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A Thermodynamic Framework for Modelling Healing of Asphalt Mixtures

Venkaiah Chowdary[1] and J. Murali Krishnan[2+]

Abstract: Asphalt mixtures exhibit complex stress and strain response. Currently asphalt mixtures are modelled as linear elastic/linear viscoelastic materials. Aging of the binder during service life, development of normal stresses during torsion, densification of the mixture due to repeated traffic loading, healing during rest periods etc., are some instances that clearly underline the need to consider the non-linear response characteristics of this material. Healing of asphalt mixtures during rest periods between load applications is one such response which requires careful consideration from the perspective of modelling. If the material is subjected to several cycles of loading and if it is subjected to rest period of sufficient duration after these loading cycles, it is seen that the response after the rest period is similar to that of a much "*stiffer*" material when compared with the same response before rest period. In short, the material exhibits "*beneficial internal structural change*" during the application of rest period. The focus of the investigation is to first record through careful experiments healing of asphalt mixtures in the laboratory and then model the same using a rigorous constitutive modelling framework. Since confinement pressure played a significant role in the development of healing, the constitutive model derived had material functions explicitly depending on pressure. The veracity of the model predictions was corroborated with the experimental results and it was found that the predictions were reasonably good.

***Key words**: Asphalt mixtures; Healing; Rest period; Thermodynamics; Viscoelasticity.*

Introduction

Asphalt mixtures exhibit complex response characteristics. Understanding the mechanical behaviour of these complex mixtures is necessary so that these materials can be effectively used through the various stages of design, construction, and maintenance of asphalt pavements. Asphalt pavements are subjected to wide range of loading and environmental conditions and asphalt undergoes micro-structural and rheological changes due to these traffic and environmental effects. During its lifetime an asphalt concrete layer is subjected to several manifestations of distress: rutting, fatigue cracking, temperature cracking, and moisture induced damage [1]. Fatigue occurring due to repeated traffic loading is one of the major distresses in asphalt pavements. Normally, the fatigue life of asphalt pavements is predicted through laboratory investigations. It is well known that the laboratory fatigue tests have limited capability in predicting the fatigue life in the field. Some of the reasons for the large difference between laboratory and field fatigue life are the differences in loading conditions, including vehicle type and axle configurations, rest periods between vehicle loads, traffic distribution (mixed traffic effect), vehicle wander, differences in the mix compaction levels achieved, and environmental factors such as seasonal temperature variations and temperature gradients that occur in the pavement [2]. Hence shift factors are used to scale up the predictions of the laboratory testing.

To realistically simulate the field traffic conditions in the laboratory, fatigue testing with rest periods between loading cycles is very much essential. The rest period may be between each loading cycle or after some cycles. As expected, the rest periods do have beneficial effect on asphalt mixtures as it allows the specimen to relax the accumulated stresses. The ability to model the evolution and interaction of the void structure (air voids as well as microcracks) within the asphalt mixture is critical to the ability to predict fatigue performance as the void structure affects the stress distribution within the mixture and affects the permeability of the mixture to both air and water which, in turn, affects the rate and level of oxidative aging and moisture damage. In fact without an understanding of the evolution of the microstructure, the understanding of the fatigue damage process would be extremely limited. There is a need to recognise the non-linear response characteristics of asphalt, its tendency to age resulting in change in its microstructure, the recognition that it is a multi-constituent mixture and factor these ideas into developing an appropriate constitutive framework within which its fatigue characteristics can be studied.

Asphalt is a complex civil engineering material. When mixed with granular aggregates, the resulting asphalt mixture exhibits wide range of responses. "Healing" is one such mechanical response wherein if the material is allowed to rest after certain number of load applications, the material exhibits a "beneficial internal structural change" [3]. That is, during the rest period between successive loading cycles of the vehicles, relaxation of stresses due to viscoelastic nature of asphalt mixtures and healing take place. While relaxation of stresses is a well understood phenomenon in viscoelastic material, healing for asphalt mixtures is quite complex and is yet to be unravelled in a convincing manner.

During every traffic load passage, the material deforms. The deformation is out of phase with the applied load due to the viscoelastic nature of the material. After the passage of the load, the deformation suffered by the material recovers with time. This recovery takes place over a period of time depending on the time of loading, temperature during load application, mixture properties etc.

[1] Assistant Professor, Transportation Division, Department of Civil Engineering, National Institute of Technology, Warangal-506004, India.
[2] Associate Professor, Department of Civil Engineering, Indian Institute of Technology, Madras, Chennai-600036, India.
[+] Corresponding Author: E-mail jmk@iitm.ac.in
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Strain recovery after load application and stress relaxation for asphalt mixtures are well understood phenomena and detailed analyses have been carried out for various linear and non-linear viscoelastic models [4, 5]. However, due to the peculiarity of asphalt mixtures, during the rest periods following load application, it is likely that the internal structure of the material changes in such a manner that the response of the material in the subsequent cycle of loading corresponds to a material whose material properties have "improved" when compared with the response during the previous cycle prior to the rest period. This "beneficial" change in the internal structure characterised through the mechanical response is normally attributed to "healing" [3]. Hence, it is not only necessary to take into account the evolution of material properties, but it is also necessary to consider the beneficial effects that take place. While this places a constraint on the manner in which the material properties can evolve with time, at this point of time, it is not known about the individual contribution of the different material parameters on the healing of asphalt mixtures. To emphasise further, while it is known that the viscosity of asphalt increases with time due to aging, the overall shear modulus of the material also increases with time due to the aggregate bonding/rebonding that takes place. Hence, the magnitude of healing of asphalt mixtures depends to a large extent on the rate at which each of the material properties change with time.

The influence of considering the healing phenomenon in the final stress analysis of the pavement is significant. For instance, one will be interested in finding answers to questions such as the lag time necessary between successive takeoffs and landings on an air field pavement that maximises the healing potential of an asphalt mixture used in the runway construction. The remaining life analysis, use of an appropriate back-calculation procedure during the pavement evaluation stage will all benefit considering the healing and the evolutionary properties of asphalt mixtures.

In the following section, we first define what is healing of asphalt mixtures. We then discuss a framework for modelling such complex response. We then illustrate the efficacy of the model by corroborating the experimental results with model predictions.

Literature Review

Healing takes place within specific class of materials due to molecular reorientation occurring within the material under certain conditions. For these class of materials, healing can occur under the influence of external pressure [6, 7], with temperature [7-10], with reduced load acting on the specimen compared to the previous loading cycle [11] or with introduction of rest period between successive loading cycles under the influence of pressure during rest periods [3]. The literary meaning of the word "healing" suggests that there should have been material "cracking" for the onset of "healing" under specified conditions of loading/unloading/rest periods.

