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Layered Flexible Pavement Studies: Challenges in Forward and Inverse Problems

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Abstract: Layered flexible pavements pose great challenges to pavement engineers and to mathematicians. Current technical issues on forward and inverse (backcalculation) problems in layered flexible pavements are first briefly reviewed. Then the newly developed forward program, *MultiSmart3D,* and backcalculation program, *BackGenetic3D,* are presented. The *MultiSmart3D* is based on the novel cylindrical system of vector functions combined with the propagator matrix method. In the forward calculation, any number of observation points can be assigned to the layered pavement in any location and the pavement can be basically made of any number of layers (while the existing programs are all limited to 26 output points and 20 sublayers/layers). In the application, the temperature-dependent modulus variation with depth is used as input, which requires that the pavement be subdivided into many sublayers (over 500). It is shown that the difference of the pavement responses based on the real modulus variation with depth and those based on the averaged moduli can be substantial.Consequently, the corresponding fatigue and rutting lives based on these two pavement profiles are remarkably different. *BackGenetic3D* is based on the forward program, *MultiSmart3D,* with backcalculation being carried out by the powerful genetic algorithm. To make this program flexible, optimal, and practical, a new dynamic parameterless genetic algorithm is developed and implemented into *BackGenetic3D*. Numerical examples show that not only is the proposed inverse program reliable and efficient, but it can also super perform over other existing backcalculation programs.

***Key words**: Backgenetic3D; forward problem; inverse problem; layered flexible pavement; MultiSmart3D.*

Introduction

It is reported that about 93% of the paved roads in the US are composed of flexible pavement [1]. Thus to improve the performance and serviceability of flexible pavement and reduce the cost to taxpayers, thorough research on flexible pavement behaviors is always highly demanded. However, due to a large number of practical factors in pavement engineering, full understanding of the mechanical behaviors of the flexible pavement is far from satisfactory.

For the flexible pavement, the total pavement structure will deflect, or flex, under loading. A typical flexible pavement structure consists of surface layer, base, and subgrade. Such a pavement structure can be modeled and studied either analytically based on the multilayered elastic theory (MET) or numerically based on the finite element method (FEM), finite difference method (FDM), or discontinuous element method (DEM). Each of these layers usually has its own elasticity parameters such as the elastic modulus and Poisson's ratio.

The forward problem is to solve the responses of the pavement under certain loading, assuming that all input parameters, such as the elastic modulus and Poisson's ratio, are known. However, these parameters, especially the elastic modulus, are unknown in advance and their exact values will never be known. Thus it is of prior importance to obtain these parameters and such an operation is called the inverse problem, also the backcalculation problem. There are different methods of estimating these unknowns, including laboratory testing and in-situ nondestructive testing such as the Falling Weight Deflectometer (FWD) test.

Burmister [2, 3] pioneered the forward problem in layered pavements. However it should be stressed that Burmister only considered pavements consisting of two or three layers and therefore his method would be limited in pavement analysis. For example, based on the Burmister's solution, the program BISAR developed by De Jong et al. [4] can only analyze a three-layer pavement system.

It has been found that the resilient modulus of the pavement materials is affected by the environment, including the temperature profile, pavement drainage and moisture, frost, and pavement compaction, which vary with depth [5]. Consequently the resilient modulus will vary with depth, and thus assuming a single modulus of elasticity for every pavement layer will be unreasonable. The FEM, FDM, and DEM are capable of solving this inhomogeneous problem, but the involved algorithms are usually very complicated and time-consuming. An efficient method is to subdivide the inhomogeneous elastic layer into many sublayers, each with a constant modulus. In so doing, the conventional two- or three-layer pavement will be expanded to a multilayered pavement. It is obvious that the more sublayers there are, the more accurately the modulus profile will be captured. To solve the resulting multilayered system, an innovative program, *MultiSmart3D*, is developed which can handle an unlimited number of layers/sublayers as compared to the current flexible pavement programs where the layered system is limited to a maximal number of 20 layers/sublayers [5].

