

# Prediction of Permanent Deformation of Pavement Base and Subgrade Materials under Accelerated Loading

Zhong Wu<sup>1+</sup> and Xingwei Chen<sup>1</sup>

**Abstract:** Results from an accelerated pavement testing (APT) experiment indicate that chemically stabilized base and subbase layers in a thin asphalt pavement can contribute significant amounts of permanent deformation to the total surface rutting. In this study, a finite element (FE) prediction model was developed to simulate the rutting performance of thin-asphalt pavements under accelerated loading. A unified, permanent deformation model was proposed as a constitutive material model for the rutting prediction of various pavement materials in the FE analysis. In general, the FE predicted that rutting developments match well with the APT measured results. However, to perform an effective FE simulation analysis and obtain close prediction results for an APT experiment, some special techniques introduced in the FE analysis of this study are deemed necessary.

**Key words:** Accelerated pavement testing; Finite element, Permanent deformation, Stabilized pavement materials, Thin asphalt pavement

## Introduction

Permanent deformation, or rutting, is one of the major distresses found in asphalt pavement. Surface rutting results from the accumulation of load-induced permanent deformation developed from all individual pavement layers, including the subgrade. When a surface asphalt layer is thin, a large percentage of surface rutting will develop from the underneath pavement layers, such as from the base, subbase and subgrade. To determine how a new material or pavement structure performs under a real roadway condition, an accelerated pavement testing (APT) experiment is usually utilized [1-3]. The main advantage of using an APT experiment lies in its ability to apply accelerated truck loads to a real pavement structure and fail the pavement in a short time period. However, running an APT experiment is expensive. It requires a costly accelerated loading device, construction of full-scale pavement test sections, and man-power. Clearly, it is neither practical nor economical to test all new pavement structures and materials using APT experiments. In order to maximize the benefit from an APT study and utilize APT results to evaluate other pavements with similar structural configurations, a finite element predictive model that can simulate the APT tests is an essential need. With such a model, pavement distress prediction functions, as well as laboratory material models, can be calibrated and verified directly based on field APT test results.

The most essential component of a finite element (FE) predictive model is the constitutive model. A constitutive model is a mathematical approximation of a particular material's stress-strain response. In literature, many permanent deformation models have been developed for hot-mix asphalt (HMA) materials, such as a three-parameter creep model [4-6], a nonlinear viscoelastic shear

deformation model [7], and a modified plastic model [8, 9]. It was reported that the three-parameter creep model was able to predict not only accumulative rut depths but also that an upheaval transverse profile developed under a single- or dual- tire APT loading [4, 5]. However, this model is indeed a theoretical approximation that uses creep strain instead of plastic strain in the prediction of permanent deformation.

For unbound granular materials, conventional plasticity models with isotropic hardening (e.g. Elastic-plastic model, Drucker-Prager model, Mohr-Coulomb model, etc.) usually work well in a FE analysis for monotonic loading. As for repeated loading, different plasticity models are required. Recently, some advanced elastic-plastic models have been studied for unbound granular materials in the permanent deformation prediction under repeated loading, such as the Shakedown model by Chazallon *et al.* [10] and the mechano-lattice model by Yandell [11].

However, very few studies have been found for the rutting prediction of chemically stabilized materials. A chemically stabilized material, such as cement stabilized soil, is generally considered to be only susceptible to cracking, not rutting. Even in the newly developed *Mechanistic –Empirical Pavement Design Guide* (M-E PDG) [12], no permanent deformation model has been introduced for chemically stabilized materials. However, the accumulation of permanent deformation has been observed for stabilized base materials under repeated loading [12-14].

This study developed an FE model to simulate the rutting performance of thin-asphalt pavements tested under an APT experiment. A unified permanent deformation model was proposed as the constitutive material model for rutting predictions of various pavement materials in the FE analysis.