In the following, we first discuss what is meant by *healing* for some engineering material. Considerable studies have been carried out on the mechanical healing of polymers [7, 12-14]. Prager and Tirrell [14] in their study on the healing process at polymer-polymer interfaces defined healing as follows: "when two samples of the same amorphous polymer are brought into contact at a temperature above the glass transition, the junction surface gradually develops increasing mechanical strength until, at very long contact times, the full fracture strength of the bulk polymer is reached. At this point the junction surface has in all respects become indistinguishable from any other surface that might be located within the bulk material: we say the junction has healed."

Chen et al. [9, 10] developed a transparent organic highly cross-linked polymeric material that can be repeatedly mended or re-mended itself under mild conditions. Here the term "mend" is used as a synonym to healing. They observed that the "intermonomer" linkages disconnected upon heating to 120^{o}C and above and reconnected upon cooling and the cracks healed or fractures mended. They found the process to be fully reversible and proposed that the process can be used to restore a fractured part multiple times without additional ingredients such as a catalyst, additional monomer, or special surface treatment of the fractured interface. Miao et al. [15] studied healing of crushed rock salt. According to them, "healing is a chemical and physical process in which the material properties evolve with time," and "healing implies that microcracks and microvoids reduce in size, with a corresponding increase in stiffness and strength." They compared healing of crushed rock salt to healing of other materials such as crack healing in geological materials, curing of concrete, sintering of ceramics, compaction of cohesive sands and clays, recovery and recrystallisation of metals, and liquid-phase-enhanced densification of granularly crystalline materials. Aldea et al. [16] investigated the healing of cracked normal strength concrete based on water permeability test and elastic wave signal transmission measurements. They observed that immediately after cracking, water permeability increased and the signal transmission decreased with increasing initial crack width. That is, the larger crack widths reduce the signal transmission. After monitoring for 100 days, they observed that the water permeability of cracked specimens decreased significantly and the signal transmission increased with time. Based on the permeability and transmission measurements, they arrived at the conclusion that the possible reasons of healing are chemical precipitation of calcium hydroxide and calcium carbonate, mechanical blocking, obstruction of narrow crack areas with ultrafine material, and swelling and rehydration of the hardened cement paste on the crack walls.

As discussed in detail, numerous studies can be found on crack healing of different class of materials. While healing for such class of materials is a reasonably well understood phenomenon, one cannot say the same thing about healing in asphalt mixtures. Considering the fact that a typical asphalt mixture has several fractions of aggregates with complex interactions taking place at different length scales, a clear and precise understanding is found lacking. Some of the closest descriptions related to healing for asphalt mixtures are "the partial recovery of mechanical properties (i.e. "strength" and "stiffness") upon resting" [17] and "beneficial internal structural change" [3].

If healing is assumed to be the rebonding of cracked surfaces, one can explain healing from the studies of Rozeveld et al. [18]. They performed fracture studies to understand the failure mechanisms in polymer (styrene-butadiene-styrene) modified asphalt mixtures using an environmental scanning electron microscopy. Philips [19] proposed that the healing of binders is a three step process consisting of: (a) the closure of microcracks due to wetting (adhesion of two cracked surfaces together driven by surface energy), (b) the closure of macrocracks due to consolidating stresses and binder flow, and (c) the complete recovery of mechanical properties due to diffusion of asphaltene structures. Step (a) was believed to be the fastest, resulting only in the recovery of "stiffness", while steps (b) and (c) were thought to occur much slower but improve both "stiffness" and "strength" of the material such that it exhibits mechanical properties similar to the original material.

Highway and airfield pavements in general are subjected to rest periods either between the loading cycles exerted by axles/gears of the same vehicle or between axles/gears of successive vehicles. The beneficial change resulting due to rest periods incorporated between successive loading cycles have been recognised by many researchers in the case of a typical fatigue testing of asphalt mixtures [11, 17, 20-26]. Based on the observations reported by several researchers, one can arrive at a firm conclusion that rest periods indeed improve the response of asphalt mixtures subjected to loading cycles with rest period when compared to the response of asphalt mixtures subjected to loading cycles without any rest periods. This improvement in response can be ascribed to "healing" of asphalt mixtures occurring due to rest periods introduced between successive loading cycles.

In their early investigation, Kim and Little [3, 27] tried to quantify healing in asphalt mixtures through several laboratory experiments and Schapery's "correspondence principle". Pronk [28] proposed a partial healing model that considers the evolution of the material properties ("stiffness and phase lag") for the asphalt concrete specimen during fatigue testing using a four point bending apparatus. At this point it is important to make a note of the comment made by Krishnan and Rajagopal [29]: "Most of the present day models for healing of asphalt mixtures borrow heavily from the ideas of Schapery [30] in which a correspondence principle is postulated between linearised elasticity and linearised viscoelasticity, under special conditions. As asphalt mix responds in a non-linear fashion, it does not seem reasonable to appeal to the correspondence principle in general." Also, it has been shown by Rajagopal and Srinivasa [31] that this correspondence principle is incorrect, models for the nonlinear viscoelastic response of solids obtained using this principle will fail to satisfy the balance of angular momentum for large deformations. That is, one cannot use this principle if large deformations involving shear, bending, torsion etc., which are commonly encountered in pavements, are involved. Schapery [30] does discuss another limitation of the model, namely that it does not satisfy frame-indifference. As most of the studies on healing of asphalt mixtures appeal to Schapery's correspondence principle, it is very important to make a note of the comment made by Rajagopal and Srinivasa [31]: "this principle (correspondence principle), which is valid if the displacement gradients are sufficiently small, has been used in several papers to develop models to describe the fracture of viscoelastic solids, and these studies need to be re-examined in the light of this note."

The complex phenomenon of healing can be investigated at several observational scales. The literary meaning of the word "healing" suggests that there should have been "cracking" for the onset of "healing" under specified condition of loading/unloading /rest periods. Most of the well known investigations on asphalt mixtures cited here have investigated the phenomenon of healing on these lines. Essentially, healing was supposed to occur when two cracked surfaces were brought together and kept under sufficient pressure such that the cracked surfaces healed. While these ideas are still valid as far as healing of cracked asphalt concrete surfaces are concerned, it is also possible for asphalt mixtures to "heal" even before the onset of clear physical cracking. If healing of asphalt mixtures is understood in the lines of definitions such as "beneficial internal structural change" [3], certain conditions of loading and rest periods can induce internal structural change such that the subsequent response of the material after rest periods exhibits improved mechanical properties. Investigations conducted by Pronk [11] are similar in these lines in that cycles of loading of low amplitude when applied after cycles of loading of high amplitude resulted in an increase of the "stiffness" of the material.

