modulus variation with depth using 125 constant modulus sublayers. Furthermore, the strain/stress discontinuity issue in layered flexible pavement is analyzed by introducing a new parameter called the "efficiency factor".

Key words*: Efficiency factor; Flexible pavement; Modulus variation; Multilayered elastic solution; Strain discontinuity; Stress discontinuity; Temperature.*

Introduction

Flexible pavement in highway engineering and construction is one of the most widely accepted practices in the world. For example, about 93% of the paved roads in the US comprised of flexible pavement (Federal Highway Administration [1]). This high percent of usage justifies the importance and need for more research and better understanding of the flexible pavement behavior. However, many parameters and boundary conditions affect the flexible pavement response, and a full understanding of the mechanical behavior of flexible pavement is not yet complete.

Modeling the flexible pavement system can be achieved using the finite difference method, finite element method, and multilayered elastic solution. The first two methods could be time consuming and could be difficult to model the high stress zone within the pavement and along the tire/pavement interface. The classical elasticity pavement programs, while being convenient for simple pavement design, cannot model the irregular pressure along the surface and the heat transfer within the pavement, and are further limited to 20 sublayers (National Research Council [2]). However, a recently developed program called ***MultiSmart3D*** (Computer Modeling and Simulation Group [3]) can be applied for multilayered elastic pavements with unlimited number of layers/sublayers and also with the capability of modeling irregular pressure distributions along the tire/pavement interface. Therefore, the ***MultiSmart3D*** program can be utilized easily to deal with material inhomogeneity in the pavement by varying the modulus with depth using thin elastic sublayers.

[1] Senior Geotechnical Engineer, Camp Dresser & McKee, Inc., 125 South Wacker Drive, Suite 600, Chicago, IL 60606, USA.
[2] Professor, Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, USA, Tel: 330-972-6739 (O), 330-972 -6020 (Fax).
[3] Structural Engineer II, PB, 999 Third Avenue, Suite 2200, Seattle, WA 98104, USA.
[4] Structural Engineer, American Bureau of Shipping, ABS Plaza, 16855 Northchase Dr., Houston, TX 77060, USA.
[5] Pavement Research Engineer, Ohio Department of Transportation, 1980 West Broad Street, Columbus, Ohio 43223, USA.
[+] Corresponding Author: E-mail pan2@uakron.edu
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Flexible Pavement

A pavement can be modeled as a multilayered elastic system; each of these layers can have its own elasticity parameters such as the modulus of elasticity and Poisson's ratio. The first elastic solution for layered pavements using the elasticity theory was by Burmister [4, 5]. His pioneer work was focused more on modeling flexible pavements assuming a fixed number of elastic layers. In the last half century, the elasticity theory has been used extensively due to its simplicity with limited number of input parameters (e.g., Huang [6]).

However, increased understanding of the pavement behavior poses an urgent need for more rigorous techniques that can model the change in modulus of elasticity either between the layers or within the same layer. The use of the average modulus of elasticity of the layer is a common accepted practice among researchers and practitioners due to the difficulty of modeling the detailed variation using conventional elasticity models. Modeling the modulus variation with depth requires a large number of elastic layers where the modulus of elasticity varies due to different factors such as temperature, moisture, etc. This limitation, so far, made the finite element method more appealing. However, the complexity of applying the finite element to pavement engineering made the modeling only a research method rather than a practical tool (Wang [7]).

In this paper, we introduce an innovative method for pavement analysis where the elastic layer can be subdivided further into many horizontal sublayers. The property and thickness of each sublayer can vary arbitrarily with depth. The multilayered elastic system can have any number of layers/sublayers as compared to the current methods used in most flexible pavement programs where the layered system is limited to a maximum number of 20 layers/sublayers [2].

Table 1. Parameters of a Typical Flexible Pavement Example.