Compared to the forward problem, the inverse problem is far from perfect since the actual modulus will never be available and one can only rely on the engineering experiences. The philosophy of the inverse problem is to match the computed responses based on the forward computation using assumed initial values to the

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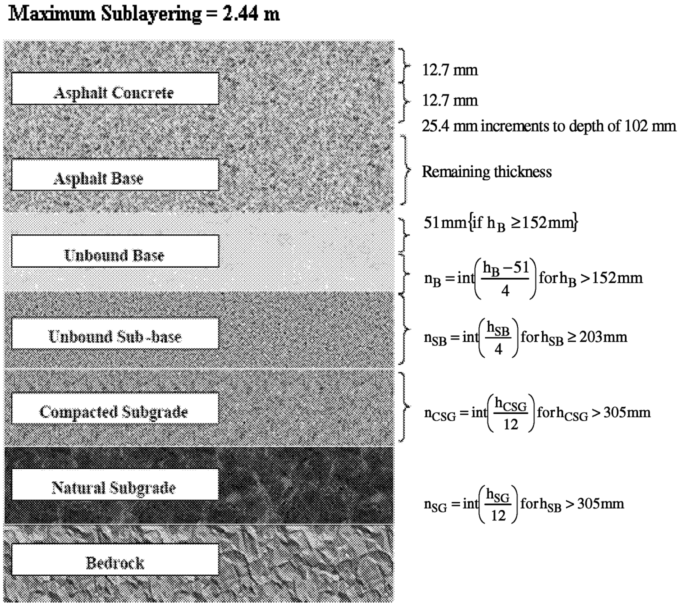

Fig. 1. Typical Sub-Layering of Flexible Pavement (Modified from MEPDG [5]).

measured responses according to certain criteria. Conventional methods for the backcalculation of the pavement moduli using data from the pavement NDT test can be grouped into three general methods [6]: (1) simplified methods, (2) gradient relaxation methods, and (3) direct interpolation methods. However, all these methods use techniques that may easily confine the search to local optima. Recently methods incorporating the genetic algorithm (GA) with its unique feature of searching global optima have emerged [7-10].

However, the backcalculation using GA is very time-consuming, and thus, a fast and efficient forward computation method will be of great help. In this paper, based on the powerful forward program, *MultiSmart3D,* and combined with GA, a new backcalculation program, *BackGenetic3D,* is developed with super performance in moduli backcalculation.

Variation of Resilient Modulus

The field data collected from the Long Term Pavement Performance (LTPP) project, a 20-year strategic project for understanding pavement performance, was analyzed and used in the newly released Mechanistic-Empirical Pavement Design Guide (MEPDG) [5] by using a climatic model. The climatic model, termed Enhanced Integrated Climatic Model (EICM), considers the effect of temperature and other environmental factors on the pavement analysis and design.

Based on the MEPDG guide and the EICM model, a software program was proposed [5]. To account for the temperature and resilient modulus variation in the pavement, the pavement layers are subdivided into many sublayers internally in the MEPDG software. A typical sub-layering for a flexible pavement section is shown in Fig. 1.

The Witczak's equation was used to calculate the resilient modulus in the asphalt concrete layer in the MEPDG software

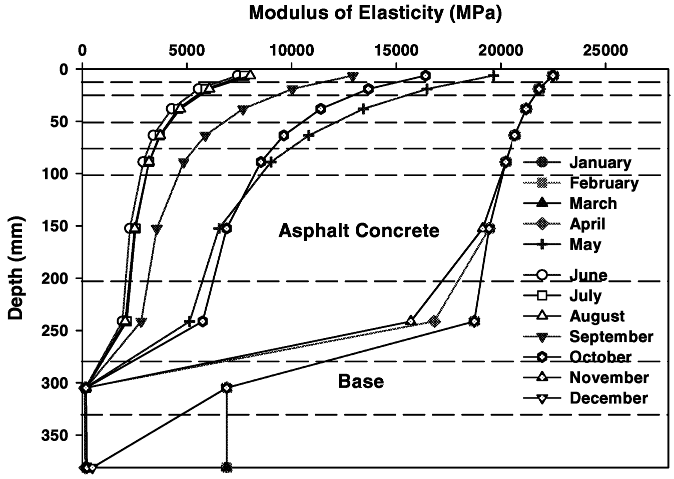

Fig. 2. Resilient Modulus vs. Depth (Data from Coree et al. [11]).

