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Determination of Allowable Differential Settlement between Bridge Abutment and Approach Embankment with Five-degree-of-freedom Vehicle Model

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Abstract: A bump develops when there is differential settlement between a bridge abutment and an approach embankment. The determination of the allowable differential settlement is important for the design and maintenance of the bridge approach, so it is studied theoretically in this paper. Because the longitudinal roughness in the bridge approach is much more serious than that in general highway sections, the pitch movement of vehicles cannot be ignored. One adds rider and seat to a half-truck model to build a five-degree-of-freedom vehicle model. The dynamic response analysis to Man-Vehicle-Road system passing over the bridge approaches with and without approach-relative slopes are carried out by means of the Laplace transform. Comparative calculations show that much difference will be produced in the maximum transient vibration value of the acceleration and the vibration frequency when different vehicle models are used. So it is more reasonable to use the five-degree-of-freedom vehicle model than to use the three-degree-of-freedom vehicle model for the determination of the allowable differential settlement. After all the required parameters are determined, the allowable differential settlement can be obtained by trial calculations.

Key words: *Bridges approach; Differential settlement; Maximum transient vibration value; Vehicles.*

Introduction

A bump develops when there is differential settlement between a bridge abutment and an approach embankment, which causes unsafe driving conditions, rider discomfort, poor public perception of the state infrastructure, structural failure of bridges, and long-term maintenance costs [1]. There are three major causes of bridge approach settlement, which are deformation of backfill, deformation of foundations soils, and poor drainage [1]. Mitigation techniques to reduce backfill deformation include more stringent backfill and compaction specifications [2], scheduling construction delays, geosynthetic reinforced earth [3-4], lightweight fills, controlled low strength materials [5-6], reinforced concrete approach slabs [7], and hydraulic fills. Techniques to improve foundation soils include, but are not limited to, removal and replacement of weak soils, ground improvement by mechanical or chemical means, surcharging with or without wick drains, and supporting the embankment on deep foundations [8-10]. But the differential settlement problems still exist. Cai et al. [11] provided a method for the structural design of the approach slab in which the effects of embankment settlements were considered.

More differential settlement between a bridge abutment and an approach embankment will produce more serious "bump at the end of the bridge" and the allowable maximum value of differential settlement is defined as the allowable differential settlement. The determination of the allowable differential settlement is important for the design and maintenance of the bridge approach [12]. When approach slabs are not used, many scholars [13-16] suggested the allowable differential settlements at the embankment-structure interface, which are between 12 and 75*mm*. When approach slabs are used, approach slabs will rotate around the abutment and a distinct approach-relative slope will obviously be produced. Even when approach slabs are not used, the approach-relative slope will be produced in the case that the post settlement of the approach embankment is not uniform. The approach-relative slope is defined as the differential settlement divided by the length over which the settlement occurs. From the standpoint of riding comfort and the safety of bridges, many scholars [14-18] suggested the allowable approach-relative slopes, which are between 1/250 and 1/50. Almost all of the allowable differential settlement criteria mentioned above are empirical and do not agree with each other very well. Lai [19] conducted a theoretical study on the allowable approach-relative slope. He simplified the vehicle and the bridge approach as a quarter-truck with two degrees of freedom and a sinusoid, respectively. The quarter-truck model is too simple and the sinusoid is far from the real shape of the bridge approach. In NCHRP 234, Briaud et al. [20] suggested the determination of the allowable differential settlement as one of the five main topics that need further research for the bump problem at the end of the bridge. For the reasons discussed above, Zhang and Hu [21] conducted a theoretical study on the allowable differential settlement in which vehicles were modeled as three-degree-of-freedom systems.

When a truck moves on the pavement, it develops several movement patterns such as bounce (up-and-down motion), pitch (out-of-phase fore-and-aft bounce), and roll (out-of-phase side-to-side bounce). Three kinds of vehicle models are commonly used in evaluation of the roughness of the pavement: (1) quarter-truck, (2) half-truck, or (3) full-truck. The quarter-truck model considers the bounce of wheels but ignores the pitch and roll movement. The next step up to improve the accuracy of the model is the half-truck model, which considers both bounce and pitch

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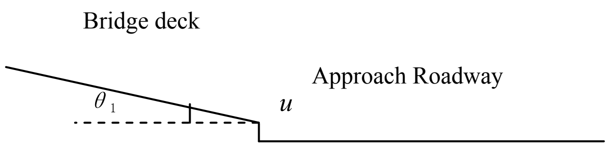

Fig. 1. Theoretical Model of Bridge Approach without Approach-relative Slopes.

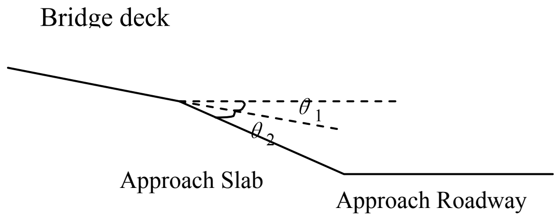

Fig. 2. Theoretical Model of Bridge Approach with Approach-relative Slopes.