Layer	Thickness (cm)	Resilient Modulus (MPa)	Poisson's Ratio
AC Layer	15	3500	0.3
Base Layer	25	700	0.3
Subbase Layer	25	300	0.3
Subgrade Layer	Infinite Half-Space	100	0.3

Modulus Variation with Depth

Material inhomogeneity within pavement layer/sublayer can be caused by the dependency of modulus of elasticity on temperature, moisture, and/or other environmental factors. This fact imposes limitations on the current analytical elastic solution which does not take into consideration the variation of modulus of elasticity with depth within the same layer. However, the newly developed multilayered program (***MultiSmart3D***) can be used to model the variation of modulus of elasticity with depth by assuming a mathematical relation between the modulus and depth, such as a quadratic relation. The modulus variation within the same layer can be modeled using several sublayers where the sublayer thickness and modulus are varied for different sublayers.

Strain/Stress Discontinuity

Discontinuity in stresses and strains at the interface between dissimilar elastic layers has been recognized by many researchers in different engineering fields [8-11]. However, when dealing with functionally graded materials such as the asphalt concrete (AC), discontinuity in stresses and strains across the "artificial" interface within the AC layer (inter-layer interface) is due to the inherited limitation of the existing analytical methods. Therefore, the stresses and strains within the functionally graded AC layer cannot be captured accurately without complex mathematical formulation. For example, in the conventional multilayer elastic analysis the vertical strains and horizontal stresses immediately above and below the interface of two artificial dissimilar elastic sublayers of constant moduli are not compatible with each other (e.g., there is a strain and stress jump across the artificial interface). In pavement engineering this problem has not attracted the attention of engineers even though rutting and fatigue predictions in flexible pavement are mostly performed using strains and stresses above and below the interface of sublayers with dissimilar elastic parameters.

Strain/stress jumps crossing interfaces in a typical flexible pavement system (see also Table 1) using conventional layered elastic solution are shown in Figs. 1 and 2 using our ***MultiSmart3D***. The contact pressure at the surface of the pavement is assumed to be 69kPa acting on a circle with a diameter of 220.3mm. It can be seen clearly that strain (ε_{zz} in Fig. 1) and stress (σ_{xx} in Fig. 2) discontinuity exists at the interfaces whilst the stress/strain variation within the same layer is smooth.

The strain/stress discontinuity at any possible artificial interface in flexible pavement can be reduced by decreasing the discontinuity in modulus of elasticity between the two layers of the interface. In a discrete layered pavement model, the modulus of elasticity in one layer can be orders higher than that in the subsequent layer. In order

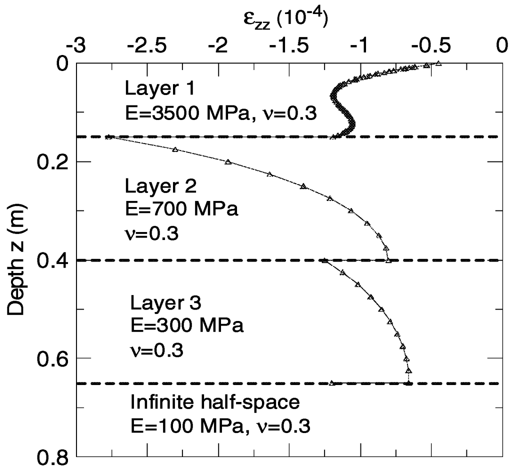

Fig. 1. Vertical Strain (ε_{zz}) Jumps at Interfaces in a Flexible Pavement System.

to ensure the continuity of strains/stresses, variation of modulus of elasticity with depth should be continuous within the layer and between both sides of the interface. Modulus of elasticity variation with depth can be achieved by subdividing each layer into a number of sublayers each with a constant modulus of elasticity. The average modulus of elasticity of all sublayers should be equal to the average modulus of elasticity of the layer.