$$\log(E_d) = \delta + \frac{\alpha}{1 + \exp\left\{\beta + \gamma\left[\log(t) - c\left(\log(\eta) - \log(\eta_{\tau r})\right)\right]\right\}} \tag{1}$$

where E_d is the dynamic modulus (psi), t the time of loading (sec), η viscosity at the temperature of interest (CPoise), $\eta_{\tau r}$ viscosity at the reference temperature (CPoise), and α, β, δ, γ, and c are specific fitting parameters.

In the MEPDG software, the resilient modulus of the unbound and subgrade materials was calculated using the Witczak-Uzan's equation

$$E = k_1 p_a \left(\frac{\theta}{p_a}\right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3} \tag{2}$$

where E is the resilient modulus, k_1, k_2, and k_3 the parameters from physical testing or estimates, p_a the standard atmospheric pressure, θ the bulk stress ($\theta = \sigma_1 + \sigma_2 + \sigma_3$), and τ_{oct} the octahedral stress.

The modulus variation using MEPDG software for a flexible pavement section in Iowa was reported by Coree et al. [11] and presented in Fig. 2. The data was calculated using input from weather stations, as well as parameters of the hot mix, unbound material and subgrade material. The Poisson's ratio for all layers was reported to be constant during the year with a value of 0.35.

As can be seen from Fig. 2, averaging the resilient modulus within the top layers for a given month might not be appropriate since the difference in the average monthly resilient modulus between two consecutive sublayers within the AC layer can be up to 30%.

Effect of the Modulus Variation

It is clear that due to the complicated temperature variation, a variety of modulus profiles with depth are possible. With increasing number of sublayers, the computational time will also increase. However, the allowed maximal number of sublayers in the MEPDG software is limited to 20 and the maximal number of evaluation points to 26. Such limitations can substantially affect the modeling of modulus vs. temperature variation with depth, especially when the variation is nonlinear.

Table 1. Parameters for the Flexible Pavement Example.

Layer	Thickness (mm)	Resilient modulus (MPa)	Poisson's ratio
AC layer	150	3,500	0.3
Base layer	250	700	0.3
Subbase layer	250	300	0.3
Subgrade layer	--	100	0.3

The pavement performance can be assessed by predicting the number of loading cycles needed to initiate cracks (fatigue cracking),which is controlled by a transfer function. Several empirical relations have been proposed; for example, MEPDG [5] uses the following calibrated Shell equation:

$$N_f = A_f K F''\left(\frac{1}{\varepsilon_t}\right)^5 E_s^{-1.4} \tag{3}$$

and the Asphalt Institute uses the following equation [12]:

$$N_f = 0.00432\ C\left(\frac{1}{\varepsilon_t}\right)^{3.291}\left(\frac{1}{E_s}\right)^{0.854} \tag{4}$$

where, N_f is the number of load repetition to fatigue cracking, ε_t the tensile strain at the bottom of the AC layer, E_s the stiffness of the material, and F'' , A_f, K and C are constants depending on the material properties. The Asphalt Institute equation can be used for asphalt concrete layers of any thickness. The equations above show that the critical tensile strain and the stiffness of the asphalt concrete layer are the key factors affecting the number of load repetition needed to initiate fatigue failure.

One pavement section is analyzed and its parameters summarized in Table 1. The contact pressure at the surface of the pavement is assumed to be 690kPa acting on a circle with a diameter of 220.3mm. The pavement responses below the center of the contact pressure are calculated using the *MultiSmart3D* program. The *MultiSmart3D* program is a fast and accurate software tool developed by the Computer Modeling and Simulation Group at the University of Akron, and is based on the innovative computational and mathematical techniques for multilayered elastic systems [13-15]. This program is capable of analyzing any pavement system regardless of the number of layers, the thickness of each layer, the number of response points, and the shape of the applied pressure on the surface of the pavement.

