Theoretical Analysis of Extreme Temperatures and Thermal Stresses in Bridge Deck Pavements

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Abstract: Thermal stresses at the extreme temperature conditions can cause various pavement distresses. These distresses may degrade the structural performance of bridge deck pavements (BDP), such as, upheaval, low-temperature cracking, reflective cracking, etc. It is essential to understand the influence of the extreme temperatures on the pavement distresses. Based on the analytical solutions of temperature in BDP, the simplified formulae for calculation of stresses for the extreme temperatures (maximum and minimum temperatures) in BDPs were developed in this study. The calculated results obtained from the developed simplified formulae were then compared with the data measured in the field. The differences between them were found to be within $2\,°C$. In addition, this paper also analyzed the thermal stresses at the extreme temperatures in BDP. The sample bridge deck and asphalt concrete pavements were analyzed. The maximum principal and shear stresses in the BDP at the wide range of temperatures were calculated using 3D finite element method. The compared results showed a strong linear correlation between the BDP maximum principal stress and the elastic modulus. This linear relationship also existed between the shear stress and the elastic modulus. From the finite element analysis, it is found that the BDP elastic modulus affects the thermal stress more than the thickness of BDP. With the consideration of thermal stresses of BDP, the thickness of BDP from 6 to 12 cm is recommended for the BDP construction.

Key words: 3D finite element method analysis, Bridge deck pavement, Extreme temperatures, Thermal stress.

Introduction

Bridge deck pavement (BDP) plays an important role in protecting the bridge deck. It prevents the bridge deck from moisture damage. BDP also disperses the concentrated load, bears the bending moment and resists the bridge deck deformation [1]. The temperature distribution in the BDP is a factor that affects the structural performance of BDP. For example, the asphalt in the asphalt concrete pavement is sensitive to the ambient temperature. The asphalt softens and the mixture strength decreases with rising temperature. The rising temperature could cause pavement distresses such as flushing, upheaval and rutting. Also the thermal stress in asphalt concrete pavement is higher at lower ambient temperatures. The BDP interlayer temperature varies with the ambient temperature. When the ambient temperature is low, the surface-layer temperature of the BDP will be lower than bottom-layer, contraction deformation will occurred under the larger temperature gradient this is known as the pavement distresses at low-temperature cracking and reflective cracking [2]. The destructive effect due to extreme temperatures (the maximum temperature and the minimum temperature in the context) needs to be considered in the BDP structure design and material selection. Most researches that have been done are related to temperature-induced stresses in pavements and load-induced stresses

in BDP. Rasmus [3] calculated the shear stress among layers of BDP and obtained a relationship between the load level and the cracking behavior of the materials. Xu [4] conducted research on vehicle load-induced stress of compound BDP and analyzed the load stress variation of BDP. Chen [5] analyzed the difference between temperature field of highway and that of BDP. According to his research, the maximum temperature of BDP was lower than that in the highway. Lu [6] simulated the temperature field of BDP and confirmed that his model can be used to calculate the temperature field of BDP accurately. However, few studies focused on the calculation of the extreme temperatures and the analysis of thermal stress in BDP. In this study, based on the analytical solutions of temperature in BDP, the simplified formulae for calculation of stresses for the extreme temperatures (maximum and minimum temperatures) in BDPs were developed. In addition, the maximum principal and shear stresses in the BDP at the wide range of temperatures were calculated using 3D finite element method. The results obtained from this study supplement and enhance the mechanics theory of BDP. They also can serve as guides in the structural and material design of BDP.

Analytical Solution for the Extreme Temperature of BDP

The analytical solution is the mathematical formula for calculating pavement temperature and sun radiation, based on the fundamental principles of meteorology and thermodynamics, with the mechanism of heat-transfer between pavement and atmosphere. Song [7] calculated the pavement temperature field with this method, and obtained the theoretical formulas for calculating the highest and the lowest pavement temperatures, the temperature gradients.

Analytical solution for the extreme temperature of BDP can provide the basis for temperature range on the Finite Element Method Analysis.