Field measurements indicate that the temperature in pavement varies with depth during the day and during the year, as shown in Fig. 3 (Ongel and Harvey [12]). Therefore, the modulus of elasticity of the layer in a flexible pavement varies with depth due to the dependency of the modulus on temperature. Pan et *al.* [13] showed that the variation of modulus of elasticity with depth due to temperature variation is complicated, which certainly cannot be described by a constant value or a simple linear function. Other conditions that should be satisfied by numerical variation of the modulus with depth include the continuity of modulus of elasticity at the interface between the subdivided layers and the equality of the average modulus in the same layer. Therefore, a nonlinear variation, instead of a linear variation, of the layer modulus with depth should

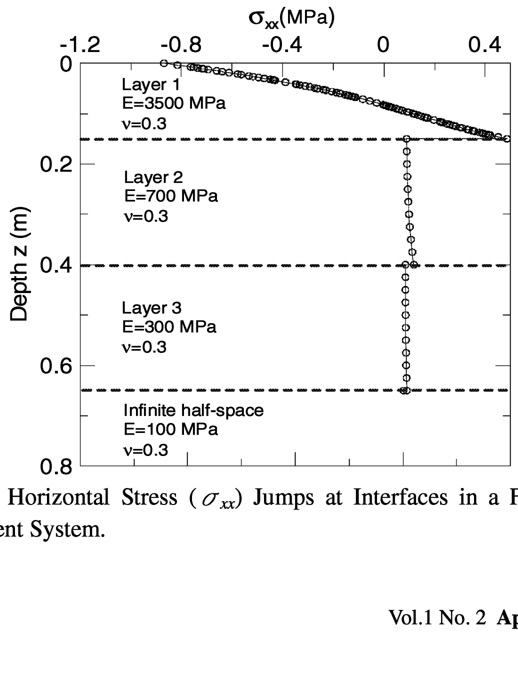

Fig. 2. Horizontal Stress (σ_{xx}) Jumps at Interfaces in a Flexible Pavement System.

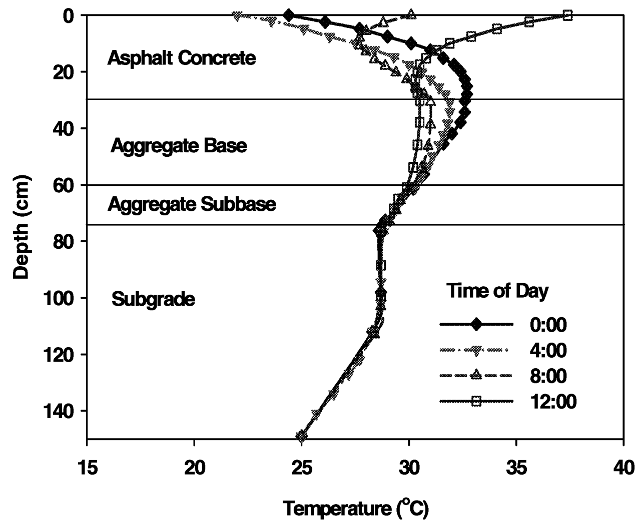

Fig. 3. An Example of Daily Temperature Variation (Modified from Ongel and Harvey [12]).

be used to satisfy the requirements. Actually, a quadratic equation can be used to describe the variation of modulus with depth, which will satisfy the three conditions described above.

A simple way to handle the strain/stress "jump" between the two sides of an (artificial) interface in a subdivided multilayered elastic material is proposed. The method is based on a "controlled" variation of modulus of elasticity within the elastic layer to ensure the lowest strain/stress "jump" across the interface of the sublayers. The proposed approach is intended to study the effect of different and approximate modulus variations within the layer on the pavement response and should not be considered as a new approach to model the change of moduli with depth. If moduli variation due to temperature variation within the elastic layer can be described accurately, the actual data can then be incorporated in the ***MultiSmart3D*** program to study the actual pavement response.

The modulus variation with depth within the same layer is described below (Note that steps (1) through (4) can be equally applied to other layers).

1. Select thickness and the corresponding elasticity parameters (modulus of elasticity and Poisson's ratio) for each main layer in the multilayered elastic system, for example, layer I in Fig. 4.
2. Use the modulus of elasticity in the upper layer (E_{I-1}) and that in the subsequent layer (E_{I+1}) as boundary conditions to control the moduli variation with depth within layer I, see Fig. 5.
3. Use the quadratic equation to describe the variation of the modulus of elasticity with depth in the layer. The resulting system of equations contains two equations and three unknowns (the three unknown constants in the quadratic equation). The third equation needed to solve the three unknowns can be obtained using:

$$\int_0^{h_I} E(z)dz = E_I h_I \tag{1}$$

where $E(z)$ is the quadratic modulus-of-elasticity function in layer I as a function of depth z and it is equal to $(a+bz+bz^2)$ with a, b, and c being the three unknowns. Eq. (1) ensures that the resulting modulus variation within layer I will always result in the same average modulus. Therefore, the three simultaneous equations can be solved to find the three unknowns.