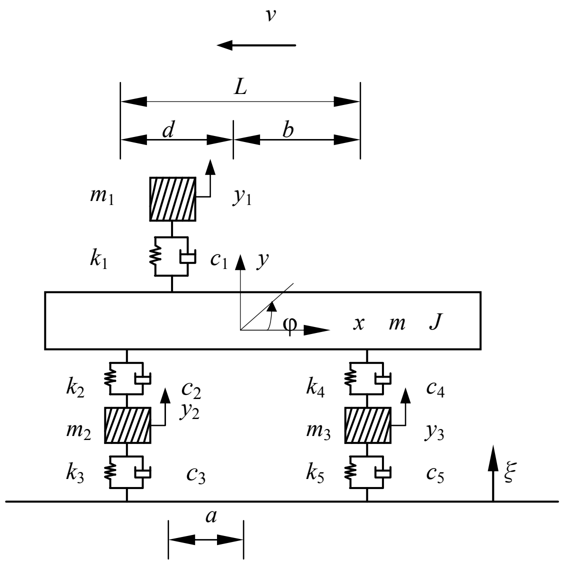

Fig. 3. Five-degree-of-freedom Vehicle Model.

movements but ignores the roll movement. More complex models can be developed to add the roll movement to describe a full-truck model. Obviously, more complicated vehicle models will significantly increase the complexity of the simulation. Therefore, car analysis can vary greatly in complexity, depending on what features are considered important from the designer's perspective. The International Roughness Index (IRI) and the Half-Car Roughness Index (HRI) are the two commonly used roughness indexes of pavement [22]. Quarter-truck and half-truck are used in calculations of IRI and HRI, respectively, which means that in evaluation of the roughness of the pavement in general sections pitch movement has been considered seriously [22-23]. The longitudinal roughness in bridge approach is much more serious than that in general highway sections, so the pitch movement should not be ignored. However, the roughness in longitudinal direction is much more serious than that in transverse direction, so roll movement is much less than pitch movement and can be ignored. Based on the discussion above, it is more reasonable to adopt a half-truck model in evaluation of the roughness of bridge approaches.

This paper adds rider and seat to a half-truck model to build a five-degree-of-freedom vehicle model and adopts it to conduct a theoretical study on the allowable differential settlement in bridge approach. In order to determine if it is reasonable to adopt a three-degree-of-freedom vehicle model for the determination of the allowable differential settlement, comparative calculations for a three-degree-of-freedom vehicle model and a five-degree-of-freedom vehicle model are carried out.

Calculating Model and Settlement Indexes of Bridge Approaches

When approach-relative slopes do not occur, the bridge approach without approach slab can be simplified as the step model shown in Fig. 1, where u = differential movement at the embankment/ structure interface and θ_1 = differential angle distortion of bridge deck whose tangent value is the differential slope of the bridge deck denoted as Δi_1. The differential angle θ_1 is so small that it can be considered to be equal to its sine value or tangent value. The step height is suggested as the differential settlement index when approach-relative slopes do not occur. When approach-relative slopes occur, the bridge approach model is shown in Fig. 2. To make the illustrations easier, approach slabs are supposed to be used. In Fig. 2, θ_2 = angle distortion between the bridge deck and the approach slab whose tangent value is the differential slope between the bridge deck and the approach slab denoted as Δi_2. Variable θ is adopted as the angle distortion of the approach slab after completion, and $\theta = \theta_1 + \theta_2$. Its tangent value is the differential slope of the approach slab, which is noted as Δi. Because θ_1, θ_2, and θ are very small, their tangent values or sine values can be seen to be equal to themselves. The allowable differential slope of the approach slab is suggested as the allowable differential settlement index and the step is assumed not to be present when an approach-relative slope occurs. In addition, the running speed of the vehicle is denoted as v', the length of the approach slab is denoted as L', and the differential settlement between the bridge abutment and the approach embankment is denoted as Δh and $\Delta h = L'\theta$.

Analytic Equations

The rider and seat are added to a half-truck model with four degrees of freedom and a linear five-degree-of-freedom system, shown in Fig. 3, is built to simulate the real vehicle. In Fig. 3, k_1, k_2, k_3, k_4, k_5 = spring constants of the seat, front suspension system, front tire, rear suspension system, and rear tire, respectively; c_1, c_2, c_3, c_4, c_5 = damping constants of the seat, front suspension system, front tire, rear suspension system, and rear tire, respectively; m_1 = mass of the rider; m = sprung mass; m_2, m_3 = unsprung masses of the front axle and rear axle, respectively; J, φ = moment of inertia and rotation angle of sprung mass for the axis passing through the center of mass of sprung mass and being vertical to running direction; a, d, b, L = distances between the centers of mass for the seat and sprung mass, for the front axle and sprung mass, for the rear axle and sprung mass, and for the front axle and rear axle, respectively; $L = d + b$; ζ, y_1, y_2, y_3, y = vertical displacements of the pavement, rider, unsprung mass of the front axle, unsprung mass of the rear axle, center of mass of sprung mass, which are all measured positive upwards from their static-equilibrium positions; ξ_1, ξ_2 = vertical displacements of the pavement under the front tire and rear tire, respectively.