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The BDP thermo-physical parameters mainly consist of thermal conductivity λ_i, thermal diffusivity α_i, thickness h_i, which determine the temperature distribution function $T_i(z,t)$.

The heat transfer in each layer of BDP can be described with Fourier's equation.

$$\frac{\partial T_i}{\partial t} = \alpha_i \frac{\partial^2 T_i}{\partial z^2} \qquad \sum_{j=1}^{i-1} h_j < z \leq \sum_{j=1}^{i} h_j \quad (h_0 = 0, i = 1, 2, 3 \ldots \ldots n) \quad (1)$$

And the boundary conditions of BDP will be（Eqs. (2) and (3)),

$$T_n(z,t) = C \qquad (z = \sum_{i=1}^{n} h_i) \qquad (2)$$

where:
z = depth of BDP (m)
t = time (s)
C = constant.

$$-\lambda \frac{\partial T_1}{\partial z} = B\left(T_a + \frac{Q_J}{B} - T_1\right) \quad (z = 0, T_a: \text{Environment Temperature}) \ (3)$$

where:
T_1 = daily mean temperature
λ = thermal conductivity
Q_J = solar radiation intensity
B = constant.

The method of separating variables was used to solve Eq. (1); the temperature field can then be expressed as,

$$T_i(z,t) = A_i \exp\left(\sqrt{\frac{\omega}{2\alpha_i}}z + j \cdot (\omega t + \sqrt{\frac{\omega}{2\alpha_i}}z)\right) + B_i \cdot \exp\left(-\sqrt{\frac{\omega}{2\alpha_i}}z + j \cdot (\omega t - \sqrt{\frac{\omega}{2\alpha_i}}z)\right)$$

$$= A_i \exp\left(\sqrt{\frac{\omega}{2\alpha_i}}z\right)\left[\cos\left(\omega t + \sqrt{\frac{\omega}{2\alpha_i}}z\right) + j\sin\left(\omega t + \sqrt{\frac{\omega}{2\alpha_i}}z\right)\right] \qquad (4)$$

$$+ B_i \exp\left(-\sqrt{\frac{\omega}{2\alpha_i}}z\right)\left[\cos\left(\omega t - \sqrt{\frac{\omega}{2\alpha_i}}z\right) + j\sin\left(\omega t - \sqrt{\frac{\omega}{2\alpha_i}}z\right)\right] \qquad (i = 1,2,3..n)$$

The boundary conditions can be substituted in Eq. (4). The analytical solution for the temperature field of BDP under natural boundary conditions can be obtained according to the superposition principle. The formula can be expressed as,

$$T_i(z,t) =$$
$$A_0 + \sum_{j=1}^{4} A_j \cdot B_j \cdot \exp\left(-\sum_{k=1}^{i-1}\sqrt{\frac{\omega}{2\alpha_k}}h_k - \sqrt{\frac{\omega}{2\alpha_i}}(z - \sum_{k=1}^{i-1} h_k\sqrt{\frac{\omega_j}{2\alpha_k}})\right)$$
$$\cdot \cos\left[\frac{\pi}{2} - \left(w_j(t - t_j) - \sum_{k=0}^{i-1}\sqrt{\frac{\omega}{2\alpha_k}}h_k - \sqrt{\frac{\omega}{2\alpha_i}}(z - \sum_{k=0}^{i-1} h_k\sqrt{\frac{\omega_j}{2\alpha_k}}) - \varphi_j\right)\right] \qquad (5)$$

where:
z = depth of BDP (m)
h = thickness of BDP (m)
ω = angular frequency (rad/h)
φ = phase angle
α = thermal diffusivity

$$\alpha = \lambda / \rho c \qquad (6)$$

where:
λ = thermal conductivity
ρ = density of material
c = specific heat.

Taking the partial derivation with time (Eq. (5)), Eq. (7) for extreme temperatures (the maximum and the minimum temperatures) of BDP were developed.