4. Subdivide layer I into a number of sublayers with the modulus of each sublayer being determined using the quadratic equation at the given depth. The constant modulus of elasticity within each sublayer should ensure a smooth transition of stresses/strains between two subsequent sublayers ((*j*-1) and (*j*) sublayers). This condition can be satisfied using:

$$E_{j-1}/E_j \approx 0.90-1.00 \tag{2}$$

where E_{j-1} and E_j are the moduli of elasticity in sublayer ($j-1$) and (j), respectively. This condition is very important since it will reduce the strain/stress discontinuity in the artificial sublayers in the layered pavement.

5. Calculate the response of the multilayered elastic system using the ***MultiSmart3D*** program.

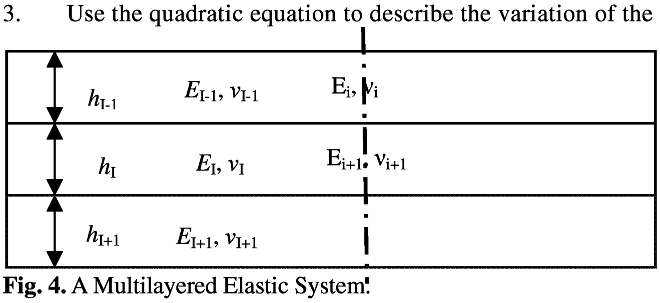

Fig. 4. A Multilayered Elastic System.

Flexible Pavement Application

Interfacial strain/stress jump in pavements is very common in discrete multilayered elastic analysis for any flexible pavement system. The present approach will be applied to a flexible pavement system to demonstrate the applicability of the new method and to establish some guidelines regarding the use of the new method.

The typical flexible pavement section is summarized in Table 1. The contact pressure at the surface of the pavement is assumed to be 690kPa acting on a circle with a diameter of 220.3mm. Pavement responses below the center of the contact pressure are calculated

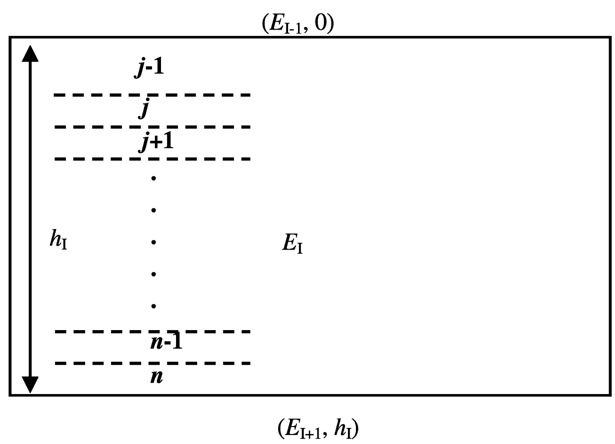

Fig. 5. Assumed Modulus Boundary Conditions for Layer I.

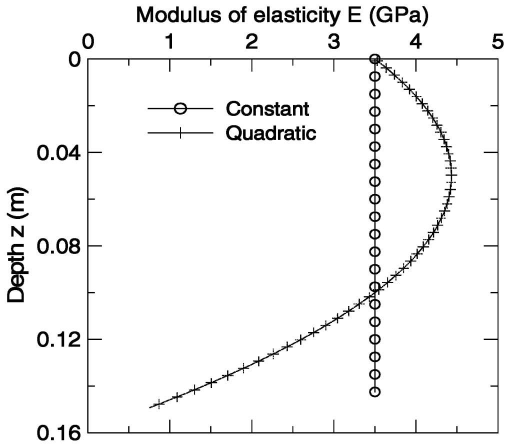

Fig. 6. Modulus of Elasticity Variation with Depth in AC Layer.