According to D'Alembert's principle, the equations of motion for the vehicle are given by differential equations:

$$m_1\ddot{y}_1 + k_1 y_1 - k_1 y + k_1 a\phi + c_1\dot{y}_1 - c_1\dot{y} + c_1 a\dot{\phi} = 0 \tag{1}$$

$$\begin{aligned} &m_2\ddot{y}_2 + (k_2 + k_3)y_2 - k_2 y + k_2 d\phi + (c_2 + c_3)\dot{y}_2 - c_2\dot{y} \\ &+ c_2 d\dot{\phi} - k_3\xi_1 - c_3\dot{\xi}_1 = 0 \end{aligned} \tag{2}$$

$$\begin{aligned} &m_3\ddot{y}_3 + (k_4 + k_5)y_3 - k_4 y - k_4 b\phi + (c_4 + c_5)\dot{y}_3 \\ &- c_4\dot{y} - c_4 b\dot{\phi} - k_5\xi_2 - c_5\dot{\xi}_2 = 0 \end{aligned} \tag{3}$$

$$\begin{aligned} &m\ddot{y} + (k_1 + k_2 + k_4)y - (ak_1 + dk_2 - bk_4)\phi - (ac_1 + dc_2 - bc_4)\dot{\phi} + \\ &(c_1 + c_2 + c_4)\dot{y} - k_1 y_1 - c_1\dot{y}_1 - k_2 y_2 - c_2\dot{y}_2 - k_4 y_3 - c_4\dot{y}_3 = 0 \end{aligned} \tag{4}$$

$$\begin{aligned} &J\ddot{\phi} - (ak_1 + dk_2 - bk_4)y - (ac_1 + dc_2 - bc_4)\dot{y} + (a^2k_1 + d^2k_2 \\ &+ b^2k_4)\phi + (a^2c_1 + d^2c_2 + b^2c_4)\dot{\phi} + ak_1y_1 + ac_1\dot{y}_1 + dk_2y_2 \\ &+ dc_2\dot{y}_2 - bk_4y_3 - bc_4\dot{y}_3 = 0 \end{aligned} \tag{5}$$

The forces P_1(t) and P_2(t), exerted on the front tire and rear tire respectively by the pavement, may be expressed as:

$$P_1(t) = k_3(y_2 - \xi_1) + c_3(\dot{y}_2 - \dot{\xi}_1) + \frac{g}{L}[b(m_1 + m) + am_1] + m_2 g \tag{6}$$

$$P_2(t) = k_5(y_3 - \xi_2) + c_5(\dot{y}_3 - \dot{\xi}_2) + \frac{g}{L}[d(m_1 + m) - am_1] + m_3 g \tag{7}$$

where, g = gravitational acceleration.

Applying Laplace transform and manipulations to Eqs. (1)-(5), one obtains:

$$[M]\{Y(s)\} = \{Q(s)\} \tag{8}$$

where,

$$\{Y(s)\} = \{\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}, \hat{\Phi}\}' \tag{9}$$

$$\{Q(s)\} = [K]\{Y(0)\} + \{Pave\} \tag{10}$$

$$\begin{aligned} \{Y(0)\} = \{&y_1(0), \dot{y}_1(0), y_2(0), \dot{y}_2(0), y_3(0), \dot{y}_3(0), y(0) \\ &, \dot{y}(0), \phi(0), \dot{\phi}(0)\}' \end{aligned} \tag{11}$$

$$\{Pave\} = \{0, (k_3 + c_3 s)\hat{\xi}_1 - c_3\xi_1(0), (k_5 + c_5 s)\hat{\xi}_2 - c_5\xi_2(0), 0, 0\}' \tag{12}$$

$$[M] = \begin{bmatrix} a_{11} & 0 & 0 & a_{14} & a_{15} \\ 0 & a_{22} & 0 & a_{24} & a_{25} \\ 0 & 0 & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \tag{13}$$

$$[K] = \begin{bmatrix} k_{11} & k_{12} & 0 & 0 & 0 & 0 & k_{17} & 0 & k_{19} & 0 \\ 0 & 0 & k_{23} & k_{24} & 0 & 0 & k_{27} & 0 & k_{29} & 0 \\ 0 & 0 & 0 & 0 & k_{35} & k_{36} & k_{37} & 0 & k_{39} & 0 \\ k_{41} & 0 & k_{43} & 0 & k_{45} & 0 & k_{47} & k_{48} & k_{49} & 0 \\ k_{51} & 0 & k_{53} & 0 & k_{55} & 0 & k_{57} & 0 & k_{59} & k_{5(10)} \end{bmatrix} \tag{14}$$

The elements which are not equal to zero in the matrixes [M] and [K] are listed in the appendix; s represents the complex variable in frequency domain; and the "^" above symbols represents Laplace transform, i.e., the Laplace transform of y_1 is denoted as $\hat{y}_1$.