Five models are discussed as summarized in Table 2. In all models the average modulus for the AC layer is kept at 3,500MPa and the Poisson's ratio is 0.3 for all layers and sublayers. The AC layer in Models 2 to 5 is subdivided into 20 sublayers, with either increasing or decreasing modulus profile(in linear or quadratic) [16].

Table 2. Fatigue Life Predication Based on Different Modulus Models.

Model	Modulus variation	Asphalt Institute equation	Shell equation
1	Constant	1.00	1.00
2	Linear increase	1.22(0.22)*	1.35(0.35)*
3	Linear decrease	0.79(0.21)*	0.70(0.30)*
4	Quadratic increase	1.16(0.16)*	1.26(0.26)*
5	Quadratic decrease	0.83(0.17)*	0.76(0.24)*

(* Relative error of the fatigue predication)

Table 3. Parameters for Four Layer Pavement.

Layer	Thickness (mm)	Actual modulus (MPa)	Poisson's ratio	Range of moduli
AC layer	152.4	3,447.38	0.35	1,000-6,000
Slab	254	31,026.40	0.25	10,000-60,000
Base	203.2	172.37	0.4	100-400
Subgrade	-	51.71	0.45	10-150

It should be stressed that the modulus variation due to the temperature could be much more complicated than this model. Here we want to show the importance of the modulus variation on the pavement response and to demonstrate the versatility of the *MultiSmart3D*.

Responses of the displacements, strains, and stresses were depicted by Alkasawneh et al. [16]. The tensile strains at the bottom of the AC layer are utilized for the fatigue life predication as mentioned above. It is found that an averaged resilient modulus could either underestimate or overestimate the horizontal strain. More specifically, increasing the modulus with depth will produce lower tensile strains at the bottom of the AC layer, while decreasing the modulus with depth will lead to higher tensile strains at the bottom of the AC layer. The difference between the horizontal strain using Model 1 and those using other models is most noticeable between 15 and 85% of the AC layer thickness.

The modulus variation as a result of the temperature variation with depth highly influences the predicted number of repeated loads (N_f) needed to initiate fatigue cracks in the AC layer. The last two columns in Table 2 show the ratio between the estimated number of repeated loads (N_f) using the modulus variation with depth (Models 2 to 5) and that using the traditional assumption of a constant modulus for the entire layer (Model 1) based on the Asphalt Institute and the Shell equation, respectively.

Since increasing the modulus with depth will produce lower tensile strains at the bottom of the AC layer, the required number of repeated loads to initiate fatigue cracks will be higher than that using the constant modulus. In this example, the increment in N_f from the modulus variation (Models 2 and 4) compared to that from constant modulus (Model 1) is approximately 22 and 16% by using the Asphalt Institute equation for the linear and quadratic modulus increment case, respectively, while it is 35 and 26% by using the Shell relation for the linear and quadratic modulus increase case, respectively.

The decrease in the modulus with depth produces higher tensile strains at the bottom of the AC layer, which leads to the reduced repeated loads required to initiate fatigue cracks. In this example, the decrease in N_f from the modulus variation (Models 3 and 5) compared to that from the constant modulus (Model 1) is approximately 21 and 17% by using the Asphalt Institute equation for the linear and quadratic modulus decrease case, respectively, while it is 30 and 24% by using the Shell equation for the linear and quadratic modulus decrease case, respectively.

The modulus variation as a result of the temperature variation with depth highly influences the predicted number of repeated loads needed to initiate fatigue cracks in the AC layer. The predicted N_f value using the constant modulus (Model 1) is far from satisfactory. To model the modulus variation with depth more accurately, increasing the number of sublayers is very critical. Therefore, this

Table 4. Parameters for GA Inputs.

Parameters	Value
Population size	512
Number of generation(G)	150
Probability of crossover(P_c)	0.5
Probability of mutation(P_m)	0

program, *MultiSmart3D*, can perform the calculation as it can deal with any number of field points and any number of layers, a challenge faced by other multilayered elastic programs.