Through using the above equations and the values of material parameters (Table 1) [8], the maximum and the minimum temperatures were calculated and listed in Table 2.

$$\begin{cases} T_{max} = \frac{1}{2}(T_1 + T_2) + 0.00036Q_d \cdot \left(\frac{\bar{t}}{t_{max}}\right)^5 + 0.250(T_1 - T_2) + 0.00079Q_d \\ T_{min} = \frac{1}{2}(T_1 + T_2) + 0.00036Q_d \cdot \left(\frac{\bar{t}}{t_{max}}\right)^5 - 0.300(T_1 - T_2) - 0.00043Q_d \end{cases} \qquad (7)$$

where:
T_1 = daily mean temperature
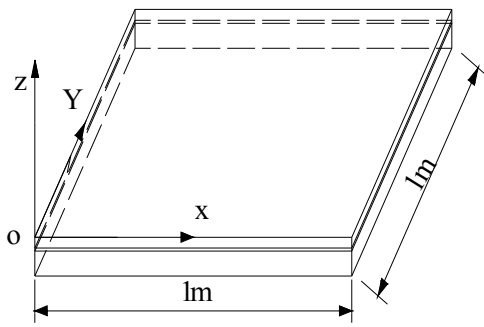T_2 = daily temperature variation amplitude

Table 1. The Typical Values of Material Parameters [8].

Material	ρ /(kg·m^{-3})	c /[J·(kg·K)$^{-1}$]	λ /[W·(m·K)$^{-1}$]	Absorption Ratio of Solar Radiation α_s
Asphalt Concrete	2500	894	1.2	0.87
Cement Concrete	2450	879	1.5	0.76

Table 2. The Calculated and Measured data of the Maximum and Minimum Temperatures of Asphalt Concrete Pavement at Xi'an Ring Road

Time	05/12/2005		05/17/2005		07/01/2005		07/17/2005		07/21/2005	
Type	T_{max}	T_{min}	T_{max}	T_{min}	T_{max}	T_{min}	T_{max}	T_{min}	T_{max}	T_{min}
Calculated T_c	29.3	40.7	40.7	23.2	56.2	27.4	52.6	27.8	54.2	26.4
Measured T_m	28.9	39.6	39.6	23.8	55.0	27.9	51.9	28.5	55.5	28.4
$\triangle T = T_c - T_m$	0.4	1.1	1.1	-0.6	1.2	-0.5	0.7	-0.7	-1.3	-2.0
Time	07/27/2005		12/27/2005		12/30/2005		04/06/2006		04/07/2006	
Type	T_{max}	T_{min}	T_{max}	T_{min}	T_{max}	T_{min}	T_{max}	T_{min}	T_{max}	T_{min}
Calculated T_c	50.8	27.7	10.7	2.4	14.5	2.7	29.8	10.8	36.5	13.9
Measured T_m	51.3	29.0	9.7	2.0	15.6	2.2	31.7	9.2	34.7	15.8
$\triangle T = T_c - T_m$	-0.5	-1.3	1.0	0.4	-1.1	0.5	-1.9	1.6	1.8	-1.9

Fig. 1. The Structure Model of Calculation.
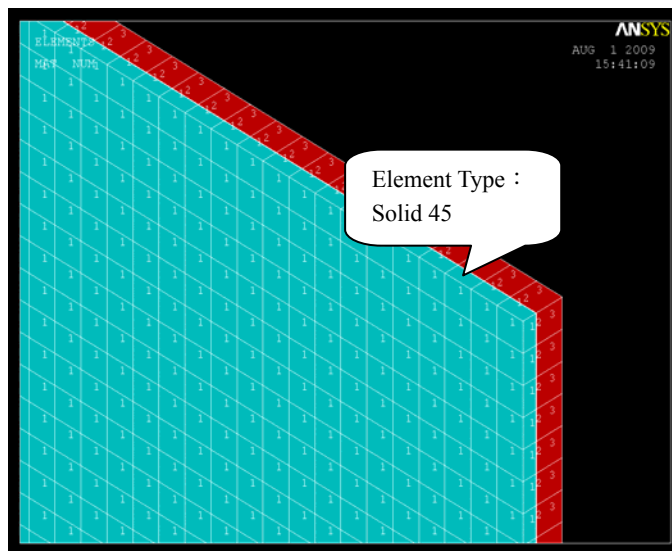

Element Type：
Solid 45

Fig. 2. The FEM Computational Model.