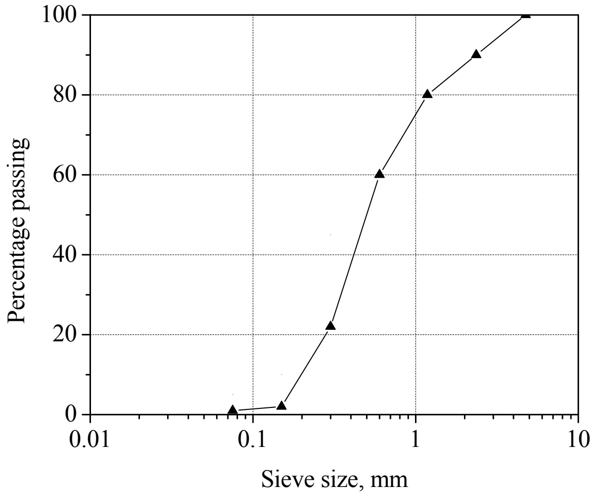

Fig. 1. Gradation of Sand Used for the Study.

Table 1. Properties of Asphalt, Sand, and Sand Asphalt Mix.

Property	Value
Penetration @ 25°C, 5*s*, 100*g*, 1/10*mm*	67
Softening Point of Asphalt, °C	41
Specific Gravity of Asphalt	1.01
Specific Gravity of Sand	2.56
Bulk Specific Gravity of Sand Asphalt Mix at 8% Air Voids	2.10

Experimental Investigations

Materials

Fine river sand passing through 4.75*mm* Indian Standard sieve with gradation as shown in Fig. 1 was mixed with 8% asphalt to prepare Marshall sized samples of 100*mm* diameter and 70*mm* height with 8% air voids. 60/70 grade straight run asphalt was used as the binder throughout the experiments. The relevant standard procedure as outlined in ASTM D1559-89 for preparing Marshall specimens for heavy traffic conditions was followed. The compacted specimens were cooled to room temperature in the moulds. The physical properties of asphalt, sand, and asphalt mix are shown in Table 1. Specimens of 35*mm* diameter and 70*mm* height as shown in Fig. 2

Fig. 2. Triaxial Test Specimen Cored from Marshall Sample.

were cored from the Marshall samples. The cored samples were allowed to cure for 24*hrs* at room temperature before the test to ensure relaxation of residual stresses developed during coring.

Testing Procedure

To characterize healing of asphalt mixtures in the laboratory, repeated triaxial tests were carried out on sand asphalt mixtures with varied confinement conditions. In reality, an asphalt pavement is confined in all directions and the cyclic loading simulates the load application due to the vehicular movement. All the tests were conducted in load controlled mode. The specimens were loaded for 7*s* and unloaded for the same time period. Lateral pressure - vertical pressure ratio of 1:5 (50 and 250*kPa*) were applied for each specimen. This specific ratio of lateral to vertical pressure was chosen after several laboratory trials. One of the main considerations in choosing this specific ratio is related to subjecting the specimen to load levels that will engender deformation capable of healing during the rest periods and yet not physically deform the specimen.

The cored specimen from the Marshall sample was covered with a rubber membrane to prevent entry of water during confinement. The specimen was placed inside the triaxial cell and was made water tight. The triaxial cell was completely filled with water without any air bubbles and the cell was pressurized with compressed air to provide confinement pressure throughout the testing period. The specimen was tested at constant load with rest periods introduced between successive loading cycles to observe the deformation response. Continuous data throughout the loading and rest periods was gathered through a data acquisition system attached to a computer. The entire testing was conducted at a room temperature of 28 ± 0.5°C.

Repeated load and rest period of same duration was applied for 100 cycles continuously as shown in Fig. 3. The material was allowed to rest for one hour after 100 cycles of loading and the same loading and rest cycles of equal duration were applied again to observe the deformation response. During rest periods, the confinement pressure was kept constant. Similar loading cycles were continued up to 500 cycles with one hour rest period after every 100 cycles. The deformation of the material with time during loading and rest periods was measured. Two samples were tested for each condition and the repeatability was found to be less than 5%. It is well known fact that during a rest period for asphalt mixes, deformation recovery and healing takes place simultaneously in a stress controlled test. If the material is allowed to rest for one hour after 100 cycles, and is loaded again as shown in Fig. 3, the decrease in total deformation in the 101^{st} cycle as compared to 100^{th} cycle shows that the material either recovers deformation along with healing or just recovers deformation. If the total deformation increases after one hour of rest period, one can conclude that the sample is approaching to failure. In order to observe whether the material is actually healing during the rest period, one has to normalize the 101^{st} cycle curve (after rest period) against the 100^{th} cycle curve. More details related to quantifying healing during creep and recovery experiments can be found from Chowdary [32].

Modelling Healing of Asphalt Mixtures

When a body is deformed, it can attain a specific state such that it can lead to conditions ripe for microstructural changes to take place. Such conditions are called by different names in different sub-fields of mechanics as yield condition, initiation condition, activation condition etc. This is essentially the case for materials not only undergoing large deformation but also for materials in which even a very small loading condition can trigger such a change. Some of the cases that come to mind are twinning, solid to solid phase transition, polymer crystallisation etc. [33].

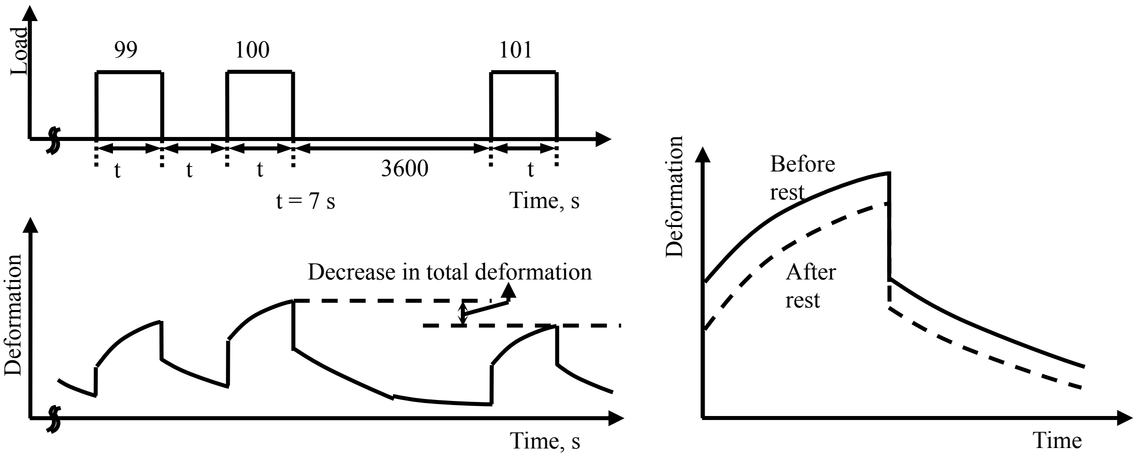

Fig. 3. Loading/Rest Periods and Corresponding Deformation.