## APT Test Sections and Performance

The APT experiment considered in this study includes six full-scale pavement test sections. Normal construction practice was followed

<sup>1</sup> Louisiana Transportation Research Center, 4101 Gourrier Ave, Baton Rouge, LA, USA 70808.

<sup>+</sup> Corresponding Author: E-mail zhongwu@ltrc.lsu.edu

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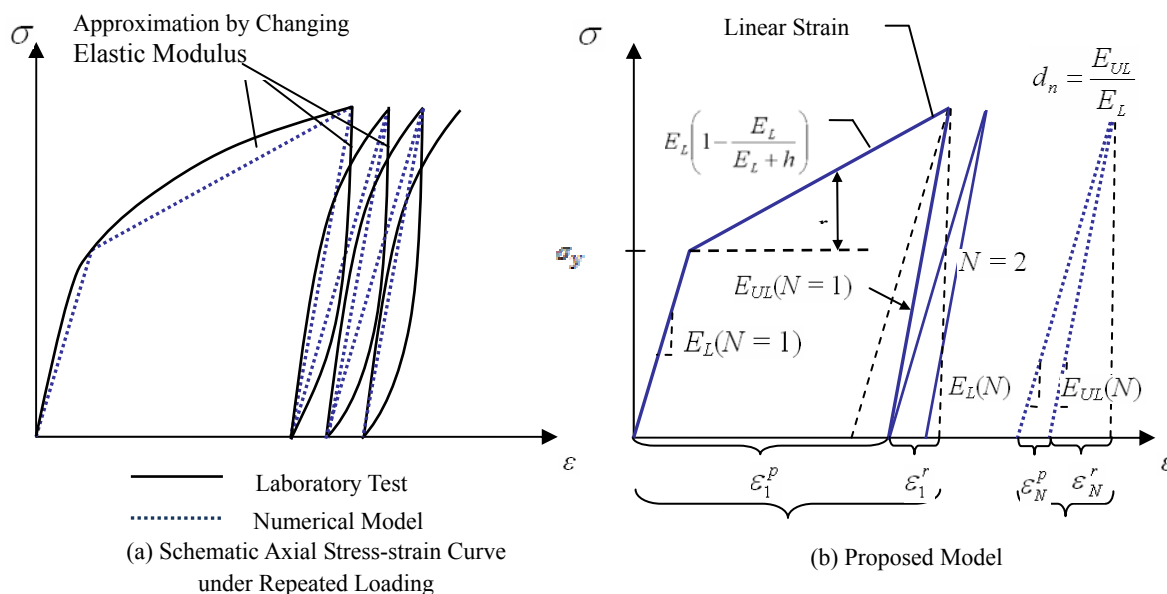


Fig. 1. Schematic Diagram of the Proposed Permanent Deformation Model.

in the construction of test sections. Despite using different base and subbase layers, as outlined below, the six APT sections shared a common pavement structure, including a 51 mm HMA layer, 216 mm base layer, 305 mm sub-base layer and 1.5 m silty-clay embankment subgrade layer (classified as A-6 soil).

Section I: a slag stabilized blended calcium sulfate (BCS) base (hereafter called BCS/Slag) plus a 15 percent lime treated soil working table layer;

Section II: a flyash stabilized BCS base (hereafter called BCS/Flyash) plus the lime treated soil working table layer;

Section III: a crushed stone base plus the lime treated soil working table layer;

Section IV: the same crushed stone base as in Section III plus an eight percent cement treated subbase;

Section V: a foamed-asphalt treated base I (hereafter called FA/50RAP) plus the same cement treated subbase;

Section VI: a foamed-asphalt treated base II (hereafter called FA/100RAP) plus the same cement treated subbase.

