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Probabilistic Model for Long-term Deformation of Subgrade Soil in Upgrading-speed Railway Lines

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Abstract: This paper presents a new empirical model for predicting the long-term deformation of subgrade soils induced by cyclic sinusoidal load and cyclic train load. Laboratory cyclic triaxial tests were conducted on compacted specimens to investigate the long-term deformation behavior of railway subgrade soils classified as CL according to Unified Soil Classification System. Multivariate regression analyses were performed on the test data and the probabilistic model for predicting long-term deformation behavior under both loading conditions was proposed. The equivalent sinusoidal load for the train load is transformed by multiplying an amplitude reduction coefficient of 0.55. The new empirical model correlates the long-term deformation with influencing factors including the relative compaction of subgrade soils, number of cyclic loading, cyclic deviatoric stress and confining stress. Another feature of the new model is that the long-term deformation is predicted with lower bound and upper bound through a probabilistic model. Three possible probabilistic applications of the new model are illustrated with example cases.

***Key words**: Empirical model; Equivalent cyclic load; Long-term deformation; Probability; Upgrading-speed railway.*

Introduction

The subgrade of most railway lines in China was constructed in the last century with relatively weak filling material and the relative compaction was low comparing with the requirement in current design code. In recent years with the rapid process of upgrading-speed in the railway engineering, the subgrade stiffness and plastic deformation control have to meet a much higher standard in the design, construction and maintenance. The increase in cyclic loading amplitude, loading frequency and cyclic number caused by the higher train speed and larger amount of passenger as well as freight have brought a larger cumulative plastic deformation to the subgrade soils. The great amount of maintenance cost is inevitable if the subgrade status deteriorates. Therefore, it is essential to characterize the long-term deformation behavior of the railway subgrade soils of the upgrading-speed railway induced by cyclic loading for maintenance and improvement to the existing railway lines.

Decades ago the dynamic problems were often solved by equivalent static load method. In the *code for design on subgrade of railway* of China, the cyclic load was simplified as an equivalent soil column. However, the simplified method applicable to relatively slow speed conditions is recently challenged by the engineering practice of upgrading-speed railway and high speed railway. Most of the previous research focused on the long-term deformation of the bed soils in the construction of subway [1-4] and road embankment [5-10]. The research on the deformation of railway ballast was also reported [11-14]. Choudhury *et al*, [15] presented an analytical model of a track-ballast-subgrade system with different formation soils such as dense uniform sand, stiff clay, loose sand, and soft clay modeled by using a mass-spring dashpot system with two degrees of freedom. Nevertheless, a simple and convenient model for long-term deformation of subgrade soil needs to be proposed in the railway engineering field.

The long-term deformation of subgrade soils induced by cyclic loading is preferably predicted with empirical equations based on laboratory experiment including cyclic triaxial test, cyclic simple shear test, resonant column test, etc. Numerous researchers have established their empirical equations for a certain loading condition or for a certain type of soils. The power model proposed by Monismith *et al*. [5] is widely used to predict the long-term deformation. It gives the correlation between the long-term deformation and number of cyclic loading and has been referenced by most of the later researcheres during the following years. Based on the power model, Li and Selig [16] introduced the ratio of cyclic deviatoric stress over static deviatoric stress at failure into the power model to estimate the cumulative plastic deformation for cohesive soils under train loading. Chai and Miura [10] further improved the model by considering initial static deviatoric stress and undrained shear strength to predict the traffic-load-induced long-term settlement of road on soft subsoil. As in the literature reported over the years numerous empirical equations obtained by applying sinusoidal load in the cyclic loading tests have been proposed and most of them correlate the long-term deformation of soils with the influencing factors such as the number of cyclic loading, amplitude of cyclic load and soil strength parameters.