Multiplying both sides of Eq. (8) by $[M]^{-1}$, one obtains:

$$\{Y(s)\} = [M]^{-1}[K]\{Y(0)\} + [M]^{-1}\{pave\} \tag{15}$$

Applying Laplace transform and manipulations to Eqs. (6) and (7), one obtains:

$$\begin{aligned} \hat{P}_1 &= k_3(\hat{y}_2 - \hat{\xi}_1) + c_3[s\hat{y}_2 - y_2(0) - s\hat{\xi}_1 + \xi_1(0)] \\ &+ \frac{g}{sL}[b(m_1 + m) + am_1] + \frac{m_2 g}{s} \end{aligned} \tag{16}$$

$$\begin{aligned} \hat{P}_2 &= k_5(\hat{y}_3 - \hat{\xi}_2) + c_5[s\hat{y}_3 - y_3(0) - s\hat{\xi}_2 + \xi_2(0)] \\ &+ \frac{g}{sL}[d(m_1 + m) - am_1] + \frac{m_3 g}{s} \end{aligned} \tag{17}$$

Once the initial displacement and velocity of each mass and the vertical displacement of pavement profile are given, $\{Y(s)\}$ can be obtained from Eq. (15), and then the variables including the acceleration of the rider and the force exerted on the vehicle in frequency domain can be given. Taking inverse Laplace transform [24] to the variables in frequency domain, one can obtain the variables in time domain.

Initial Conditions and Pavement Height

Whether or not approach-relative slopes occur, the initial conditions and the vertical displacement of pavement profile when down bridges are similar to those when up bridges, so driving direction is supposed to be down bridges. As a result, the initial conditions and pavement height are discussed in the following two cases.

Case I, without Approach-Relative Slopes

The whole process of a vehicle's passing over the bridge approach without approach-relative slopes is divided into the following two phases linked up end to end.

When the Front Tire and Rear Tire Are Running on the Approach Pavement and the Bridge Deck, Respectively

The time when the front tire gets to the approach pavement is suggested as the initial time and the approach pavement is suggested as the datum plane. The initial displacements of the pavement are obtained as:

$$\xi_1(0) = 0 \tag{18a}$$

$$\xi_2(0) = L\theta_1 + u \qquad (18b)$$

The displacements of the pavement in this phase are obtained as:

$$\xi_1 = 0 \qquad (19a)$$

$$\xi_2 = L\theta_1 + u - vt\theta_1 \qquad (19b)$$

Previous studies indicated that the allowable differential settlement usually was very small (about 1.5-5*cm*) at the embankment-structure interface. During the course of vehicles' running down the step, the velocity's change of each mass is small enough to be ignored. Therefore, the initial velocity of each mass is the same with those when the front tire arrives at the end of the bridge deck, and they are obtained as:

$$\dot{y}_1(0) = -v\theta_1 \qquad (20a)$$

$$\dot{y}_2(0) = -v\theta_1 \qquad (20b)$$

$$\dot{y}_3(0) = -v\theta_1 \qquad (20c)$$

$$\dot{y}(0) = -v\theta_1 \qquad (20d)$$

$$\dot{\phi}(0) = 0 \qquad (20e)$$

The initial displacement of each mass vary with the step height, but generally

$$u \le \Delta k_3 \qquad (21)$$

where, Δk_3 is the compression of the front tire. It is given by:

$$\Delta k_3 = \frac{g}{k_3(b+d)}[b(m_1+m) - am_1 + m_2] \qquad (22)$$

Thus, the initial displacement of each mass are obtained as:

$$y_1(0) = u + (d-a)\theta_1 \qquad (23a)$$

$$y_2(0) = u \qquad (23b)$$

$$y_3(0) = u + L\theta_1 \qquad (23c)$$

$$y(0) = u + d\theta_1 \qquad (23d)$$

$$\phi(0) = \theta_1 \qquad (23e)$$

The displacement and velocity of each mass when the rear tire gets to the approach pavement can be computed.

When Both the Front Tire and Rear Tire are Running on the Approach Pavement

The time when the rear tire gets to the approach pavement is considered as the initial time and the approach pavement is considered as the datum plane. The initial values of the variables including the displacement and velocity of each mass are the same with those at the end of the first phase. Both the displacement of the pavement under the front or rear tire and its Laplace transform are equal to zero.