This APT experiment used an accelerated loading device called the Accelerated Loading Facility (ALF). The ALF device is a 33 m (100 ft) long accelerated loading device originally developed in Australia. The ALF wheel assembly models one-half of a single axle with dual tires and a loading speed of 17 km (10.5 miles) per hour. The dual tires mounted on the ALF machine for this study were the MICHELIN radial 11R22.5 tires with an inflation pressure of 724 kPa (105 psi). The beginning ALF load used was 43.4 kN (9,750 lbs). During ALF loading, each test section was monitored using one Multi Depth Deflectometer (MDD) and two pressure cells to measure load-induced layer deformations and compressive stresses, respectively. The instrumentation measurement data were collected at every 25,000 ALF repetitions. Note that the entire APT experiment was conducted under natural southern Louisiana conditions. No environmental control was applied. The APT results generally indicated that all sections except Section VI failed due to

excessive surface rutting. No visible surface cracks were observed on those sections at the end of testing. Post-mortem trench results further indicate that there are no signs of any fatigue in the underneath stabilized base or subbase base layers. Section VI was classified as a fatigue cracking failure in the HMA layer with a corresponding average rut depth of 9 mm. Therefore, the above APT results generally indicate that chemically stabilized materials, such as those various base/subbase used in test sections, are not plasticity-free materials. Those materials did contribute a large amount of permanent deformation to the surface rutting. Therefore, how to model the plasticity of those materials and predict their permanent deformation performance in a pavement structure needs to be studied.

### FE Simulation of APT Test

#### FE Material Models

For the purpose of an FE simulation analysis of an APT pavement section, a unified permanent deformation (PD) material model was developed [15]. This model can be used for chemically stabilized layers, as well as for other unbound or bounded pavement layers.

As outlined in Fig. 1, the developed PD model was formulated based on the stress-strain response of a material under repeated loading in a permanent deformation test [16], in which the unloading stress-strain path is usually steeper than the loading path during the same loading cycle. It consists of a conventional elastic-plastic model with linear strain hardening for the first cycle of loading. The Von Mises equivalent stress and a linear strain hardening rule are used in the determination of the initial yielding condition and hardening during the first load cycle. The subsequent loading and unloading cycles after yielding are simulated by linear loading/unloading paths with different secant modulus values. The secant modulus, as shown in Fig. 1, is defined by a set of linear