In railway engineering practice, however, the wave of train load is different from that of sinusoidal load. The long-term deformation under train load and under sinusoidal load would be inconsistent. In this regard, Gong *et al*. [17] conducted a series of cyclic triaxial tests on compacted specimen by applying the simplified train load; they confirmed that the long-term deformation trend under the train load agreed well with the power model. As the follow-up research, a

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Table 1. Models for Predicting Long-term Deformation.

Investigator	Model	Parameters
Barksdale [4]	$\varepsilon_p = a + b \cdot \log N$	*a, b*
Allen and Deen [18]	$\log \varepsilon_p = C_0 + C_1(\log N) + C_2(\log N)^2 + C_3(\log N)^3$	C_0, C_1, C_2, C_3
Wolff [19]	$\varepsilon_p = (mN + a)(1 - e^{-bN})$	*a, b, m*
Behzadi and Yandell, [20]	$\varepsilon_p = Ae^{Bq_d}N^S$, q_d = Deviatoric Stress, S = Slope of log(ε_p) versus log(N)	*A, B*
Lekarp and Dawson, [21]	$\frac{\varepsilon_p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p}\right)^b_{\max}$, $L = \sqrt{q^2 + p^2}$, $p_0 = 100$ kPa	*a, b*, N_{ref}
Ling, *et al*. [22]	$\varepsilon = (2.59r_c^2 - 0.45r_c + 0.48)\ln N - 20.14r_c^2 + 9.18r_c - 2.92$ $0.18 < r_c < 0.71$	$r_c = \sigma_d / 2c_u$ c_u = Undrained Shear Strength

new empirical model for predicting long-term deformation under the sinusoidal load and train load is proposed in this paper. The new model correlates the long-term deformation with three influencing factors: (1) relative compaction of subgrade soils, (2) number of cyclic loading and (3) ratio of cyclic deviatoric stress over confining stress. Moreover, an equivalent method to simulate the train load with sinusoidal load is proposed. The equivalent method transforms the train load to the sinusoidal load by multiplying a reduction factor of cyclic load amplitude to predict the long-term deformation of subgrade soils under the train load. It is also advisable to conduct relevant laboratory test using the simple sinusoidal load with the proposed equivalent method.

Brief Review of Previous Empirical Models

The power model proposed by Monismith *et al*. [5] gives the most commonly used equation to predict long-term deformation of subgrade soils under cyclic loading:

$$\varepsilon_p = AN^b \tag{1}$$

where, ε_p is the cumulative plastic strain (%); N is the number of cyclic loading; A and b are regression coefficients. Li and Selig [16] ameliorated the power model by considering both the traffic loading and strength of soils in their prediction equation:

$$\varepsilon_p = a\left(\frac{q_d}{q_f}\right)^m N^b \tag{2}$$

where, q_d is the cyclic deviatoric stress induced by traffic load; q_f is the static deviatoric stress at failure; a, b and m are regression coefficients. Chai and Miura [10] further developed the model by taking into account the effect of initial static deviatoric stress to predict the traffic-load-induced permanent deformation of soft subsoil:

$$\varepsilon_p = a\left(\frac{q_d}{q_f}\right)^m \left(1 + \frac{q_s}{q_f}\right)^n N^b \tag{3}$$

where, q_s is the initial static deviatoric stress; a, b, m and n are regression coefficients. Some researchers found that the deformation induced by the first cycle of loading has a significant influence on the long-term deformation. Besides, the amount of deformation induced by the first cycle of loading is always measured and recorded with error in the laboratory observation. Thus, another approach to predict the long-term deformation was proposed by separating the deformation induced by the first cycle of loading from the Monismith model. Qiu and Sun [18] proposed a model which independently calculates the deformation of the first cycle to predict the long-term deformation of road subsoil induced by traffic loading:

$$\varepsilon_p = \varepsilon_p(1) f(N) \tag{4}$$

where, ε_p (1) is the cumulative plastic strain induced by the first cycle of loading (%) and $\varepsilon_p(1) = p_1 r_c^{p_2}$; *f(N)* is the cumulative function and $f(N) = N^{p_3 r_c^{p_4}}$. In the model r_c is the ratio of deviatoric stress over the strength of material and p_1, p_2, p_3, p_4 are material parameters. In addition to the models cited above, many other investigators proposed their models for predicting long-term deformation for a certain sort of soils. Some other models [19-23] for predicting long-term deformation are summarized in Table 1. It should be noted that most of these research focused on the traffic loading in road engineering.