Given a natural configuration of the body with its attendant details about its microstructure, deforming the material results in initiation of any of the above mentioned conditions and this results in the material occupying a new configuration when unloaded. Elastic response, an idealisation, results in the material occupying the same original configuration since during such process no microstructural changes are introduced. The microstructure can change and evolve with time as the material is deformed and once the forces acting on the body are removed, the body occupies a new configuration. This configuration could be called as the relaxed or stress free configuration of the material. Eckart [34] proposed a theoretical framework to study such evolving natural configurations, which he called as "variable relaxed states". His analysis concerned with the kinematics in a non-Euclidean geometrical structure. A clear exposition on the role of natural configuration, evolution of natural configurations, link between microstructure and natural configurations, and solution related to several engineering applications can be found in the works of Rajagopal and co-workers. This approach has helped in explaining multinetwork theory [35], twinning [36], classical metal plasticity [37], solid to solid phase transition [38], viscoelastic liquids [39], anisotropic liquids [40], and asphalt mixtures [5, 29, 41-44].

The framework used is essentially based on ideas related to natural configurations. The dissipative response of the material results in the material occupying different stress-free configurations as load cycles are applied and removed. The key question here is related to specifying field equations for evolution of the natural configuration. This is determined based on the maximisation of the rate of dissipation function. It is also understood here that this maximisation should be carried out subject to appropriate constraints. It has been shown by Rajagopal and Srinivasa [39] that a variety of thermodynamically consistent rate type models can be developed for viscoelastic materials by the appropriate choice of different forms for rate of dissipation function and stored energy function.

In the following, the notations and the kinematical quantities are detailed, the model for asphalt concrete is then derived and this model is used to predict the experimental data.

Preliminaries

Consider a body B in a configuration $\kappa_R(B)$. For convenience of notation, κ_R will be used to denote $\kappa_R(B)$. Let $\mathbf{X}$ denote a typical position of a material point in κ_R. The configuration occupied by the body at time t will be denoted by κ_t. The motion of the body is given as follows:

$$\mathbf{x} = \chi_{\kappa_R}(\mathbf{X}, t) \tag{1}$$

This is essentially a one-to-one mapping which assigns to each point $\mathbf{X}$ in the configuration κ_R a corresponding point $\mathbf{x}$ in the configuration κ_t.

The deformation gradient $\mathbf{F}_{\kappa_R}$ is defined through:

$$\mathbf{F}_{\kappa_R} = \frac{\partial \chi_{\kappa_R}}{\partial \mathbf{X}} \tag{2}$$

The left and right Cauchy-Green stretch tensors $\mathbf{B}_{\kappa_R}$ and $\mathbf{C}_{\kappa_R}$ are defined through:

$$\mathbf{B}_{\kappa_R} = \mathbf{F}_{\kappa_R}\mathbf{F}^{\mathrm{T}}_{\kappa_R} \tag{3}$$

$$\mathbf{C}_{\kappa_R} = \mathbf{F}^{\mathrm{T}}_{\kappa_R}\mathbf{F}_{\kappa_R} \tag{4}$$

There are several ways in which asphalt mixtures can be modelled. Considering the complexity of its behaviour and widely disparate ways in which it can exhibit mechanical response, it can be modelled as an incompressible material or a compressible material. As asphalt mixtures consists of aggregate matrix, asphalt mastic, and air voids, it is possible that one can model it using the continuum theory of mixtures [45]. Krishnan and Rao [46, 47] used continuum theory of mixtures to model asphalt mixtures. One could also model asphalt mixtures as a constrained two constituent mixture of aggregate matrix and asphalt mastic (see for instance [41]). Taking into account all the complexities of asphalt mixtures such as influence of shape, size, and distribution of aggregate particles, distribution of air voids, physical chemistry of asphalt, aging of asphalt etc., in one model is a Herculean task.

As of now, the role of air voids reduction on the healing potential of asphalt mixtures is not very clear. While it is intuitive to observe that asphalt mixtures with more air voids can "heal" better (under specific confinement conditions), it is not clear how the healing rate versus rate of air voids reduction changes. It is possible that from an initial air voids, repeated loading can densify the mixture in such a way that the flexibility of the internal structure of the aggregate matrix and asphalt mastic in reorienting and exhibiting better mechanical response after rest period is slowly lost. Hence at this juncture, the asphalt mixtures will be modelled as an incompressible material. Also, since not much information is available related to the interaction among the various constituents, asphalt mixtures will be modelled as a single constituent mixture. Suffice to say that one is really looking at one of the most complex phenomena exhibited by an engineering material.

The conservation of mass is given by,

$$\dot{\rho} + \rho\,\mathrm{div}(\mathbf{v}) = 0 \tag{5}$$

where ρ is the density of the material and $\mathbf{v}$ is the velocity. For an incompressible material, the conservation of mass reduces to,

$$\mathrm{div}(\mathbf{v}) = 0 \tag{6}$$

The balance of linear momentum is given by,

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\nabla\mathbf{v})\mathbf{v}\right] = \mathrm{div}(\mathbf{T}) + \rho\mathbf{g} \tag{7}$$

where $\mathbf{T}$ is the Cauchy stress tensor and $\mathbf{g}$ is the acceleration due to gravity.

The balance of angular momentum for a body implies that the stress tensor $\mathbf{T}$ is symmetric in the absence of internal couples and hence can be written as,

$$\mathbf{T} = \mathbf{T}^{\mathrm{T}} \tag{8}$$

The conservation of energy is given as follows:

$$\rho\dot{\varepsilon} + \mathrm{div}(\mathbf{q}) = \mathbf{T}\cdot\mathbf{L} + \rho r \tag{9}$$

where ε is the specific internal energy, **q** is the heat flux vector, **L** is the velocity gradient and r is the radiant heating.