Case II, with Approach-Relative Slopes

The whole process of a vehicle's passing over the bridge approach with approach-relative slopes is divided into the following four phases linked up end to end.

When the Front Tire and Rear Tire Are Running on the Approach Slab and the Bridge Deck, Respectively

The time when the front tire gets to the approach slab is suggested as the initial time and the level at the end of the bridge deck is suggested as the datum plane. If the vibration of the vehicle caused by the differential slope of the bridge deck at the initial time is ignored, one obtains:

$$y_1(0) = (d-a)\theta_1 \qquad (24a)$$

$$\dot{y}_1(0) = -v\theta_1 \qquad (24b)$$

$$y_2(0) = 0 \qquad (24c)$$

$$\dot{y}_2(0) = -v\theta_1 \qquad (24d)$$

$$y_3(0) = L\theta_1 \qquad (24e)$$

$$\dot{y}_3(0) = -v\theta_1 \qquad (24f)$$

$$y(0) = d\theta_1 \qquad (24g)$$

$$\dot{y}(0) = -v\theta_1 \qquad (24h)$$

$$\phi(0) = \theta_1 \qquad (24i)$$

$$\dot{\phi}(0) = 0 \qquad (24j)$$

$$\xi_1(0)=0 \qquad (24k)$$

$$\xi_2(0) = L\theta_1 \qquad (24l)$$

The displacements of the pavement in this phase are obtained as:

$$\xi_1 = -vt\theta \qquad (25a)$$

$$\xi_2 = L\theta_1 - vt\theta_1 \qquad (25b)$$

The time when the rear tire gets to the approach slab is denoted as τ_1, and $\tau_1 = L/v$. Based on Eq. (25), the displacements of the pavement at this time can be obtained as:

$$\xi_1(\tau_1) = -L\theta \qquad (26a)$$

$$\xi_2(\tau_1) = 0 \qquad (26b)$$

The velocity of each mass at this time can also be obtained.

When Both the Front and Rear Tires Are Running on the Approach Slab

The time when the rear tire gets to the approach slab is considered as the initial time and the level at the end of the bridge deck is suggested as the datum plane. The initial displacement and velocity of each mass are the same with those at the end of the first phase. Similarly, $\xi_1(0) = -L\theta$ and $\xi_2(0) = 0$.

The displacements of the pavement in this phase are obtained as:

$$\xi_1 = -(vt + L)\theta \tag{27a}$$

$$\xi_2 = -vt\theta \tag{27b}$$

The time when the front tire begins to leave the approach slab is denoted as τ_2, and $\tau_2 = (L' - L)/v$. Based on Eq. (27), the displacements of the pavement at this time can be obtained as:

$$\xi_1(\tau_2) = -L'\theta \tag{28a}$$

$$\xi_2(\tau_2) = -v\tau_2\theta \tag{28b}$$

The velocity of each mass at this time can also be obtained.

When the Front Tire and Rear Tire Are Running on the Approach Pavement and the Approach Slab, Respectively

The time when the front tire begins to leave the approach slab is considered as the initial time and the approach pavement is considered as the datum plane. The displacement and velocity of each mass at the initial time have been computed in the second phase. But it should be noted that the datum planes in the second and third phases have changed, and the differential height is the post settlement of the approach embankment denoted as Δh. The initial velocity of each mass and the initial rotation angle of the sprung mass in this phase are the same with those at the end of the second phase. The initial vertical displacement of each mass in this phase is equal to the sum of Δh and that at the end of the second phase. Similarly, $\xi_1(0) = 0$ and $\xi_2(0) = L\theta$.

The displacements of the pavement in this phase are obtained as:

$$\xi_1 = 0 \tag{29a}$$

$$\xi_2 = L\theta - vt\theta \tag{29b}$$

The time when the rear tire begins to leave the approach slab is denoted as τ_3, and $\tau_3 = L/v$. Based on Eq. (29), the displacements of the pavement at this time can be obtained as:

$$\xi_1(\tau_3) = 0 \tag{30a}$$

$$\xi_2(\tau_3) = 0 \tag{30b}$$

The velocity of each mass at this time can also be obtained.

When Both the Front and Rear Tires Are Running on the Approach Pavement

The time when the rear tire begins to leave the approach slab is considered as the initial time, and the approach pavement is considered as the datum plane. The initial displacement and velocity of each mass are equal to those at the end of the third phase, $\xi_1(0) = 0$, $\xi_2(0) = 0$, $\dot{\xi}_1 = 0$, and $\dot{\xi}_2 = 0$.

It should be noted that only if $L' > L$ the whole process of a vehicle's passing over the bridge approach can be divided into the four phases above. Instead, the third phase does not exist but the initial conditions and the pavement height in other phases remain unchanged.