Backcalculation of Modulus

Backcalculation of flexible pavement moduli uses the deflections measured via the Falling Weight Deflectometer (FWD), the most common NDT equipment for the evaluation of flexible pavements. The mechanical field (strain, stress, etc) is required for estimating the lifetime of the pavement which is always a complex task due to the dependence on the seed value and the high probability of converging to local optima.

The genetic algorithm method is robust and very reliable in finding the global optima, making it more attractive than other available backcalculation procedures. However, the genetic algorithm is time consuming, and its operation and strategy can differ from one problem to another. Tuning the genetic parameters and operators are also complex and experience dependent, which further limits the wide use of this method. The sensitivity analysis of the genetic operators on the search space and interaction among the genetic operators was carried out using the newly developed *BackGenetic3D* program [17] so that backcalculation of moduli in any pavement system can be performed with no restrictions on the number of layers, thickness of layers, location of the response points, number of loading circles, the shape of the loaded area (circle, triangle, etc.), and the configuration of the applied loading (uniform or nonuniform).

The *BackGenetic3D* is based on the forward multilayer elastic program, *MultiSmart3D,* and it is a very advanced and fast program capable of backcalculating the elastic moduli accurately. The program searches the domain of the elastic moduli for possible solutions (moduli) and determines the optimal solutions using a guided stochastic search technique. The search technique is based on the genetic algorithm technique proposed by Holland [18] with many improvements to handle the complexity associated with the backcalculation of moduli. To make this program more flexible and practical, the Dynamic Parameterless Genetic Algorithm (DPGA) based on Harik and Lobo [19] and Lobo [20] is employed with the following steps:

1. Run population P_i for a number of generations until the average fitness of two consecutive generations is less than a specified tolerance value;
2. Insert the fittest individual from run P_i to parents' pool of population P_{i+1} which is twice of P_i. The remaining individuals of the parents' pool of P_{i+1} are generated randomly;
3. Inspect the average fitness of the population after several evaluations. If the average fitness of two consecutive generations is less than a specified tolerance value, stop the run; otherwise, continue the run;
4. Repeat step 2 through 3 until the change in the average fitness between the two populations (P_i and P_{i+1}) is less than a tolerance value.

The DPGA can increase the population size of the same run dynamically, and the diversity of the population can also be increased substantially without destroying the schema of the individuals in the population.

One backcalculation example for a four-layer pavement [7] is analyzed, with the pavement parameters shown in Table 3 and the GA parameters in Table 4. Also, the search ranges for the pavement moduli, based on the engineering experience, are listed in Table 3.

Table 5. Backcalculation Results Using Different Program.

Program	AC (MPa)	Slab (MPa)	Base (MPa)	Subgrade (MPa)	Deflection RMSE (%)
MICHBACK	3443.03 (0.13)*	31131.9 (0.34)*	158.59 (7.99)*	51.75 (0.08)*	0.007
MODULUS	3639.74 (5.58)*	30827.1 (0.64)*	67.57 (60.80)*	52.4 (1.33)*	0.068
EVERCALC	10908.93 (216.44)*	15838 (48.95)*	90.95 (47.24)*	51.83 (0.23)	1.526
BackGenetic 3D	3445.66 (0.05)*	31036 (0.031)*	172.8 (0.25)*	51.61 (0.2)*	0.0014

(* Relative error (%) of the backcalculated moduli)

Table 5 compares the results using *BackGenetic3D* with those from MICHBACK, MODULUS and EVERCALC [7]. Notice that EVERCALC performs the worst with serious discrepancies in all the layers except for the subgrade, and that MODULUS and MICHBACK also show great discrepancies for the base layer. However, the *BackGenetic3D* can backcalculate the most accurate moduli for every layer in this four-layer pavement with the maximum relative error of moduli being only 0.25% and deflection error only 0.0014%.

Conclusions

To account for the modulus variation with depth, caused by temperature variation and other factors shown in LTPP data, an innovative and powerful program, *MultiSmart3D*, was developed. Numerical results using this program showed that moduli variation with depth affected greatly on the pavement performance. Based on this forward program, *MultiSmart3D*, a new inverse program powered with the genetic algorithm, *BackGenetic3D*, was also developed for the moduli backcalculation and showed superior performance to other existing backcalculation programs.

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