The second law of thermodynamics is used in continuum mechanics in the form of the Clausius-Duhem inequality [48]. In the current investigation, the second law is introduced in the form of an equality (see [49, 50]). The balance law for entropy is written in the following form:

$$\rho\dot{\zeta} + \mathrm{div}\left(\frac{\mathbf{q}}{\theta}\right) = \rho\frac{r}{\theta} + \rho\Xi, \quad \Xi \geq 0 \tag{10}$$

where ζ is the entropy, θ is the absolute temperature and Ξ is the rate of entropy production.

Combining the balance of energy, Eq. (9), and the balance of entropy, Eq. (10), one gets the following reduced energy-dissipation equation [49, 50]:

$$\mathbf{T}\cdot\mathbf{L} - \rho\dot{\psi} - \rho\zeta\dot{\theta} - \frac{\mathbf{q}\cdot\mathrm{grad}(\theta)}{\theta} = \rho\theta\Xi = \xi \geq 0 \tag{11}$$

where ψ is the Helmholtz potential and is given by $\psi = \varepsilon - \theta\zeta$ and ξ is the rate of dissipation. It is a normal practice to split rate of dissipation into two parts. The first part pertaining to heat conduction (ξ_c) and the second part pertaining to the rate at which work is converted into energy in its thermal form (ξ_d) , i.e., $\xi = \xi_c + \xi_d$.

The rate of dissipation due to heat conduction is given as [49],

$$\xi_c = -\frac{\mathbf{q}\cdot\mathrm{grad}(\theta)}{\theta} \geq 0 \tag{12}$$

Using the above equation one can rewrite Eq. (11) as,

$$\mathbf{T}\cdot\mathbf{L} - \rho\dot{\psi} - \rho\zeta\dot{\theta} = \xi_d \geq 0 \tag{13}$$

The above equation will be used to constrain the acceptable constitutive equation in this study.

Modelling of Asphalt Mixtures

With reference to each current configuration $\kappa_{c(t)}$, a natural configuration $\kappa_{p(t)}$ is associated. Fig. 4 shows a very simple case. With reference to Fig. 4, κ_R is a reference configuration, $\kappa_{c(t)}$ is the configuration currently occupied by the material at time t, and $\kappa_{p(t)}$ is the natural configuration associated with the material that is currently in the configuration $\kappa_{c(t)}$. The subscript "p" in $\kappa_{p(t)}$ refers to the "preferred" stress free configuration associated with the configuration $\kappa_{c(t)}$. This stress free configuration is different from the reference configuration κ_R. One can associate with the change in the natural configuration, changes effected in the material microstructure due to the loading history to which it is subjected.

For a classical elastic solid, $\kappa_{p(t)}$ and κ_R are one and the same. Detailed discussion on natural configurations and their role in continuum mechanics can be found from Rajagopal [51].

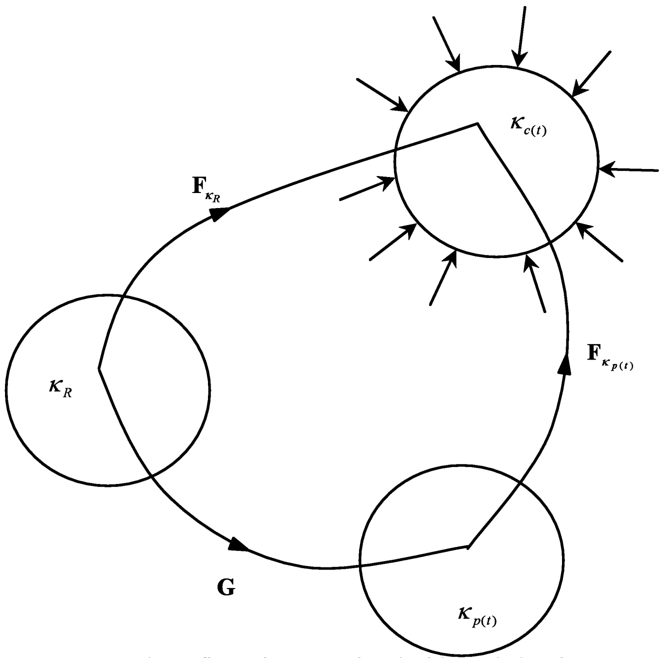

Fig. 4. Natural Configuration Associated with Asphalt Mixtures.

Many real materials exist in a variety of stress-free configurations and these configurations need not have to be related through rigid body motion. It is also possible for biological materials to exist in a "stressed natural configuration", for instance, the case of a cell born in a stressed tissue. Details related to such complex phenomenon can be found from Humphrey and Rajagopal [52].

It is well known that there are a variety of polymers with multiple relaxation mechanisms. One could associate with each relaxation mechanism, a natural configuration. In the study by Krishnan and Rajagopal [41], asphalt concrete is modelled as a two constituent constrained mixture and with each constituent, a relaxation mechanism is assigned. Krishnan and Rajagopal [42] used the same framework and proposed a model in which pure asphalt is modelled as a mixture of amorphous and crystalline phases and assigned separate relaxation mechanism for the same.

In this investigation, the loading conditions engender in the material two types of changes. These loading conditions will be discussed within the purview of triaxial loading. The first change entails considerable change in the material microstructure such that unloading from such a state involves a permanent change in the material microstructure such that subsequent loading conditions evoke a different response. This change could be called "destructive internal structure" change. Typically, the material is subjected to continuous cycles of loading with no rest period between cycles for this change to occur. The second change relates to the reformation of internal structure. This reformation could happen only if particular conditions of confinement are induced. As discussed in the literature review, one could associate physical and chemical healing characteristics of asphalt binder and mastic as some of the possible reasons, for this change.

It is to be reiterated here that not every asphalt mixture can exhibit such behaviour under confinement. The magnitude of confinement pressure, the aggregate interlock, the constituent properties of asphalt binder, the age of the material, temperature, etc., are some of the factors necessary for such reformation to occur.

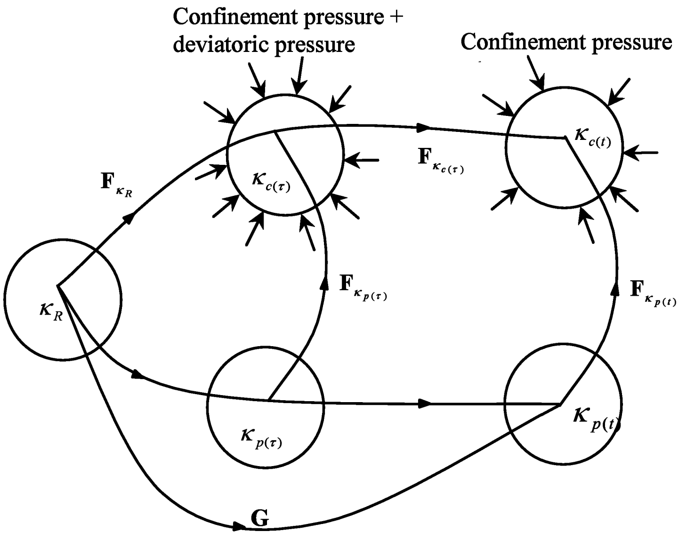

Fig. 5. Natural Configuration Associated with Asphalt Mixtures during Loading and Rest Period.