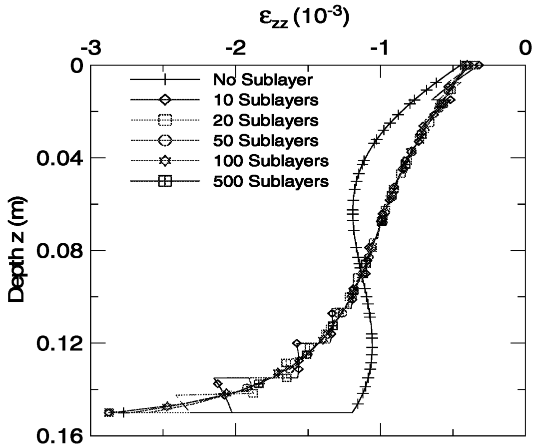

Fig. 8. Vertical Strain (ε_{zz}) Variation with Depth in AC Layer.

using the ***MultiSmart3D*** program. The coordinate system is chosen such that the *x*- and *y*-axes are on the surface of the pavement (*z=0*) whilst the *z*-axis is vertical and extends along the depth direction.

The interfacial strain/stress jump within the AC layer is studied by the proposed method. The AC layer is subdivided into 10, 20, 50, 100, and 500 sublayers. The modulus of elasticity within each sublayer is constant and equal to the average modulus of elasticity of the sublayer. For each sublayering model, the pavement response within the AC layer is calculated with 120 points along the *z*-axis using both the constant (average) and the quadratic modulus variation methods. Field response is also calculated with an additional point (a total of 121 points) which is located immediately below the interface between the AC layer and base layer in order to study the strain/stress jump between the two materials (layers). The modulus variation with depth using constant and quadratic modulus variation functions is shown in Fig. 6. The pavement responses using different models are shown in Figs. 7 through 11.

Fig. 7 shows that the use of an average constant modulus (no sublayers) can underestimate the displacement at any point within the AC layer except at the AC/base interface, as compared to the quadratic model with different sublayers. In addition, it can be seen that the number of sublayers has negligible effect on the calculated displacement in the AC layer. The maximum difference of the displacements using the quadratic distribution compared to those using the average modulus is approximately 1.5%.

Fig. 8 shows the variation of the vertical component of strain (ε_{zz}) with depth. It is clear that the average modulus of elasticity can overestimate the strain approximately within the upper 60% of the AC layer and underestimate the strain in the lower 40% of the AC layer. In addition, the strain jump between the two sides of the AC/base interface can be up to 130% in the constant modulus case while it ranges between 42% and 1% in the quadratic variation using 10 and 500 sublayers, respectively. On the other hand, the maximum differences of strains in the upper 60% and lower 40% of the AC layer between the quadratic and constant models are approximately 40% and 58%, respectively. The influence of the number of sublayers is clearly seen in Fig. 8 especially near the interface.

Fig. 9 shows the variation of the horizontal component of strain (ε_{xx}) with depth. The strains in the upper 30% of the AC layer are almost identical regardless of the number of sublayers used within the layer. However, using the constant modulus can underestimate the strain within the lower 70% of the AC layer. The maximum difference of the strains in the lower 70% of the AC layer based on the quadratic and constant models is about 40%, whilst the number of sublayers has negligible effect on the strain variation with depth.

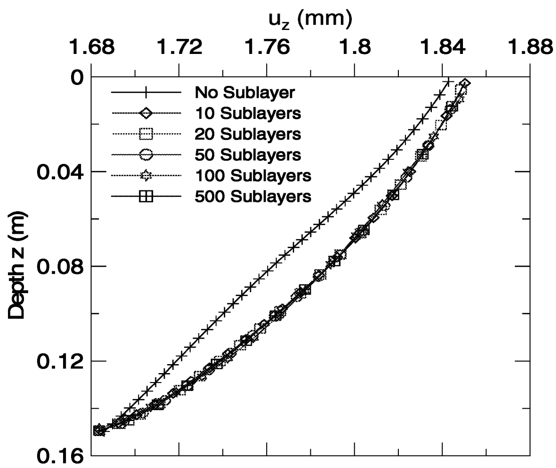

Fig. 7. Displacement (u_z) Variation with Depth in AC Layer.