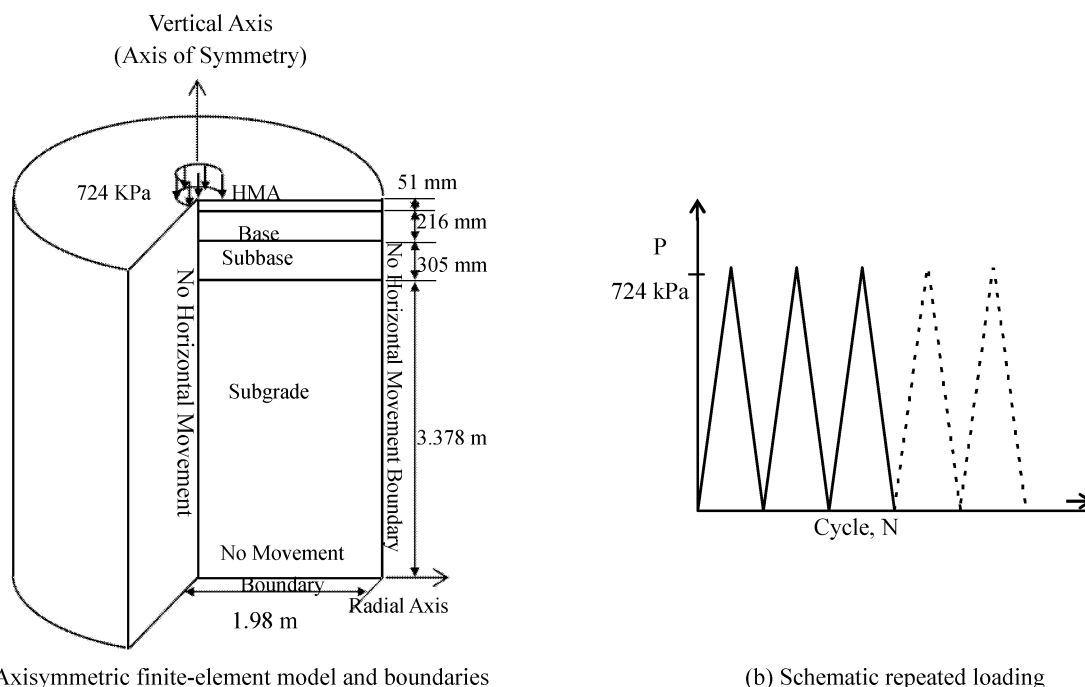


Fig. 2. Axisymmetric Finite Element Model.

slopes in a stress-strain diagram, in which the straight lines are used to simplify the curved paths of loading or unloading. If no yielding is reached during the first cycle, the material element stays in an elastic condition and no plastic strain will be predicted.

The proposed PD model was implemented into the ABAQUS through a user-defined UMAT FORTRAN subroutine. It requires four sets of input parameters (as illustrated in Fig. 1): initial yield stress ( $\sigma_y$ ), hardening constant ( $h_c$ ), loading modulus ( $E_L$ ), and modulus ratio function ( $d_N$ ). Note that all input parameters can be obtained from a permanent deformation test. More details may be referred to elsewhere [15, 17]. Specifically, the modulus ratio function,  $d_N$ , is expressed in the following exponential form based on the PD test results [15]:

$$d_N = \frac{a}{N^b} + 1 \quad (1)$$

where  $a$  and  $b$  are permanent deformation parameters, and  $N$  is the cycle number.

### The Developed FE Model

Fig. 2(a) presents the FE model developed for the rutting prediction of APT test sections in this study. As shown in the figure, the FE model has a two-dimensional size of 3.95 m by 1.98 m and uses an eight-node axisymmetric CAX8R element available in ABAQUS [18]. Both horizontal and vertical boundaries were restrained by roller supports, and the horizontal boundary used one-half of the APT section width. An axisymmetric pavement model was selected primarily due to its advantageous fast processing, as it does not require an excessive computational time. It was also adopted by other researchers in simulating the performance of the asphalt pavement [19-21].

As shown in Fig. 2(b), a repeated load with a triangle shape was applied sequentially to simulate the APT repeated loading in the FE rutting simulation analysis. Note that using the sequential loading scheme in an FE pavement analysis can simulate a moving load based on the number of load repetitions instead of loading times. A similar approach was adopted by Desai [22] and Saad *et al* [23].