Taking into account all these factors, one could express the evolution of internal structure of the material in terms of the natural configuration as given in Fig. 5. This will be explained with respect to repeated cycles of loading with a rest period introduced after several cycles of loading. It is assumed that the magnitude of rest period as well as the confinement pressure given is sufficient for the material to show a reformed internal structure.

With reference to Fig. 5, let κ_R be the reference configuration associated with the material before any load application. Repeated loading of several cycles changes the microstructure of the material. $\kappa_{c(\tau)}$ is one such configuration of the material at time τ. If the material is completely unloaded at time τ, the natural configuration associated is given by $\kappa_{p(\tau)}$. The material has undergone some "damage" during the load application and since any rest period with sufficient pressure is not given, the material associated with the stress free configuration $\kappa_{p(\tau)}$ when compared with the material associated with the reference configuration κ_R exhibits deterioration in the mechanical properties. On the other hand, let the material be subjected to confinement pressure alone for a time period of $(t - \tau)$ [the so called rest period]. The natural configuration associated with the material at time t is given by $\kappa_{p(t)}$. Now during this "rest period", the material undergoes possible reformation of the internal structure. Experimental investigation and the microstructural studies show that the application of confinement pressure alone for a sufficient amount of time results in a beneficial internal structural change. Within the context of the natural configurations introduced here, it essentially means that the material at configuration $\kappa_{p(t)}$ exhibits a much better mechanical response when compared with material associated with configuration $\kappa_{p(\tau)}$.

Considering the difficulties associated with precisely pinpointing the various micro-mechanisms that aid in the healing process, one can only hypothesise lacking clear and precise experimental evidence, the transition occurring in the material during the time period $(t - \tau)$. For instance, one can state that at time τ, some portion of the material microstructure is damaged while the remaining portion is undamaged. It is also not very clear here as to what one means by "damage" as far as "healing" is concerned. As discussed earlier, the current state-of-the-art related to healing of asphalt mixtures presupposes the presence of a crack. The objective of this investigation is to, however, experimentally investigate and model the healing of asphalt mixtures without subjecting the material to any clear physical damage. One can assume that some portion of the material of volume fraction "α" heals during $(t - \tau)$ whereas the volume fraction $(1 - \alpha)$ remains as it is. It is then possible to consider the material from τ to t as a constrained mixture of two constituents; one constituent that is actively reforming and the other constituent that is inert to such changes. In such a case, the internal energy and entropy of the material can be assumed to be additive during τ to t.

$$\varepsilon = \int_{\tau}^{t} \varepsilon_d \frac{d\alpha}{d\tau} d\tau + i_\varepsilon + (1-\alpha)\varepsilon_{ud} \tag{14}$$

Here ε_d is the internal energy associated with the volume fraction α of the material that is reforming and ε_{ud} is the internal energy associated with the volume fraction $(1 - \alpha)$ of the material that is inert to such reformation. Here i_ε is the interfacial energy per unit mass of the mixture.

The entropy of the mixture during the time period $(t - \tau)$ is given as,

$$\zeta = \int_{\tau}^{t} \zeta_d \frac{d\alpha}{d\tau} d\tau + i_\zeta + (1-\alpha)\zeta_{ud} \tag{15}$$

Here ζ_d is the entropy associated with the volume fraction α, ζ_{ud} is the entropy associated with the volume fraction $(1 - \alpha)$ and i_ζ is the interfacial entropy per unit mass of the mixture. The interfacial components are added taking into account the presence of the constituent boundary.

The above formulation associated with Fig. 5 is by far the most complete treatment of modelling of healing one can associate with asphalt mixtures. The lack of information related to proceeding further along these lines is apparent. In the current study, the material is modelled as a single constituent mixture only. However, the reformation associated with the mixture during $(t - \tau)$ is taken care of through appropriate material functions to be discussed later.

Let $\mathbf{G}$ be the mapping between the tangent space κ_R and the natural configuration $\kappa_{p(t)}$, i.e.,

$$\mathbf{G} = \mathbf{F}_{\kappa_R \to \kappa_{p(t)}} = \mathbf{F}^{-1}_{\kappa_{p(t)}} \mathbf{F}_{\kappa_R} \tag{16}$$

The left Cauchy-Green stretch tensor is defined as,

$$\mathbf{B}_{\kappa_{p(t)}} = \mathbf{F}_{\kappa_{p(t)}} \mathbf{F}^{\mathrm{T}}_{\kappa_{p(t)}} \tag{17}$$

The velocity gradient $\mathbf{L}$ and the velocity gradient $\mathbf{L}_{\kappa_{p(t)}}$ associated with $\kappa_{p(t)}$ are defined through [39]:

$$\mathbf{L} = \dot{\mathbf{F}}_{\kappa_R} \mathbf{F}^{-1}_{\kappa_R} \tag{18}$$

$$\mathbf{L}_{\kappa_{p(t)}} = \dot{\mathbf{G}} \mathbf{G}^{-1} \tag{19}$$

Similarly the symmetric part of the velocity gradient is defined through,

$$\mathbf{D} = \frac{1}{2}\left(\mathbf{L} + \mathbf{L}^{\mathrm{T}}\right) \tag{20}$$

$$\mathbf{D}_{\kappa_{p(t)}} = \frac{1}{2}\left(\mathbf{L}_{\kappa_{p(t)}} + \mathbf{L}^{\mathrm{T}}_{\kappa_{p(t)}}\right) \tag{21}$$

For a proper frame-invariant formulation, appropriate frame invariant derivatives have to be used. For instance, the upper convected Oldroyd derivative of any quantity is given as [53]:

$$\overset{\triangledown}{\mathbf{A}} = \dot{\mathbf{A}} - \mathbf{LA} - \mathbf{AL}^{\mathrm{T}} \tag{22}$$