Cyclic Triaxial Test

A series of stress-controlled cyclic triaxial tests using user-defined cyclic wave were performed on the compacted specimens of Shanghai clay (Layer α-1 and α-2) in order to investigate the deformation behavior of the railway subgrade soils. Laboratory soil mechanical tests were conducted on the soil sample before the specimens were compacted. The soil selected to make the compacted specimens indicates a group symbol of CL in the Unified Soil Classification System. Its liquid limit ranges from 30.1% to 43.8%, and plastic index from 11.5 to 21. The optimum moisture content and maximum dry unit weight of the soil sample are 20% and 1.67g/cm^3 respectively from the standard Proctor compaction test. The specimens were applied static load with the principal stress ratio of σ_1/σ_3=1.25 for 24 hours before starting the cyclic test to simulate the actual stress state in field. All the cyclic loading tests are performed under undrained condition.

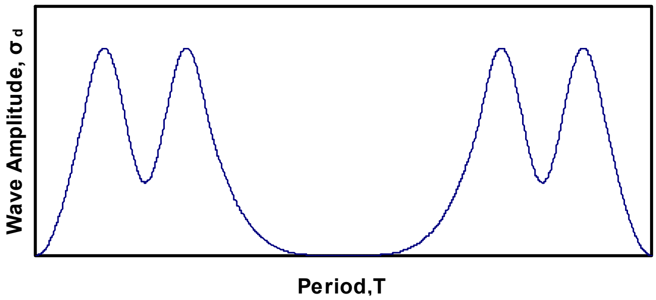

Fig. 1. Simplified Train Wave Based on Field Measurement and Railway Engineering Codes.

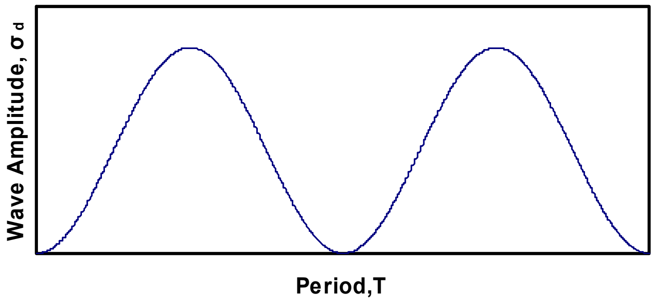

Fig. 2. Sinusoidal Wave (One Sinusoidal Period Simulates the Loading of One Bogie in a Train).

Table 2. Specimen Information and Test Plan.

Case No.	Load Form	Moisture Content, w/%	Relative Compaction, K	Confining Stress, σ_c /kPa	Amplitude of Cyclic Stress, σ_d /kPa	Number of Cyclic Loading, N
1	Train Load*	22.6	0.93	30	30	10000
2		22.6	0.93	30	80	9000
3		21.6	0.98	12	80	9985
4	Sinusoidal Load**	22.4	0.96	30	30	7465
5		23.2	0.86	30	30	9000
6		22.6	0.92	30	24	10000
7		23.0	0.96	30	19.5	8561
8		22.5	0.97	30	13.5	10000
9		21.8	0.96	30	9	9000

*Range of influencing factors: $\lim: 1.0 \le \sigma_d / \sigma_c \le 6.7, \quad 0.93 \le K \le 0.98, \quad N \le 10^4$