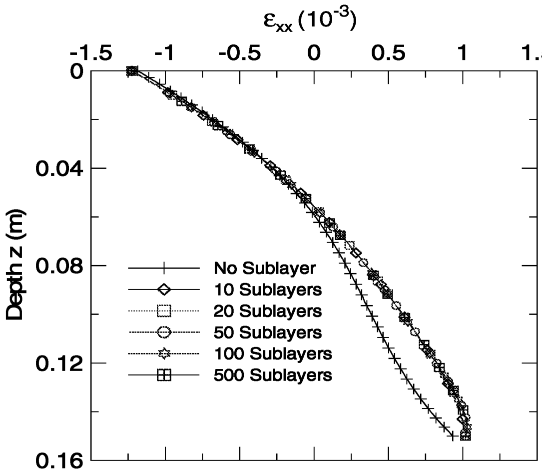

Fig. 9. Horizontal Strain (ε_{xx}) Variation with Depth in AC Layer.

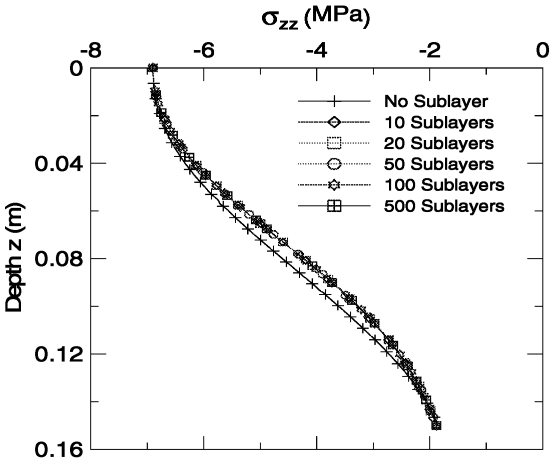

Fig. 10. Vertical Stress (σ_{zz}) Variation with Depth in AC Layer.

Similar behavior can be observed for the other horizontal strain (ε_{yy}) due to symmetry.

Fig. 10 shows the variation of the vertical component of stress (σ_{zz}) with depth. The difference between the stresses using the quadratic and constant models is less than 10% in the upper 20% and the lower 80% of the AC layer, whilst it is almost the same outside the 20% to 80% thickness range. The number of sublayers has negligible effect on the vertical stress variation with depth.

Fig. 11 shows the variation of the horizontal component of stress (σ_{xx}) with depth. The difference between stresses using the quadratic and constant models is between 3% and 480%, with the maximum difference being at the interface between the AC and base layer. It is further noticed that the number of sublayers has noticeable effect on the stress variation with depth.

Effect of Sublayer Number

The number of sublayers, when used to vary the modulus of elasticity within the elastic layer, has a considerable effect on the strain/stress jump between the two sides of the interface. In order to investigate the effect of modulus of elasticity on the response between the two sides of the interface, the changes in the strain (ε_{zz})

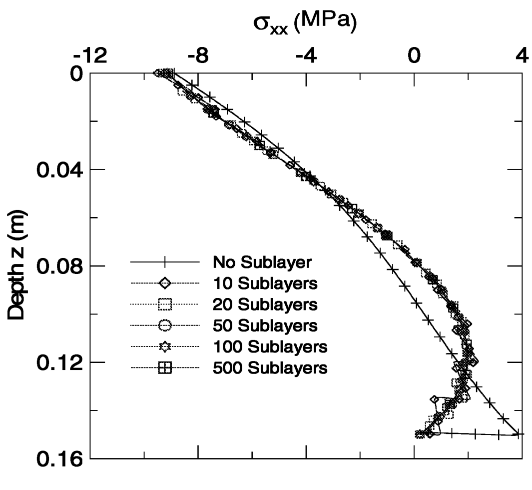

Fig. 11. Horizontal Stress (σ_{xx}) Variation with Depth in AC Layer.