T_{max} = the maximum temperature of asphalt concrete pavement
T_{min} = the minimum temperature of asphalt concrete pavement
Q_d = daily solar radiation.

In order to validate Eq. (7), Wang [9] measured the maximum and the minimum temperatures of actual roads of 12 bridges in Xi'an Ring Road that were used to compare with the calculated results. The comparisons were tabulated in Table 2.

As shown in Table 2, the range of $\left|\Delta T\right|_{max}$ is -1.9℃ to 0.4℃, and the range of $\left|\Delta T\right|_{min}$ is -2.0℃ to 1.6℃. The difference between the calculated and the field measured maximum and the minimum temperatures are within 2℃.
where:
$\left|\Delta T\right|_{max}$ = Calculated $T_{c,max}$ − Measured $T_{m,max}$
$\left|\Delta T\right|_{min}$ = Calculated $T_{c,min}$ − Measured $T_{m,min}$

Finite Element Computation Model in This Study

The structure of BDP can be considered as an elastic system (Fryba, [10]). The system consists of an asphalt concrete pavement layer, a waterproof layer and the bridge deck. Eight-node solid elements were used to simulate the asphalt concrete pavement and the bridge deck, and the three-dimensional membrane element is used for the waterproof layer. Finite element software ANSYS is applied for the calculation. The structure model of calculation is shown in Fig. 1. The following assumptions were used as the basis for the calculation:

1. Each layer is homogeneous, continuous, isotropic and elastic;
2. Interlayer is continuous in both horizontal and vertical directions;
3. The bottom of the bridge deck is restricted from movement in all directions, and the side of bridge deck has the constraints in the horizontal direction;
4. Net weight of the pavement structure is not considered.

These assumptions were considered reasonable and were used by Lu [6] universal in calculation of BDP.

The contact condition between asphalt concrete pavement and bridge deck largely affects the thermal stress (Lu [6]). The thermal stress in asphalt concrete pavement reduced to minimum for smooth interfaces between the deck and BDP. The maximum temperature stress is developed when interfaces are rough. The thermal stress is maximum when completely continuous interfaces in both horizontal and vertical directions are assumed.

Xu [11] concluded that 1m×1m and 20mm×20mm were the reasonable size of structure models and elements used in the calculation of thermal stresses including the maximum principal stress and the maximum shear stress. The type of element is solid 45. The finite element method (FEM) computational model is shown in Fig. 2. These sizes were used in this study. Based on the literature [8], other parameters of each layer used in the calculation are listed in Table 3.

It is more complex to do calculation when both parameters (thickness and elastic modulus) are varied. In this study, during the calculation of the thermal stress of asphalt concrete pavement with varied modulus, the thickness of asphalt concrete pavement assigned is 6cm. During the calculation of the thermal stress of asphalt concrete pavement with varied thickness, the asphalt concrete elastic modulus assigned is 2000 MPa.

Calculation Results and Analysis

The calculation results for thermal stresses are shown in Tables 4 to 7. According to the results of Tables 4 to 7, the thermal stresses of BDP with different moduli and 6cm thickness at 60℃ and -20℃ are illustrated in Figs. 3 and 4, respectively. The thermal stresses of BDP with different thickness and 2000 MPa modulus at 60℃ and -20℃ are illustrated in Figs. 5 and 6, respectively.

Table 3. The Model Parameters for the Calculation Thermal Stresses.

Structure Layers	Thickness (cm)	Elastic Modulus (MPa)	Poisson Ratio	Thermal Conductivity (W/m℃)	Linear Expansion Coefficient (1/℃)
Asphalt Concrete	2,4,6,8,10	8,001,000,150,020,000,000	0.25	1.2	2.1×10^{-5}
Waterproof Layer	0.3	150	0.45	1	1.0×10^{-5}
Bridge Deck	30	30000	0.15	1.5	1.5×10^{-5}

Table 4. Stress of BDP with Different Moduli at 60℃

Modulus of BDP (MPa)	First Principal Stress (MPa)	Third Principal Stress (MPa)	Maximum Shear Stress (MPa)
800	0.22	-1.41	0.82
1000	0.27	-1.76	1.02
1500	0.42	-2.64	1.53
2000	0.54	-3.52	2.03
3000	0.80	-5.28	3.04

Table 5. Stress of BDP with Different Moduli at -20℃.