**Range of influencing factors: $\lim: 0.3 \le \sigma_d / \sigma_c \le 1.0, \quad 0.86 \le K \le 0.97, \quad N \le 10^4$

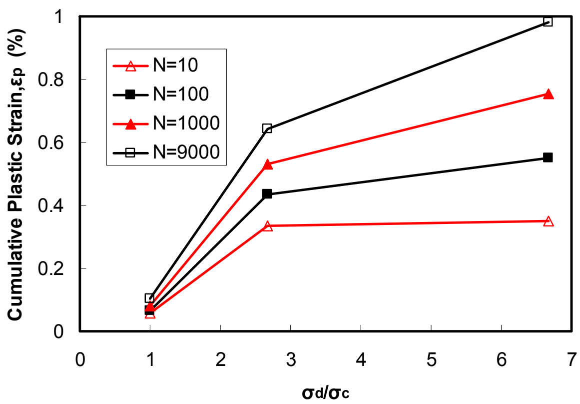

Fig. 3. Relationship between Plastic Strain ε_p and Cyclic Deviatoric Stress Ratio σ_d/σ_c.

Gong *et al*. [17] proposed a simplified train wave to simulate the train wave in field based on field measurement and design codes. The train and sinusoidal waves defined as one period in the cyclic loading are shown in Fig. 1 and Fig. 2 in which T represents one period of loading. The intervals of the peaks of the train wave in Fig. 1 can be obtained according to axles distance. In the Chinese design codes, the amplitude of cyclic stress at the surface of the subgrade is $\sigma_d = 0.26P(1\pm0.004v)$, in kPa, which determines the extreme amplitudes of the train wave in Fig. 1. In the above equation, P is the axle weight (kN), and v is the velocity of the train (200 km/h for upgrading speed train). In their proposed simplified method to obtain equivalent sinusoidal wave, one sinusoidal period loading simulates the loading effect of one bogie in a train as illustrated in Fig. 2. The equivalent sinusoidal load induces the same amount of long-term deformation as train load does if the amplitude of sinusoidal load is obtained by multiplying a reduction coefficient to the amplitude of the train load. In those cyclic triaxial tests the amplitude of cyclic load applied on the specimens was calculated by the design codes and the amplitude attenuation with depth was also taken into account. The wave frequency estimated based on the field condition is about 2.1Hz. Since a high relative compaction of the subgrade of the upgrading-speed railway is required in order to minimize the deformation, all specimens were compacted to a relatively high density with moisture content close to the optimum moisture content. Table 2 summarizes the specimen information and test plan.

Probabilistic Model for Long-Term Deformation Prediction

The test results confirm that the relationship between long-term deformation (denoted as cumulative plastic strain ε_p hereafter) and the number of cyclic loading (N) conforms to the power model proposed by Monismith *et al*. [5]. Besides, the long-term deformation correlates nonlinearly with the ratio of cyclic deviatoric stress over confining stress, as shown in Fig. 3. The internal factors like moisture content and relative compaction also have a significant influence on the long-term deformation. The moisture content is closely correlated to the relative compaction, which was already verified by standard Proctor compaction test. The dry unit weight

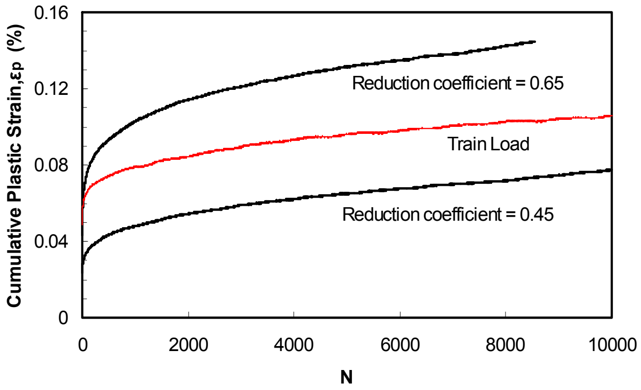

Fig. 4. Range of Reduction Coefficient Proposed by Gong *et al.* [16].