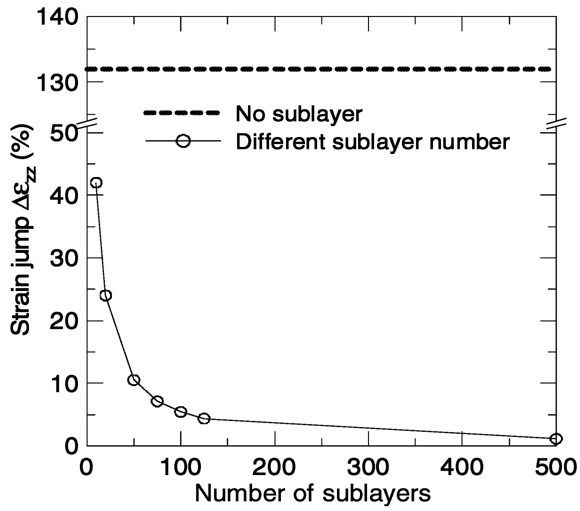

Fig. 12. Vertical Strain (ε_{zz}) Jumps Above and Below the Interface.

and stress (σ_{xx}) are plotted against the number of sublayers in Figs. 12 and 13, respectively.

It is observed that the quadratic modulus variation with depth can be used effectively to reduce the stress jump across the interface of the sublayers when the elastic layer is subdivided to capture the modulus variation with depth within the layer. This jump in pavement response between the sublayers within the AC layer can be reduced by increasing the number of sublayers (which is to reduce the ratio of the modulus of elasticity between two subsequent sublayers). Furthermore, from Figs. 12 and 13, it can be seen that using 125 sublayers can produce smooth pavement responses between the sublayers and can effectively reduce the jump in the strain (ε_{zz}) and stress (σ_{xx}) as compared to the pavement responses using the average elastic modulus. The effect of the moduli ratio between the two sides of the sublayer interface and the resulting vertical strains (ε_{zz}) and horizontal stresses (σ_{xx}) can be further understood by using the "***efficiency factor (EF)***" as suggested below:

$$EF_{Strain} = \left(\frac{Strain_{j+1}}{Strain_j}\right) \Big/ \left(\frac{E_j}{E_{j+1}}\right) \tag{3}$$

and

$$EF_{Stress} = \left(\frac{Stress_{j+1}}{Stress_j}\right) \Big/ \left(\frac{E_j}{E_{j+1}}\right) \tag{4}$$

The efficiency factor can be used to easily demonstrate the relation between the number of sublayers, the change in the pavement response across the interface, and the required ratio to achieve a smooth response through the sublayer interface instead of the "jump" in the response. It can be seen from Fig. 14 that as the number of sublayers increases, the efficiency factors for the strains and stresses approach unity, indicating a smooth transition in the pavement response between the two sides of the sublayer interface.

Fig. 14 further indicates that the horizontal stress σ_{xx} is more sensitive to the efficiency factor than the vertical strain ε_{zz}. In practice, the horizontal stress σ_{xx} plays a major role in pavement fatigue cracking whilst the vertical strain ε_{zz} contributes substantially to pavement rutting. From Fig. 14, we also observe

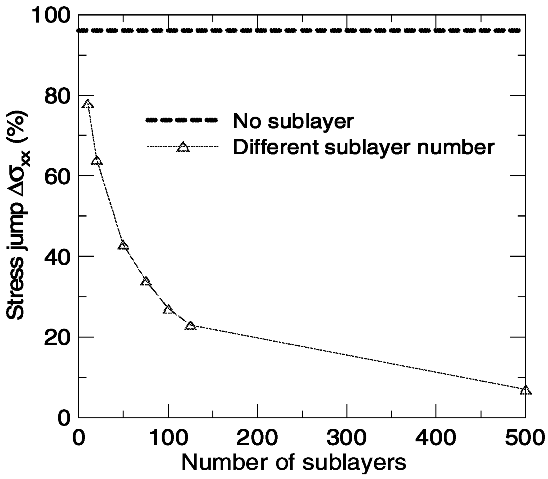

Fig. 13. Horizontal Stress (σ_{xx}) Jumps Above and Below the Interface.