### Material Model Inputs

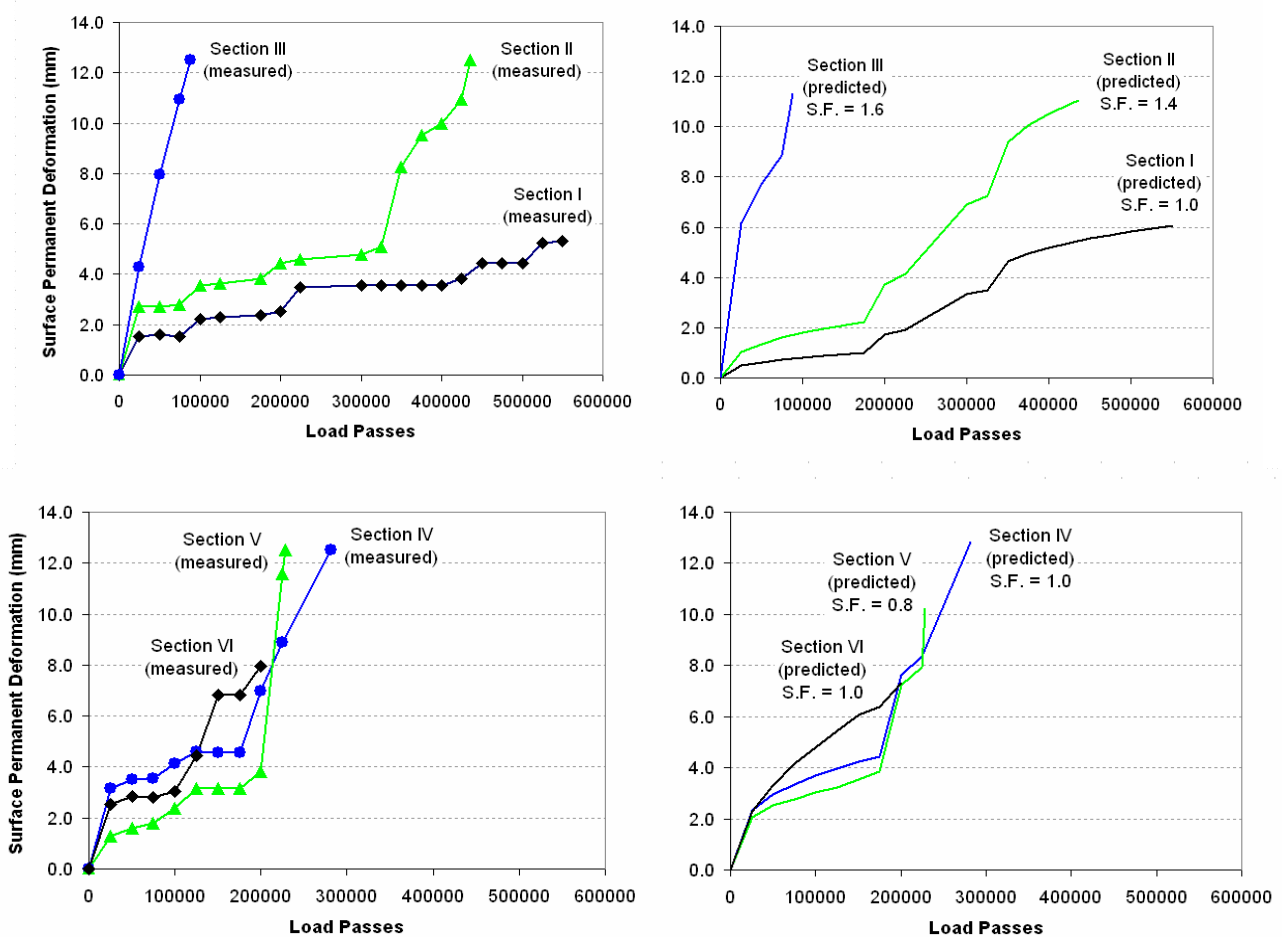
The aforementioned PD material model was applied for all pavement materials/layers of APT test sections in the FE analysis. Table 1 presents the corresponding material model input parameters for each pavement materials considered.

For the rutting prediction of an HMA layer, an ideal material model should include both visco-elasticity and plasticity components. However, due to the following considerations the proposed plasticity model was also used for this layer: 1) The main focus of this study is on the prediction of permanent deformation developed in the base and subbase layers; 2) Inclusion of another HMA type of permanent deformation model in the FE analysis would require a lot more computational time; 3) The HMA layer is only 51 mm thick, thus its rutting contribution to the total surface rutting should be very limited (as already proven by the APT results). Therefore, a set of model input parameters were selected using a trial-and-error process for the HMA layer to make sure that the FE predicted rut depths of this layer are considerably small and similar to the field observation.