corresponds to two moisture content values except at the optimum moisture content from the typical compaction curves. The relative compaction is an important field index for engineers in design and construction. Thus, by introducing relative compaction only, the new model is given as Eq. (5):

$$\varepsilon_p = a\left(\frac{\sigma_d}{\sigma_c}\right)^m K^n N^b \tag{5}$$

where ε_p is the cumulative plastic strain (%); K is the relative compaction of the subgrade soils; N is the number of cyclic loading; σ_d is the amplitude of cyclic deviatoric stress and σ_c is the confining stress; a, b, m, n are regression coefficients. Eq. (5) is a multivariate power model. The advantages of the power model are: (1) it could represent linear or nonlinear relationship between each of the independent variables and dependent variable by adjusting the power value; (2) it is easy to transform the power equation into linear equation by taking logarithm to conduct multivariate regression analysis.

Gong *et al.* [17] concluded that the amplitude of equivalent sinusoidal load is much smaller than that of the train load if the same amount of permanent deformation is induced, given that all the other factors are the same. The amplitude of the equivalent sinusoidal load is obtained by multiply a reduction coefficient to the amplitude of the train load. The reduction coefficient ranges from 0.45 to 0.65 based on the experiment results, as shown in Fig. 4. The interval of reduction coefficient was proposed roughly without considering the variation of the influencing factors. There is inevitably a small deviation of the input of cyclic load amplitudes and relative compactions among all the specimens. For instance, it is not feasible (sometimes unnecessary) to compact all the specimens to exactly the same compaction coefficient with the same moisture content. Therefore, a serial of multivariate regression was conducted in order to narrow down the interval of reduction coefficient and simultaneously take into account the variations of those influencing factors. The reduction coefficients selected in the regression are 0.45, 0.50, 0.55, 0.60 and 0.65 which are within the range proposed by Gong *et al.* [17]. It should be noted that even though it will help search the best reduction coefficients if more reduction coefficients between 0.45 and 0.65 are selected, a simple reduction coefficient which already meets the prediction need would be preferred for convenient application in the engineering practice.

Based on Eq. (5) for long-term deformation and the multivariate regression on the current test results, the following model for long-term deformation prediction were established, in which the reduction coefficient was selected with least mean square method:

$$\ln(\varepsilon_p) = -2.822 + 1.293\ln\left(\frac{\sigma_d}{\sigma_c}\right) - 5.097\ln(K) + 0.126\ln(N) \pm 0.192 \tag{6}$$

In Eq. (6), 0.192 represents the standard deviation of the model. The reduction coefficient for train load is selected as 0.55 in this model. The term σ_d is the amplitude of cyclic deviatoric stress for sinusoidal load. If train load is applied, $\sigma_d = 0.55\sigma_{d,train}$.

The proposed model for predicting the long-term deformation induced by sinusoidal load and train load presented in Eq. (6) has an error term which is ± one standard deviation of model error. The new regression model correlates relative compaction (K), number of cyclic loading (N) and the ratio of cyclic deviatoric stress over confining stress. The relative compaction has been available since the railway subgrade was designed. The other two factors can be estimated by design code according to field condition. The variations of the input variables are also considered in this empirical model. Therefore it is convenient to predict the long-term deformation of subgrade soils with the new model which gives lower bound and upper bound of the prediction limit obtained through ± one, two, or three standard deviations. Figs. 5 and 6 show the prediction intervals of the long-term deformation based on Eq. (6) (± one standard deviation) for the train load and the sinusoidal load conditions, respectively (Case No. 3 and Case No. 5 in Table 2). With the equivalent method proposed herein, it is feasible to use the sinusoidal load in the future laboratory testing and numerical simulation of the train load. Especially when the laboratory test is limited by the equipment (e.g., no user defined wave option in the cyclic triaxial test device), it is advisable to employ the equivalent method to simulate the train load in order to predict the long-term deformation.