Effects on Vehicles' Vibration of Compression of Approach Roadway

If the approach slab is not present and the approach pavement is flexible, the compression of the approach roadway cannot be ignored arbitrarily. Its effects on the vehicle's vibration are discussed as follows:

1. The spring constant and damping constant of the approach roadway are denoted as k and c, respectively. The spring of the tire linked in series with that of the approach roadway is considered as the spring of the new tire. Similarly, the dash pot of the new tire can be obtained. The spring constant and damping constant of the front tire are denoted as k_3' and c_3', respectively. And the spring constant and damping constant of the rear tire are denoted as k_5' and c_5', respectively. Zhang [12] proved that the effect on the allowable differential settlement of the damping constant of the approach roadway is significantly small. So the hypothesis of $c/c_3 = k/k_3$ and $c/c_5 = k/k_5$ can be accepted safely. If the spring and dash pot of the new tires are supposed to be independent of each other, let $n_1 = k/k_3$ and $n_2 = k/k_5$, one can obtain:

$$k_3' = \frac{n_1}{n_1 + 1} k_3 \tag{31a}$$

$$c_3' = \frac{n_1}{n_1 + 1} c_3 \tag{31b}$$

$$k_5' = \frac{n_2}{n_2 + 1} k_5 \tag{32a}$$

$$c_5' = \frac{n_2}{n_2 + 1} c_5 \tag{32b}$$

2. An iterative method is used to compute the allowable differential settlement in which the two new tires are used and the compression of the approach roadway plus the step height is looked upon as the new step height.

Vibration Comfort Index

In British Standard (BS) 6841 [25] and International Organization for Standardization (ISO) 2631 [26], the calculation of the root-mean-square (RMS) of accelerations is recommend as the basic method to evaluate human exposure to whole-body vibration. But the two standards give no guidance on the assessment of exposures that last less than 1*min* [27]. Besides, the two standards indicate that RMS magnitudes will underestimate the effects of vibration in some

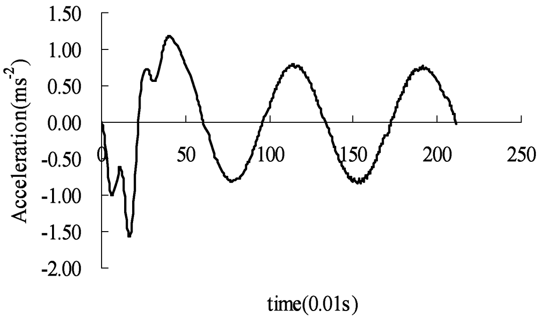

Fig. 4. Time History of Acceleration without Relative Slopes.

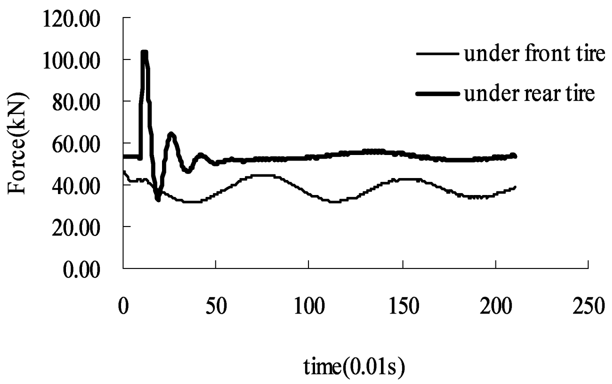

Fig. 5. Time History of Vertical Force without Relative Slopes.

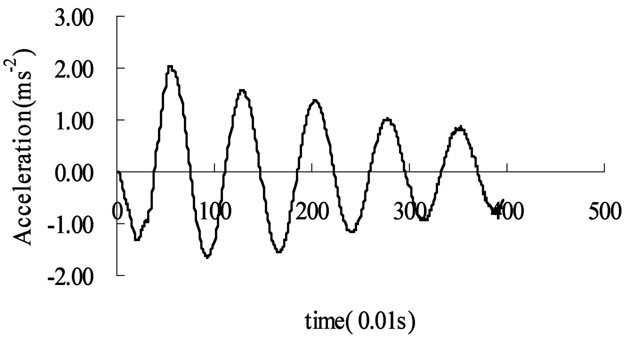

Fig. 6. Time History of Acceleration with Relative Slopes.

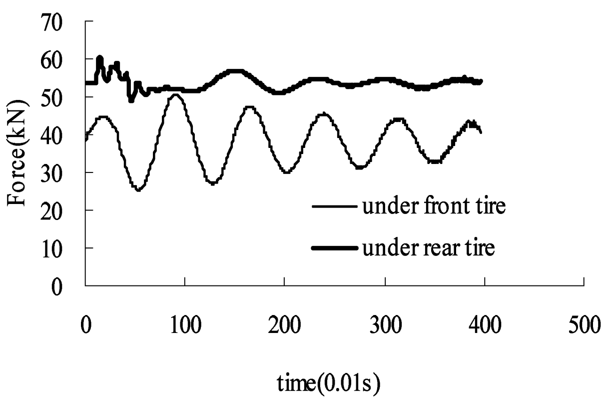

Fig. 7. Time History of Vertical Force with Relative Slopes.

conditions (such as high crest factors, occasional shocks, and transient vibration).