Modulus of BDP (MPa)	First Principal Stress (MPa)	Third Principal Stress (MPa)	Maximum Shear Stress (MPa)
800	0.47	-0.07	0.27
1000	0.59	-0.09	0.34
1500	0.88	-0.14	0.51
2000	1.17	-0.18	0.68
3000	1.76	-0.26	1.02

Table 6. Stress of BDP with Different Thickness at 60℃.

Thickness of BDP (cm)	First Principal Stress (MPa)	Third Principal Stress (MPa)	Maximum Shear Stress (MPa)
2	0.14	-2.58	1.36
4	0.26	-2.64	1.45
6	0.40	-2.64	1.52
8	0.53	-2.60	1.56
10	0.64	-2.52	1.58

Table 7. Stress of BDP with Different Thickness at -20℃.

Thickness of BDP (cm)	First Principal Stress (MPa)	Third Principal Stress (MPa)	Maximum Shear Stress (MPa)
2	0.86	-0.05	0.46
4	0.88	-0.09	0.48
6	0.88	-0.13	0.51
8	0.86	-0.18	0.52
10	0.84	-0.21	0.52

Maximum Principal Stress = Max (｜First Principal Stress｜, ｜Third Principal Stress｜).

From above Figures, it can be seen that when the surface temperature of asphalt concrete pavement is 60℃ and the elastic modulus is 800 MPa, the maximum principal stress and the maximum shear stress were calculated to be 1.41 MPa and 0.82 MPa respectively. When the elastic modulus increases from 800 MPa to 3000 MPa, the maximum principal stress and the maximum shear stress increase by 275% and 271%. When the surface temperature of asphalt concrete pavement is -20℃ and the elastic modulus is 800MPa, the maximum principal stress and the maximum shear stress are 0.47 MPa and 0.27 MPa respectively. When the elastic modulus increases from 800 MPa to 3000 MPa, the maximum principal stress and the maximum shear stress increase by 274% and 278%. When the surface temperature of asphalt concrete pavement is 60℃ and the thickness increases from 2 cm to 10 cm, the
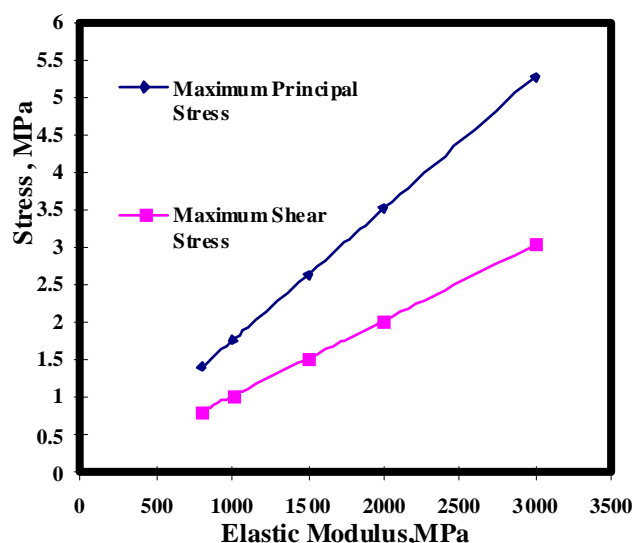