All other model input parameters shown in Table 1 were determined through laboratory permanent deformation tests [15, 16]. Due to stress-dependency [24], different material inputs were selected for the crushed stone base used in sections III and IV. It should be pointed out that temperature dependency of the asphaltic

**Table 1.** FE Model Inputs Used for Simulation of APT Test Sections.

Materials	Proposed Permanent Deformation Model						Accelerated Analysis Parameter B
	$E_L$ (MPa)	$\sigma_y$ (kPa)	$h_c$ (MPa)	$d=a/N^b+1$		Poisson's Ratio	
				$a$	$b$		
HMA	689	89.6	1,302	0.0100	0.60	0.35	0.2
BCS/Slag	882	86.5	1,069	0.0059	0.67	0.3	0.31
BCS/Fly Ash	593	84.2	1,028	0.0073	0.56	0.3	0.41
Limestone (Section III)	130	73.7	487	0.1200	0.62	0.3	0.31
Limestone (Section IV)	228	73.7	487	0.0960	0.64	0.3	0.31
FA/50RAP	191	70.6	317	0.1250	0.62	0.35	0.32
FA100 RAP	174	69.2	335	0.1560	0.63	0.35	0.32
Cement-treated Soil	446	34.2	465	0.0027	0.56	0.3	0.24
Lime-treated Soil	210	31.9	675	0.0052	0.63	0.3	0.36
Subgrade	49	29.0	50	0.0340	0.59	0.45	0.47



**Fig. 3.** Measured and Predicted Surface Permanent Deformation of APT Test Sections.

materials were also considered in the FE simulation analysis, and the following model [25] was used in the temperature correction of both HMA and foamed asphalt base layers:

$$E_{T_w} = E_{T_c} / [(1.8T_w + 32)^{2.4462} * (1.8T_c + 32)^{-2.4462}] \quad (2)$$

where  $T_w$  is the temperature to which the modulus of elasticity is adjusted ( $^{\circ}C$ ),  $T_c$  is the mid-depth temperature at the time of FWD data collection ( $^{\circ}C$ ),  $E_{T_w}$  is the adjusted modulus of elasticity at  $T_w$  (MPa), and  $E_{T_c}$  is the measured modulus of elasticity at  $T_c$  (MPa).

**Results and Discussions**

Fig. 3 presents the measured and predicted total rut depths (or surface permanent deformations) of the six APT test sections. As can be seen in the figure, the predicted permanent deformations were in reasonably good agreement with the field measured results when a “shift factor” (ratio between the measured and FE predicted permanent deformation) was applied to each predicted value. Note that the field measured total rut depths were the average values of eight measurement stations. In this study, the shift factors (S.F.)

ranged from 1.0 to 1.6 with an average value of 1.13, indicating a slight difference between lab and field conditions. Fig. 4 presents the correlation between the measured and predicted surface rut depths for all data points evaluated. It was found that a linear relationship with no intercept ( $Y = \beta X$ ) existed between the measured and predicted rut depths with R-square of 0.86, indicating no bias, shift or offset from the line of equality.

Fig. 5 presents the percentage comparison between the MDD measured and FE predicted individual layer contribution to total rut depth for different base/sub-base and subgrade materials. Note that only four MDD data were available for this analysis. As shown in the figure, the predicted rutting contribution due to a base layer ranges from 61% to 79.5%, compared to the measured range of 63% to 76.5%. Similarly, the predicted PD contribution due to a lime treated soil layer and subgrade matched closely to the measured contribution. However, the FE model tends to slightly under-predict the rutting contribution due to a cement treated layer, as shown in Fig. 5.

In summary, both the predicted layer rutting contribution and total rutting progression were found to closely match those measured values. This indicates that the selected PD material model, as well as the developed axisymmetric FE model, can work well in the prediction of permanent deformation for various base/subbase and subgrade materials, including chemically stabilized materials, under the APT loading condition of this study.

### Key Techniques Considered in FE Simulation Analysis

It is worthy noting that, to complete the above FE analysis and obtain reasonably close prediction results, the following simulation techniques must be used.