Where the basic evaluation method may underestimate the effects of vibration, "the running RMS method" is defined in ISO 2631, which is defined by using linear averaging

$$a_w(t_0) = \left[\frac{1}{\tau} \int_{t_0-\tau}^{t_0} a_w^{\ 2}(t)dt \right]^{1/2} \tag{33}$$

where, τ = integration time; t_0 = instantaneous time; and $a_w(t)$ = frequency weighted acceleration at time t. The standard recommends the use of 1*s* as the integration time when calculating the running RMS value. A quantity called the "maximum transient vibration value (MTVV)" is defined as the highest magnitude of the running RMS obtained during the measurement period. The vibration caused by the differential settlement in bridge approaches lasts a short time and is a kind of transient vibration [12], so MTVV is accepted as a vibration comfort index. In addition, the vibration frequency also has some effects on discomfort, so the MTVV required in the determination of the allowable differential settlement should vary with the vibration frequency.

Calculation Results

The vehicle properties are considered as follows: m_1=46*kg*, m_2=480*kg*, m_3=945*kg*, m=7885*kg*, J=37432$kg{\cdot}m^2$, k_1 = 2100*N/m*, k_2= 95×10^4*N/m*, k_3 = 48×10^4*N/m*, k_4 = 17 ×10^4*N/m*, k_5= 190×10^4*N/m*, c_1 = 1.8×10^3$N{\cdot}s/m$, c_2 = 7×10^3$N{\cdot}s/m$, c_3 = 2×10^3$N{\cdot}s/m$, c_4 = 14×10^3$N{\cdot}s/m$, c_5 = 3×10^3$N{\cdot}s/m$, a=05*m*, b=1.5*m*, and d=2.0*m* [28]. The vehicle runs down a bridge at a speed of 33.33 *m/s*. The parameters for the bridge approach are considered as follows: (1) where approach-relative slopes do not occur, θ_1 = 0, k = 6 ×10^6*N/m*, c = 5×10^4$N{\cdot}s/m$, and u = 0.01*m* and (2) where approach-relative slopes occur, Δi = 0.5%, L'= 10*m*, and θ_1 = 0.

Besides MTVV, the vibration frequency and the maximum vertical force exerted on the pavement by the vehicle, denoted as F_{max}, are also included in results. Through the MATLAB program, one obtains the time history of the rider's acceleration and the vertical force exerted on the vehicle by the pavement, as shown in Figs. 4-7.

If approach-relative slopes do not occur, the computed vibration frequency is 1.39*Hz*, and MTVV = 0.74m/s^2, and F_{max} = 103.67*kN*. Instead, the computed vibration frequency is 1.35*Hz*, and MTVV = 1.2 0m/s^2 and F_{max} = 60.57*kN*.

If the allowable MTVV is 1.0m/s^2 for the vibration frequencies of 1.39*Hz* and 1.35*Hz*, the allowable step height of 2.0*cm* and the allowable differential slope of the approach slab of 0.42% can be obtained.

Comparative Calculations with Different Vehicle Models

When the five-degree-of-freedom vehicle model is used, the results against the various moments of inertia of vehicles are listed in Table 1 in which other parameters are as shown in the example above. The table indicates that, whether or not approach relative slopes occur,

Table 1. Effect of Moment of Inertia of Vehicles.

J ($kg \cdot m^2$)	MTVV (m/s^2)		Vibration Frequency (Hz)		F_{max} (kN)	
	Without Relative Slope	With Relative Slope	Without Relative Slope	With Relative Slope	Without Relative Slope	With Relative Slope
0	0.61	0.61	5.55	1.16	74.79	63.89
1000	0.46	0.55	4.55	1.04	75.28	64.17
37342	0.35	1.20	1.39	1.35	72.10	60.57

Table 2. Effect of Vehicle Model.

Vehicle Model	MTVV (m/s^2)		Vibration Frequency (Hz)		F_{max} (kN)	
	Without Relative Slope	With Relative Slope	Without Relative Slope	With Relative Slope	Without Relative Slope	With Relative Slope
Three-degree-of-freedom System	0.56	1.94	1.47	1.47	45.95	48.49
Five-degree-of-freedom System	0.28	0.61	5.55	1.16	42.92	63.89

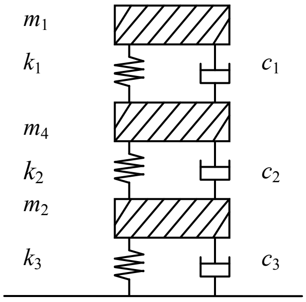

Fig. 8. Three-degree-of-freedom Vehicle Model.

the moment of inertia has great effects on MTVV and the vibration frequency. For example, when J = 37342$kg \cdot m$, if it is supposed that J = 0, the relative errors for MTVV and the vibration frequency will be up to 74 and 299%, respectively, for the bridge approach without approach relative slopes. When approach relative slopes occur, the two relative errors will be up to 49 and 14%, respectively. So, for the determination of the allowable differential settlement in the bridge approach, the moment of inertia of the five-degree-of-freedom vehicle cannot be ignored.