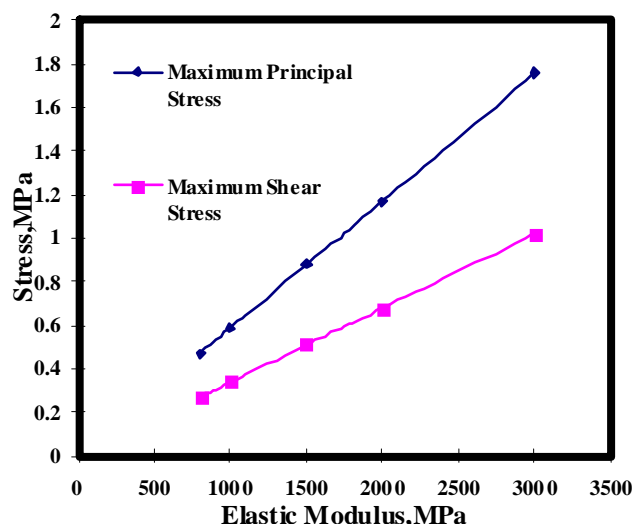**Fig. 3.** Stress of BDP with Different Moduli at 60℃

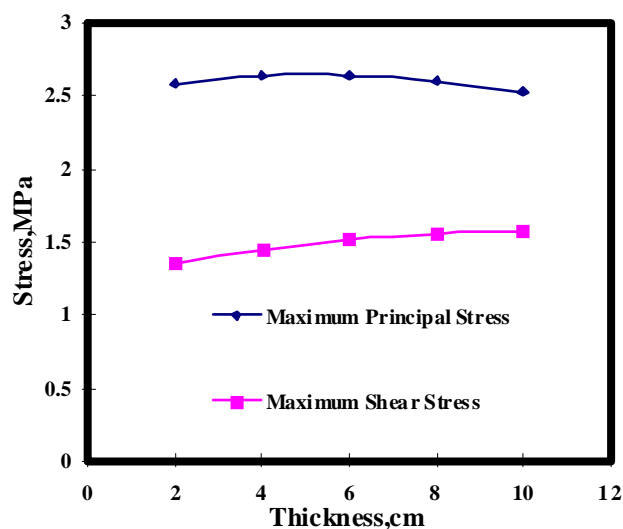**Fig. 4.** Stress of BDP with Different Moduli at -20℃.

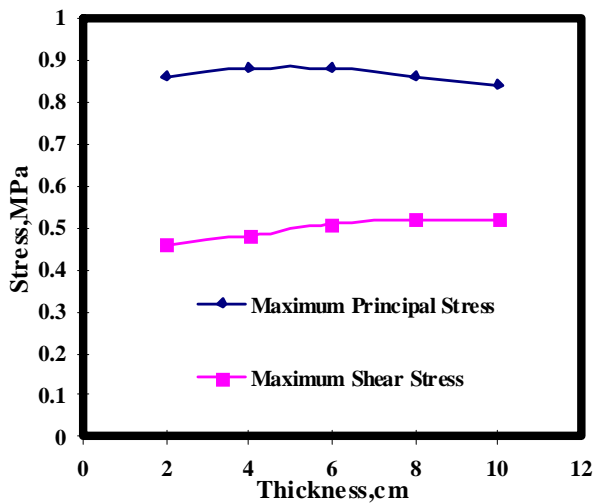**Fig. 5.** Stress of BDP with Different Thickness at 60℃.

Fig. 6. Stress of BDP with Different Thickness at -20℃.

Fig. 7. Maximum Principal Stress of BDP with 6cm Thickness at -20℃.

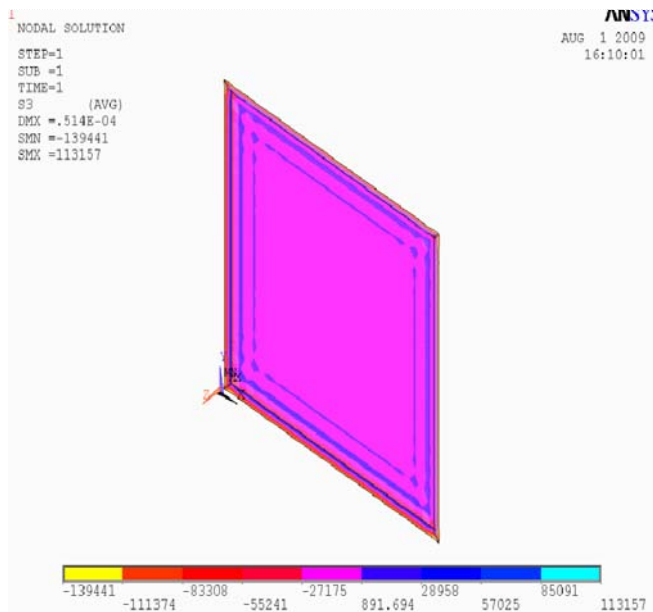**Fig. 8.** Maximum Principal Stress of BDP with 6cm Thickness at 60 ℃.