### Accelerated Analysis Procedure

The FE analysis of a pavement structure under repeated loading is a time-consuming process. Therefore, an accelerated analysis procedure is desired. Desai and Whitenack [26] developed a procedure to accelerate the FE analysis based on an assumption that no inertia under dynamic loading would occur during the repeated load application. In the procedure, the FE analysis would be performed only for selected initial cycles, and the growth of plastic strains after the initial cycles could be evaluated by extension based on an empirical relationship between plastic strains and the number of cycles obtained from laboratory test data.

A similar accelerated analysis procedure was employed in this study. Basically, only the permanent deformation developed at a reference cycle number ( $N_r$ ) needs to be predicted using the FE analysis. After that, the growth of permanent deformation may be predicted through the following Eq. (3). Note that the parameter B in Eq. (3) is a material-specific parameter determined from the laboratory permanent deformation test. The B-values for each material used in the FE analysis are presented in the last column of Table 1. The reference number  $N_r$  can be determined by choosing a connection point in the FE predicted permanent deformation curve where the growth rate at that point is equal to the parameter B.

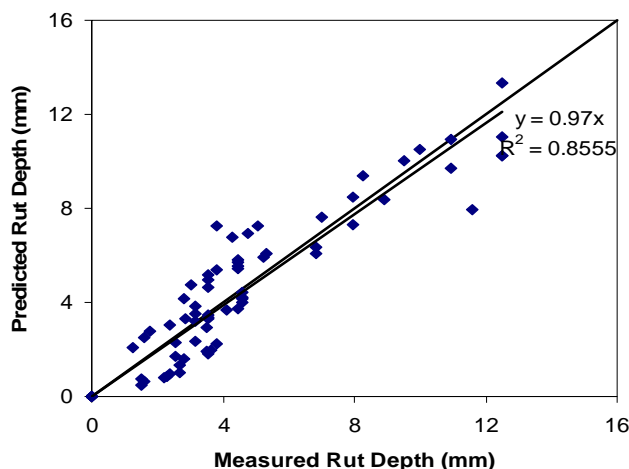


Fig. 4. Correlation between Measured and Predicted Surface Rut Depths for APT Sections.

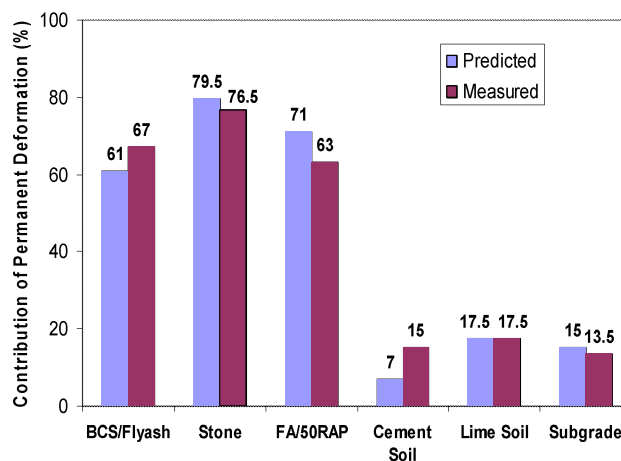


Fig. 5. Measured and Predicted Layer Contribution of Permanent Deformation.

$$PD(N) = PD(N_r) \left( \frac{N}{N_r} \right)^B \tag{3}$$

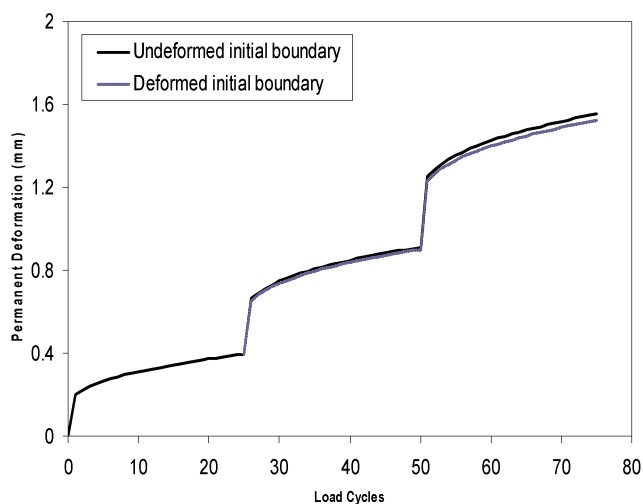
where  $PD(N)$  is the permanent deformation for pavement layer/sublayer after  $N$  cycles,  $N_r$  is reference cycles,  $N$  is the number of load cycles,  $PD(N_r)$  is the ABAQUS calculated permanent deformation for pavement layer/sub-layer after  $N_r$  cycles.