The inverted triangle denoted the upper convected Oldroyd derivative and the superposed dot signifies the material time derivative. Using the above Eq. (22), one can write $\overset{\triangledown}{\mathbf{B}}_{\kappa_{p(t)}}$ as follows:

$$\overset{\triangledown}{\mathbf{B}}_{\kappa_{p(t)}} = \dot{\mathbf{B}}_{\kappa_{p(t)}} - \mathbf{L}\mathbf{B}_{\kappa_{p(t)}} - \mathbf{B}_{\kappa_{p(t)}}\mathbf{L}^{\mathrm{T}} = -2\mathbf{F}_{\kappa_{p(t)}}\mathbf{D}_{\kappa_{p(t)}}\mathbf{F}^{\mathrm{T}}_{\kappa_{p(t)}} \tag{23}$$

As discussed earlier, it will be assumed here that the motion is isochoric, i.e.,

$$\mathrm{tr}\left(\mathbf{D}_{\kappa_{p(t)}}\right) = 0 \tag{24}$$

It is well known that asphalt mixtures exhibit response akin to that of an anisotropic material. The anisotropy depends on the aggregate characteristics, aggregate gradation, compaction method used in the manufacture of the material etc. The orientation of the aggregates also keeps evolving during loading and it is possible that the healing of the mixtures depends to a considerable extent on the aggregate orientation. Essentially, the flexibility related to reforming of the material depends to a large extent on the existing aggregate matrix orientation. The anisotropy can be built into the model by making the stored energy and the rate of dissipation function depend on anisotropy. For simplicity, asphalt mixtures are modelled as an isotropic material here.

The internal energy and the entropy are assumed to be functions of temperature θ and the first two invariants of $\mathbf{B}_{\kappa_{p(t)}}$, and they are given as follows:

$$\varepsilon = \varepsilon\left(\theta, I_{\kappa_{p(t)}}, II_{\kappa_{p(t)}}\right) \tag{25}$$

$$\zeta = \zeta\left(\theta, I_{\kappa_{p(t)}}, II_{\kappa_{p(t)}}\right) \tag{26}$$

Here, $I_{\kappa_{p(t)}} = \mathrm{tr}\left(\mathbf{B}_{\kappa_{p(t)}}\right)$ and $II_{\kappa_{p(t)}} = \mathrm{tr}\left(\mathbf{B}^2_{\kappa_{p(t)}}\right)$. The Helmholtz potential then has the following form:

$$\psi = \psi\left(\theta, I_{\kappa_{p(t)}}, II_{\kappa_{p(t)}}\right) \tag{27}$$

All the above forms essentially underline the response of an incompressible, isotropic material with an instantaneous elastic response. The evolution of the material's internal structure also depends on the confinement pressure. The experimental investigation reported here as well as in the literature discussed underlines the necessity to have a specific confinement pressure for healing to take place. Since sand asphalt is a mixture of fine grained particles of various sizes bound together by asphalt, a higher confinement pressure will increase the resistance to deformation. The pressure in the lateral direction will mobilise finer grained particle interlock and hence the deformation resistance of sand asphalt is a function of the confinement pressure. As past experiments have confirmed, healing depends to a large extent on the confinement pressure that is applied during the rest period. Since asphalt concrete is a mixture of aggregate particles of various sizes bound together by asphalt, a higher confinement pressure will increase the healing potential of the mixture during rest period. The fact that the confinement pressure will increase the resistance to deformation is well known [54, 55]. Since the rate of dissipation is a measure of how work is converted to energy in its thermal form, it should in some way reflect this dependence on confinement pressure. The following form is assumed for the rate of dissipation function based on the above discussion:

$$\xi = \xi\left(\theta, \mathbf{B}_{\kappa_{p(t)}}, \mathbf{D}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T})\right) \tag{28}$$

The specific form for the constitutive equation will be derived now. The internal energy is assumed to be a linear function of temperature and that the change in internal energy due to deformation is dependent on the first invariant of $\mathbf{B}_{\kappa_{p(t)}}$:

$$\varepsilon = \mathrm{C}\theta + \mathrm{A} + f(I_{\kappa_{p(t)}}) \tag{29}$$

$$\zeta = \mathrm{C}\ln(\theta) + \mathrm{B} \tag{30}$$

Here, A and B are constants, C is the specific heat. The following specific form for the rate of dissipation is assumed:

$$\xi = \eta_1\left(\mathbf{B}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T})\right)\mathbf{D}_{\kappa_{p(t)}} \cdot \mathbf{B}_{\kappa_{p(t)}}\mathbf{D}_{\kappa_{p(t)}} \tag{31}$$

Here, $\eta_1\left(\mathbf{B}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T})\right)$ is the viscosity of asphalt mixtures.

Substituting Eqs. (29) and (30) into Eq. (11), one gets the following expression:

$$\mathbf{T} \cdot \mathbf{L} - \rho\dot{f}(I_{\kappa_{p(t)}}) = \xi \tag{32}$$

The natural configuration $\kappa_{p(t)}$ can be taken such that,

$$\mathbf{F}_{\kappa_{p(t)}} = \mathbf{V}_{\kappa_{p(t)}} \tag{33}$$

This form is possible due to the assumption of isotropy associated with natural configuration of asphalt mixtures. Using Eqs. (23) and (33), one gets the following from Eq. (32):

$$\left(\mathbf{T} - 2\rho\frac{\partial f}{\partial I_{\kappa_{p(t)}}}\mathbf{B}_{\kappa_{p(t)}}\right) \cdot \mathbf{D} = \left(\eta_1\left(\mathbf{B}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T})\right)\mathbf{B}_{\kappa_{p(t)}}\mathbf{D}_{\kappa_{p(t)}} - 2\rho\frac{\partial f}{\partial I_{\kappa_{p(t)}}}\mathbf{B}_{\kappa_{p(t)}}\right) \cdot \mathbf{D}_{\kappa_{p(t)}} \tag{34}$$

One is essentially looking at a form sufficient to satisfy the above Eq. (34). Due to the incompressibility assumption made here, one can stipulate that,

$$\mathbf{T} = -p\mathbf{1} + 2\rho \frac{\partial f}{\partial I_{\kappa_{p(t)}}} \mathbf{B}_{\kappa_{p(t)}} \quad (35)$$

Where p is the Lagrange multiplier due to the constraint of incompressibility and **1** is the identity tensor.