Applications of the Proposed Model

The subgrade soils of high-speed railway are generally compacted to a high relative compaction (K) in order to support larger load and control the cumulative deformation. In engineering practice the subgrade will be maintained periodically in case the subgrade status deteriorates and cumulates a large amount of plastic deformation which will jeopardize the safety of railway transportation. Therefore it is more practical to predict the probability of the long-term deformation exceeding a certain allowable deformation value under a relatively low number of cyclic loading. To illustrate the probabilistic use of the models proposed in this paper, an allowable cumulative plastic strain ($\varepsilon_{p,\,\mathrm{lim}}$) value of 0.05%, 0.075%, 0.1%, 0.15% and a cyclic number of 5000 for train loading are selected for demonstration herein. Three probabilistic applications of the new empirical model are presented as follows.

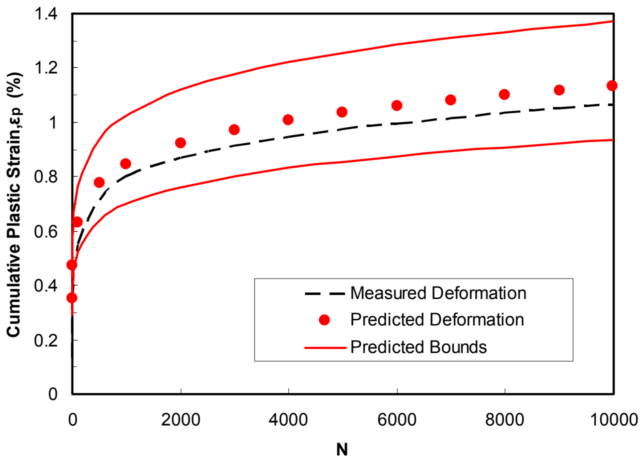

Fig. 5. Prediction Intervals of Long-term Deformation (Case No.3 in Table 2).

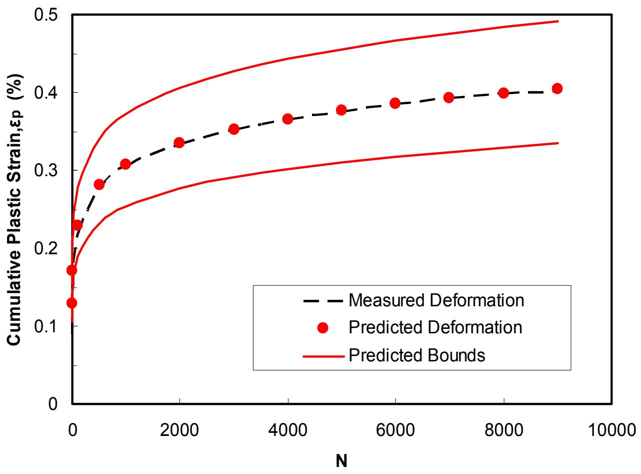

Fig. 6. Prediction Intervals of Long-term Deformation (Case No.5 in Table 2).

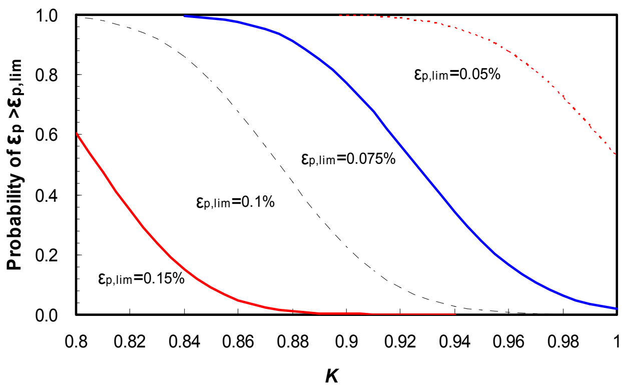

Fig. 7. Chart for Determination of Relative Compaction (EA 1).