### Load Level Effects

Four different ALF load levels were used during the APT experiment of this study. Load I of 43.4 kN was applied for the first 175,000 cycles, Load\_II of 53.6 kN was used between 175,000 and 225,000 cycles, Load\_III of 63.8 kN was set between 225,000 and 325,000, and Load\_IV of 74.1 kN was run after 325,000 cycles [14]. In the FE simulation analysis of each test section, the permanent deformation curves under different load levels were first predicted separately using the built FE simulation model, and then extended based on Eq. (3). To obtain the cumulative pavement damage,

**Table 2.** Wander Adjustment Factor for APT Sections (Percentage).

Load Level	Section I	Section II	Section III	Section IV	Section V	Section VI
Load_I	71.6	70.9	68.7	68.9	68.3	68.3
Load_II	76.3	75.8	72.2	72.8	71.6	71.2
Load_III	79.7	79.1	75.0	75.9	74.4	73.7
Load_IV	82.2	81.6	77.2	78.2	76.6	75.7

**Fig. 6.** Predicted Permanent Deformation with Un-deformed and Deformed Initial Boundaries.

the predicted permanent deformation curves of different load levels were made direct connections (in a tail-to-head fashion) at specified loading cycles where the load magnitudes were increased. It is understood that such a direct connection of load-deformation curves can underestimate the effect due to the change of load levels on the deformed FE meshes. To investigate this effect, twenty-five cycles of FE analysis were conducted on both deformed and un-deformed boundary conditions under different load levels. As shown in Fig. 6, the permanent deformation results from the un-deformed meshes, which represent the direct connection of permanent curves as used in the FE simulation analysis, were found slightly higher (about 2 percent) than those from deformed meshes after three different load levels were applied. However, the difference was relatively small.

### Wander Effects

During the APT experiment, a 381 mm normally distributed wander function was applied to the ALF loading. Epps *et al.* [27] presented the effect of lateral wander on test sections at WesTrack and concluded that the wander effect must be accounted for when quantifying the rutting performance under APT loading. In this study, the following equation was developed to compute an adjusted factor for the predicted permanent deformation due to the wander effect:

$$R = \sum F_x \cdot RD_x \quad (4)$$

where  $R$  is the wander adjustment factor,  $x$  is the load offset from the center line,  $F_x$  is the load frequency with offset  $x$ , and  $RD_x$  is the

ratio of rut depth with load offset  $x$  and offset zero.

Table 2 presents the wander adjustment factors under different load levels for six APT sections. Note that the wander adjustment factors have been included in the precedent FE simulation results.

### Summary and Conclusions

Finite element simulation analysis was performed on six thin asphalt sections under accelerated loading. These sections include five base materials and two treated soil mixtures. A unified permanent deformation material model was developed for the rutting prediction of various pavement materials in an FE pavement analysis. In general, the FE predicted surface permanent deformations were found in reasonably good agreement with the field measured results with an average shift factor of 1.13. In addition, the predicted base layer rutting and the corresponding permanent deformation contribution of individual layers were also found to closely match the field measurement results. To perform an effective FE simulation analysis for an APT experiment, some special techniques were recommended. Overall, the developed permanent deformation model shows promising results in an FE analysis of the rutting prediction for various pavement materials, including chemically stabilized materials.

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