Fig. 9. Maximum Principal Stress of BDP with 10cm Thickness at -20℃.

maximum principal stress increases from 2.58 MPa (2 cm) to 2.64 MPa (6 cm), then decreases to 2.53 MPa (10 cm), and the maximum shear stress increases from 1.36 MPa to 1.58 MPa. The distribution of BDP maximum principal stresses with 6 cm thickness at -20℃ and 60℃ are illustrated in Figs. 7 and 8, respectively. The distribution of BDP maximum principal stresses with 10cm thickness at -20℃ and 60℃ are illustrated in Figs. 9 and 10, respectively. When the surface temperature of asphalt concrete pavement is -20℃ and the thickness increases from 2 cm to 10 cm, the maximum principal stress increases from 0.86 MPa (2 cm) to 0.88 MPa (6 cm), then decreases to 0.84 MPa (10 cm), and the maximum shear stress increases from 0.46 MPa to 0.52 MPa.

According to the thermo mechanical properties of asphalt concrete, the thermal strain is proportional to the temperature change; the stress is proportional to strain is the modulus keeps a constant; and the stress is 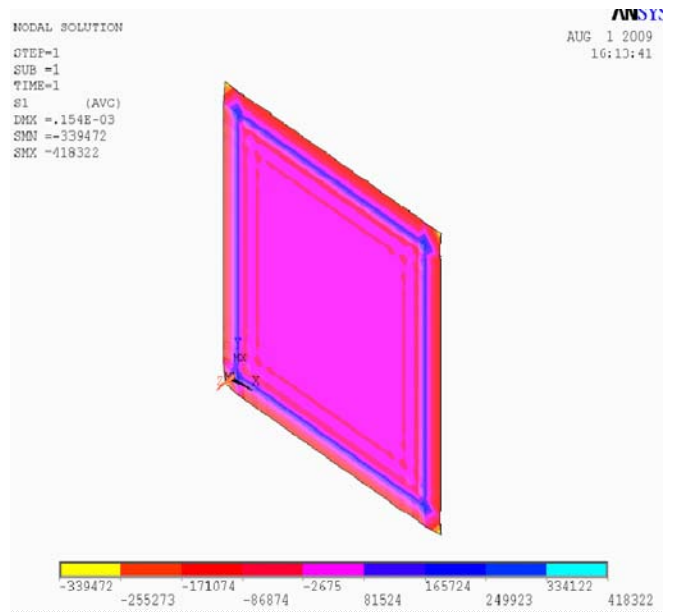proportional to modulus at the same strain. The temperature of BDP is higher when the surface temperature is 60℃, it causes the larger strain and the higher thermal stress. When the elastic modulus of asphalt concrete is larger, the thermal stress with the same strain is larger [12]. It is clear that the maximum principal stress and the maximum shear stress at 60℃ are much larger than those at -20℃. According to Figs. 3 and 4, the maximum principal stress and the maximum shear stress have a linear relationship with the elastic modulus of asphalt concrete pavement as shown in Eq. (8).
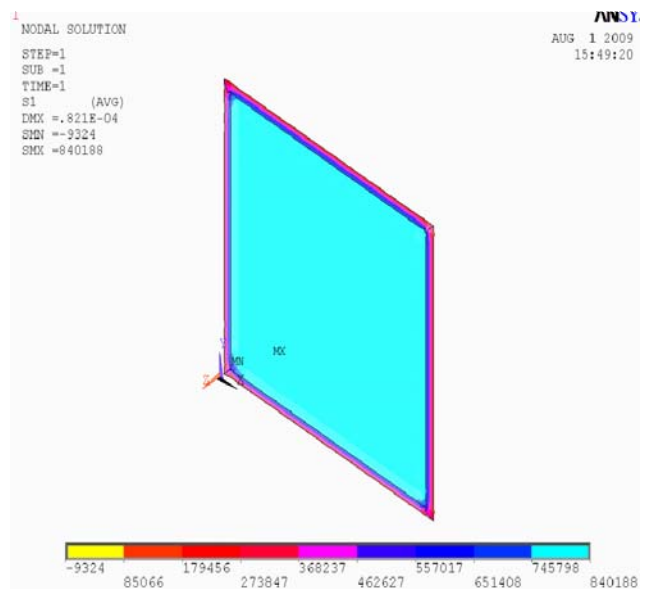
$$\begin{cases} S_{MP}=0.0018E+0.0013 \quad, \quad S_{MS}=0.001E+0.0099 \quad (60°C) \\ S_{MP}=0.0006E+0.0022 \quad, \quad S_{MS}=0.0003E-0.0013 \quad (-20°C) \end{cases} \quad (8)$$
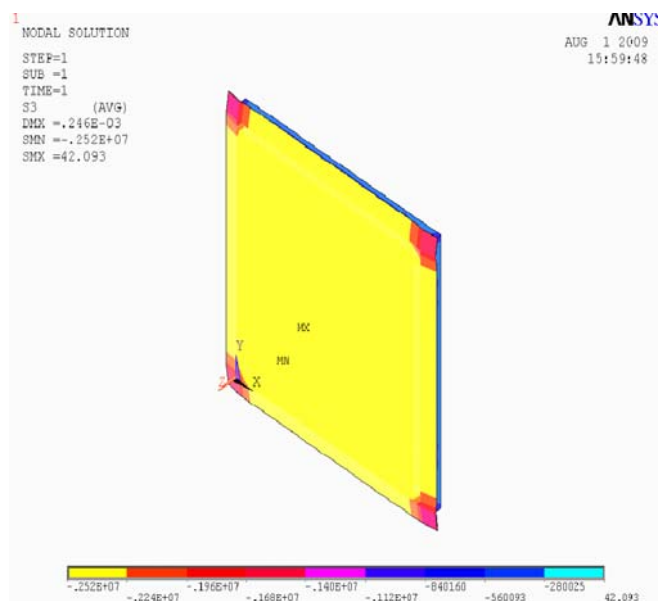
where:

Fig. 10. Maximum Principal Stress of BDP with 10 cm Thickness at 60℃.

S_{MP} = the maximum principal stress (MPa)
S_{MS} = the maximum shear stress (MPa)
E = the elastic modulus (MPa).

It can be seen that the maximum principal stress varies with the thickness of asphalt concrete pavement (Figs. 5 and 6). It initially increases, peaks at thickness of 6 cm, then decreases with the thickness. The maximum shear stress increases with the thickness of asphalt concrete pavement. There are two reasons here. First, the stress resistance increases with the BDP thickness, then the thermal stress decreases. Second, the temperature gradient increases with the BDP thickness, then the thermal stress increases. The interaction of the two sides causes the variation of thermal stress. Considering the BDP construction requirements, Zhang [13] suggested the BDP thickness should be not less than 4cm, and it is not economical for the deck with the BDP thickness larger than 12cm. Based on the consideration of BDP mechanics, construction and economy, the thickness of BDP between 6 and 12 cm is considered effective in controlling the thermal stress of BDP.

Conclusions

The simplified calculation formulas for the maximum and minimum temperatures in BDP were developed by solving the Fourier heat transfer equation. The difference between the calculated temperatures and the field measured temperatures was less than 2℃.

The BDP thermal stress under different moduli and thicknesses of BDP were presented in the paper. The BDP thermal stress varies with the ambient temperature. The BDP elastic modulus affects the thermal stress more than the thickness of BDP. From the finite element analysis, a linear relationship exists between the BDP maximum principal stress and the elastic modulus. This linear relationship was also found between the shear stress and the elastic modulus.

Based on the consideration of BDP stress and its distribution, the

thickness of BDP between 6 cm to 12 cm is considered effective in controlling the thermal stress of BDP.

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