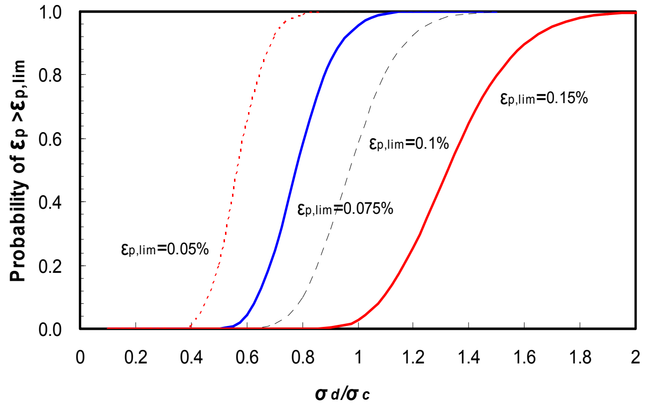

Fig. 8. Chart for Stress Ratio Control (EA 2).

Engineering Application One (designated as EA1): Determine the relative compaction of subgrade soils

Given that the number of cyclic loading by train is 5000, and the ratio of cyclic deviatoric stress over confining stress is 0.7 (within the limit of stress ratio in Table 2), the probability of the cumulative plastic strain exceeding the allowable cumulative plastic strain ($\varepsilon_{p,\text{lim}}$) at a certain value of relative compaction K is written in Eq. (7):

$$P(\varepsilon_p \geq \varepsilon_{p,\text{lim}}) = P(\ln(\varepsilon_p) \geq \ln(\varepsilon_{p,\text{lim}}))$$

$$= 1 - \Phi\left[\frac{\ln(\varepsilon_{p,\text{lim}}) - \begin{pmatrix} -2.822 + 1.293\ln(0.55\times 0.7) \\ -5.097\ln(K) + 0.126\ln(5000) \end{pmatrix}}{0.192}\right] \quad (7)$$

It should be noted that the reduction coefficient of 0.55 for equivalent sinusoidal load is used here. The function $\Phi(\ )$ denotes the cumulative probability in standard normal distribution and can be easily obtained by using the function NORMSDIST in Excel spreadsheet. Fig. 7 shows the probability of exceeding cumulative plastic strain of 0.05%, 0.075%, 0.1% and 0.15% versus the relative compaction. According to this chart, the subgrade soils should be compacted to a relative compaction greater than 0.84, 0.91, and 0.96, when the probability of exceeding the allowable cumulative plastic strains of 0.15%, 0.1%, and 0.075% respectively is less than 20%. The probability is 53% even though the relative compaction reaches 1.0, when the allowable cumulative plastic strain is 0.05%. That means a cumulative plastic strain of 0.05% is inevitable in the field.

Engineering Application Two (designated as EA2): Control the cyclic deviatoric stress

Given that the number of cyclic loading by train is 5000, and the relative compaction is 0.95 (within the limit in Table 2), the probability of the cumulative plastic strain exceeding the allowable cumulative plastic strain ($\varepsilon_{p,\text{lim}}$) at a certain ratio of cyclic deviatoric stress over confining stress is expressed in Eq. (8). Fig. 8 gives the probability of exceeding cumulative plastic strain of 0.05%, 0.075%, 0.1% and 0.15% versus the stress ratio. In Fig. 8, the probability of

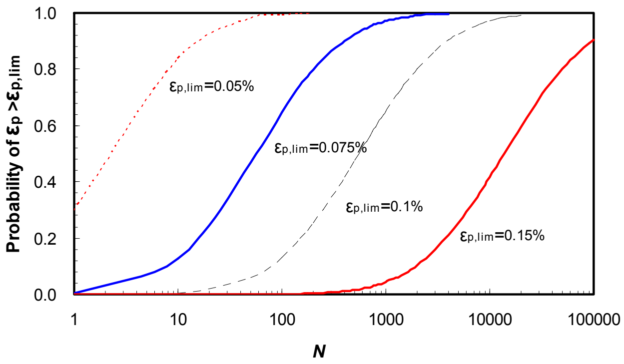

Fig. 9. Chart for Determination of Maintenance Interval (EA 3).