The five-degree-of-freedom system shown in Fig. 3 is simplified into the three-degree-of-freedom system shown in Fig. 8. Because the front wheel is more close to rider than the rear wheel, the three-degree of-freedom system is made up of the rider, a portion of the sprung mass in Fig. 3 and the front axle. It can be obtained that m_4=$m \cdot b/L$ from the moment of force balance principle and other variables in Fig. 8 are the same with those in Fig. 3. When both the spring constant and the damping constant of the approach pavement are supposed to be infinite, the results for the five-degree-of-freedom system in which J = 0 and the three-degree-of-freedom system are shown in Table 2. This table indicates that even if the moment of inertia of the vehicle is ignored, whether or not approach-relative slopes occur, the MTVV and vibration frequencies for the five-degree of-freedom system are very different from those for the three-degree-of-freedom system.

From the discussions above, it can be concluded that, for the determination of the allowable differential settlement in the bridge approach, it is more reasonable to use the five-degree-of-freedom vehicle model than to use the three-degree-of-freedom vehicle model.

Summary and Conclusions

The determination of the allowable differential settlement is important for the design and maintenance of the bridge approach. In this paper, a dynamic response analysis for vehicles passing over the bridge approach is carried out by means of the Laplace transform in which the vehicle is modeled as a five-degree-of-freedom system, and the bridge approaches with and without approach-relative slopes are simplified as broken lines model and step model, respectively. Comparative calculations with different vehicle models are carried out. Results show that, whether or not approach-relative slopes occur, the MTVV and vibration frequency for the five-degree of-freedom system are very different from those for the three-degree-of-freedom system. So it is more reasonable to use the five-degree-of-freedom vehicle model than to use three-degree-of-freedom vehicle model for the determination of the allowable differential settlement.

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Appendix

The elements which are not equal to zero in the matrixes [M] and [K] are as follows:

$a_{11} = m_1 s^2 + k_1 + c_1 s$; $a_{14} = -(k_1 + c_1 s)$; $a_{15} = k_1 a + c_1 as$;

$a_{22} = m_2 s^2 + k_2 + k_3 + c_2 s + c_3 s$;

$a_{24} = -(k_2 + c_2 s)$; $a_{25} = dk_2 + dc_2 s$;

$a_{33} = m_3 s^2 + k_4 + k_5 + c_4 s + c_5 s$; $a_{34} = -(k_4 + c_4 s)$;

$a_{35} = -(k_4 b + c_4 bs)$; $a_{41} = -(k_1 + c_1 s)$; $a_{42} = -(k_2 + c_2 s)$;

$a_{43} = -(k_4 + c_4 s)$;

$a_{44} = ms^2 + k_1 + k_2 + k_4 + c_1 s + c_2 s + c_4 s$;

$a_{45} = -(ak_1 + dk_2 - bk_4 + ac_1 s + dc_2 s - bc_4 s)$;

$a_{51} = ak_1 + ac_1 s$; $a_{52} = dk_2 + dc_2 s$; $a_{53} = -(bk_4 + bc_4 s)$;

$a_{54} = -(ak_1 + dk_2 - bk_4 + ac_1 s + dc_2 s - bc_4 s)$;

$a_{55} = Js^2 + a^2 k_1 + d^2 k_2 + b^2 k_4 + a^2 c_1 s + d^2 c_2 s + b^2 c_4 s$.

$k_{11} = m_1 s + c_1$; $k_{12} = m_1$; $k_{17} = -c_1$; $k_{19} = c_1 a$;
$k_{23} = m_2 s + c_2 + c_3$; $k_{24} = m_2$; $k_{27} = -c_2$; $k_{29} = c_2 d$;
$k_{35} = m_3 s + c_4 + c_5$; $k_{36} = m_3$; $k_{37} = -c_4$; $k_{39} = -c_4 b$;
$k_{41} = -c_1$; $k_{43} = -c_2$; $k_{45} = -c_4$; $k_{47} = ms + c_1 + c_2 + c_4$;
$k_{48} = m$; $k_{49} = -(ac_1 + dc_2 - bc_4)$; $k_{51} = ac_1$; $k_{53} = dc_2$;
$k_{55} = -bc_4$; $k_{57} = -(ac_1 + dc_2 - bc_4)$;
$k_{59} = Js + a^2 c_1 + d^2 c_2 + b^2 c_4$; $k_{5(10)} = J$.

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