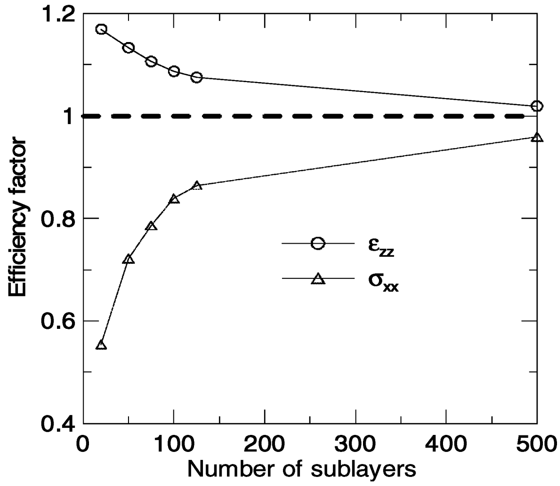

Fig. 14. Efficiency Factor (EF) vs. Number of Sublayers.

that substantial reduction in the strain/stress "jump" on the two sides of the artificial interface can be achieved using, say 125 sublayers, and that further reduction can be obtained by increasing the number of sublayers beyond the 125 sublayers threshold. It should be noted that the sublayer thickness in this example was constant for simplicity; however, variation of the sublayer thickness with depth is possible if needed.

Discussions and Conclusions

The average modulus of elasticity is not recommended for the analysis and design of complicated flexible pavements. Continuous variation of the modulus in the AC layer can reduce the "jump" in the stresses and strains on the interface of the sublayers when the layer is subdivided into many sublayers to capture the modulus variation with depth. Moduli variation within any layer in the pavement system can be performed using a quadratic relation in which the average elasticity moduli are the boundary conditions. The applicability of this method, when no actual moduli variation data are available, is demonstrated using the ***MultiSmart3D*** program, recently developed at University of Akron under the sponsorship of ODOT/FHWA. This new program is superior to any available multilayered flexible pavement program since unlimited number of layers can be used. The efficiency factor is introduced as a new way of measuring the needed number of sublayers and the effectiveness of the modulus variation with depth in order to reduce the jump in the vertical strain and horizontal stress between the sublayers.

The use of the quadratic modulus variation in the pavement also shows that the inhomogeneity of the AC layer can be modeled using the effective multilayered elastic approach. By increasing the number of sublayers, any realistic modulus variation in pavement (due to temperature, moisture, or other environmental factors) can be accurately simulated. However, modeling variation of the resilient modulus using sublayering can be difficult using most of the commercially available programs since most existing multilayered elastic programs limit the number of input layers and the thickness of each layer.

The stress/strain jumps exist between two distinct (or discrete) material layers. The intent of this paper is to show the significant influence of the moduli variation on stress/strain responses in pavement. In addition, approximation of modulus variation with depth by a few sublayers encountered difficulties since stress/strain jumps exist between the sublayers because of the moduli differences between the two adjacent sublayers. When the actual field moduli variation with depth, within the elastic pavement layer, is not available, selecting the moduli variation due to temperature variation can be challenging. By choosing the quadratic modulus variation, we demonstrate the difference in pavement responses (deformation, strain, and stress fields) when the elastic layer is subdivided into different sublayers to model the elastic moduli variation due to the temperature variation.

Acknowledgements

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Notation

The following symbols are used in this paper

a, b, c: Fitting constantsE_I: Modulus of elasticity of layer *I*
E_j: Modulus of elasticity of sublayer *j*
EF_{Strain}: Strain efficiency factor
EF_{Stress}: Stress efficiency factor
E(z): Modulus of elasticity at depth *z*
h: Depth
$Strain_j$: Average strain of sublayer *j*
$Stress_j$: Average stress of sublayer *j*
u_z: Displacement in vertical (z-) direction
x: Horizontal coordinate
y: Horizontal coordinate
z: Depth
ε_{xx}: Horizontal strain (*x*-direction)
ε_{yy}: Horizontal strain (*y*-direction)
ε_{zz}: Vertical strain (*z*-direction)
v_I: Poisson's ratio of layer *I*
v_j: Poisson's ratio of sublayer *j*
σ_{xx}: Horizontal stress (*x*-direction)
σ_{zz}: Vertical stress (*z*-direction)