cumulative plastic strain exceeding 0.05% 0.075%, 0.1% and 0.15% is already 100% when the stress ratio reaches 0.8, 1.1, 1.4, and 1.9 respectively, which means maintenance to the subgrade needs to be conducted immediately. As stated above, the stress ratio is correlated with the axle weight, the velocity of the train and the lateral earth pressure at rest in the subgrade. Therefore it is feasible to estimate the stress ratio for an existing railway line according to the field condition. With the stress ratio estimated and the probabilistic model proposed, the probability of the cumulative plastic strain of the subgrade soils exceeding the allowable plastic strain will be obtained.

$$P(\varepsilon_p \geq \varepsilon_{p,\lim}) = P(\ln(\varepsilon_p) \geq \ln(\varepsilon_{p,\lim})) = 1-\Phi\left[\frac{\ln(\varepsilon_{p,\lim}) - \begin{pmatrix} -2.822+1.293\ln(0.55\times\sigma_d/\sigma_c) \\ -5.097\ln(0.95)+0.126\ln(5000) \end{pmatrix}}{0.192}\right] \tag{8}$$

Engineering Application Three (designated as EA3): Control the number of cyclic loading to decide the optimum interval of subgrade maintenance

Given that the relative compaction is 0.95 and the stress ratio is 1.2 for train loading, Eq. (9) gives the probability of the cumulative plastic strain exceeding the allowable cumulative plastic strain ($\varepsilon_{p,\lim}$) at a certain number of cyclic loading. The interval of maintenance to the subgrade (expressed as the number of cyclic loading herein) could be estimated according to Eq. (9) or Fig. 9 based on the probabilistic analysis. For instance, after 200, 2000 and 20000 times of cyclic loading respectively, the probability that the long-term deformation of greater than 0.05%, 0.075% and 0.1% is already 100%, which calls for countermeasure since the allowable cumulative plastic strain will be exceeded.

It is convenient to make the decision on design or maintenance (for example, determining a target relative compaction, controlling the stress ratio, and estimating the maintenance interval) to the railway subgrade to meet the plastic deformation requirement with the probabilistic analysis illustrated above. Although the input parameters in Eqs. (7)-(9) and Figs. 7-9 are hypothetical, it is quite straightforward and advisable to obtain similar charts based on field information with the proposed empirical model.

$$P(\varepsilon_p \geq \varepsilon_{p,\lim}) = P(\ln(\varepsilon_p) \geq \ln(\varepsilon_{p,\lim})) = 1-\Phi\left[\frac{\ln(\varepsilon_{p,\lim}) - \begin{pmatrix} -2.822+1.293\ln(0.55\times 0.7) \\ -5.097\ln(0.95)+0.126\ln(N) \end{pmatrix}}{0.192}\right] \tag{9}$$

Conclusions

A probabilistic model for predicting the long-term deformation (denoted as cumulative plastic strain) of subgrade soils under the sinusoidal load and train load is proposed through multivariate regression analyses of the cyclic triaxial test data. In this model, the train load is expressed as an equivalent sinusoidal load. The amplitude of the equivalent sinusoidal load for the train load is obtained by an amplitude reduction method. The amplitude reduction coefficient of 0.55 is selected based on the regression analysis. The equivalent sinusoidal method is useful when the user-defined wave is not available in the testing equipment system. The new model correlates the long-term deformation with influencing factors including relative compaction of subgrade soils, number of cyclic loading, and ratio of cyclic deviatoric stress over confining stress. It is convenient to predict the long-term deformation of subgrade soils with the new model, given the above factors which are generally available in the design and construction as specified by the codes.

Generally speaking it is necessary to collect a large data base from cyclic tests to generate a model for the long-term deformation prediction. Yet the probabilistic approach makes it possible to use a smaller data base for developing the empirical model proposed in this paper. It is advisable to use the new model with prediction error (± one standard deviation of model prediction) to estimate the deformation with lower upper bounds. In addition, three example applications of the new model were presented in detail, which would contribute to an improved engineering practice on the subgrade maintenance of the upgrading-speed railway lines.

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