A specific form associated with the neo-Hookean elastic solid is chosen here [56] and is given as:

$$f(I_{\kappa_{p(t)}}) = \frac{1}{2\rho} \mu(I_{\kappa_{p(t)}})(I_{\kappa_{p(t)}} - 3) \quad (36)$$

Then Eq. (35) becomes,

$$\mathbf{T} = -p\mathbf{1} + \mu(I_{\kappa_{p(t)}})\mathbf{B}_{\kappa_{p(t)}} \quad (37)$$

Now it is necessary to stipulate how the natural configurations evolve with time. There are three issues to be considered while taking into account the evolution of the natural configurations. The first issue is related to the constraint of incompressibility. The second constraint is related to the fact that not every admissible process should involve in the change in material microstructure leading to a different natural configuration, or in other words κ_R is also one of the possible configuration which the material can occupy. The third constraint which is specific to the problem under consideration is related to the possibility in $\kappa_{p(t)}$ being also one possible choices for $\kappa_{p(\tau)}$. This means that even under no load and with sufficient confinement pressure, it is possible for the material not to exhibit healing in the sense in which it is defined in this work.

In this work, for a fixed $\mathbf{B}_{\kappa_{p(t)}}$ and **D**, $\mathbf{D}_{\kappa_{p(t)}}$ is picked in such a way that it maximises the rate of entropy production. The third constraint is not taken into account explicitly here. The implicit form for the material functions chosen is enough to satisfy this constraint as will be shown in the simulation. The above mentioned constrained maximisation procedure results in [39],

$$\mathbf{V}_{\kappa_{p(t)}} \mathbf{D}_{\kappa_{p(t)}} \mathbf{V}_{\kappa_{p(t)}} = \frac{\mu_1(I_{\kappa_{p(t)}})}{\eta_1(\mathbf{B}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T}))} \left[\mathbf{B}_{\kappa_{p(t)}} - \lambda \mathbf{1}\right] \quad (38)$$

Taking the dot product of the above Eq. (38) with $\mathbf{B}^{-1}_{\kappa_{p(t)}}$ and noting that $\mathrm{tr}(\mathbf{V}_{\kappa_{p(t)}} \mathbf{D}_{\kappa_{p(t)}} \mathbf{V}^{-1}_{\kappa_{p(t)}}) = \mathrm{tr}(\mathbf{D}_{\kappa_{p(t)}}) = 0$, one gets,

$$\lambda = \frac{3}{\mathrm{tr}(\mathbf{B}^{-1}_{\kappa_{p(t)}})} \quad (39)$$

and Eq. (38) becomes,

$$-\frac{1}{2} \overset{\triangledown}{\mathbf{B}} = \frac{\mu_1(I_{\kappa_{p(t)}})}{\eta_1(\mathbf{B}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T}))} \left[\mathbf{B}_{\kappa_{p(t)}} - \lambda \mathbf{1}\right] \quad (40)$$

This completes the development of the model. It is now necessary to assume appropriate forms for the $\mu_1(I_{\kappa_{p(t)}})$ and $\eta_1(\mathbf{B}_{\kappa_{p(t)}}, \mathrm{tr}(\mathbf{T}))$.

Corroboration of the Model with the Experimental Data

The deformation is assumed to be homogeneous. In a cylindrical polar coordinate system, a point in the reference configuration is determined by (R, Θ, Z) and the same point in the current configuration is denoted by (r, θ, z). Thus the kinematics of deformation are,

$$r = \frac{1}{\sqrt{\Lambda(t)}} R, \quad \theta = \frac{1}{\sqrt{\Lambda(t)}} \Theta, \quad z = \Lambda(t) Z \quad (41)$$

where $\Lambda(t)$ denotes stretch in the Z direction. The deformation gradient for this motion is given by,

$$\mathbf{F}_{\kappa_R} = \mathrm{diag}\left(\frac{1}{\sqrt{\Lambda(t)}}, \frac{1}{\sqrt{\Lambda(t)}}, \Lambda(t)\right) \quad (42)$$

Note that $\det(\mathbf{F}_{\kappa_R}) = 1$ for the assumed deformation field. The left Cauchy-Green stretch tensor and the velocity gradient are given by,

$$\mathbf{B}_{\kappa_R} = \mathrm{diag}\left(\frac{1}{\Lambda(t)}, \frac{1}{\Lambda(t)}, \Lambda^2(t)\right) \quad (43)$$

$$\mathbf{L} = \frac{\dot{\Lambda}(t)}{\Lambda(t)} \mathrm{diag}\left(-\frac{1}{2}, -\frac{1}{2}, 1\right) \quad (44)$$

The velocity gradient **L** is diagonal and hence the symmetric part of the velocity gradient, **D**, is the same as **L**. The components of $\mathbf{B}_{\kappa_{p(t)}}$ are assumed to be $\mathrm{diag}\left(\frac{1}{B(t)}, \frac{1}{B(t)}, B^2(t)\right)$. This assumption is consistent with the stipulation that the stress free state for the material is achieved via a motion of the form given by Eq. (40). The constitutive equation can now be written as,

$$T_{rr} = -p + \mu_1(I_{\kappa_{p(t)}}) B_{\kappa_{p(t)} rr} \quad (45)$$

$$T_{\theta\theta} = -p + \mu_1(I_{\kappa_{p(t)}}) B_{\kappa_{p(t)} \theta\theta} \quad (46)$$

$$T_{zz} = -p + \mu_1(I_{\kappa_{p(t)}}) B_{\kappa_{p(t)} zz} \quad (47)$$

Eliminating p from Eqs. (45)- (47), one can obtain,

$$T_{rr} - T_{\theta\theta} = \mu_1(I_{\kappa_{p(t)}}) \left(B_{\kappa_{p(t)} rr} - B_{\kappa_{p(t)} \theta\theta}\right) \quad (48)$$

$$T_{zz} - T_{rr} = \mu_1(I_{\kappa_{p(t)}}) \left(B_{\kappa_{p(t)} zz} - B_{\kappa_{p(t)} rr}\right) \quad (49)$$

Since $\det(\mathbf{B}_{\kappa_{p(t)}}) = 1$, $\mathrm{tr}(\mathbf{B}^{-1}_{\kappa_{p(t)}})$ can be written as,

$$\begin{aligned} \mathrm{tr}(\mathbf{B}^{-1}_{\kappa_{p(t)}}) &= \frac{1}{\mathbf{B}_{\kappa_{p(t)} rr}} + \frac{1}{\mathbf{B}_{\kappa_{p(t)} \theta\theta}} + \frac{1}{\mathbf{B}_{\kappa_{p(t)} zz}} \\ &= \mathbf{B}_{\kappa_{p(t)} rr} \mathbf{B}_{\kappa_{p(t)} \theta\theta} + \mathbf{B}_{\kappa_{p(t)} rr} \mathbf{B}_{\kappa_{p(t)} zz} + \mathbf{B}_{\kappa_{p(t)} \theta\theta} \mathbf{B}_{\kappa_{p(t)} zz} \end{aligned} \quad (50)$$

From Eq. (40), the evolution equation along the z direction is

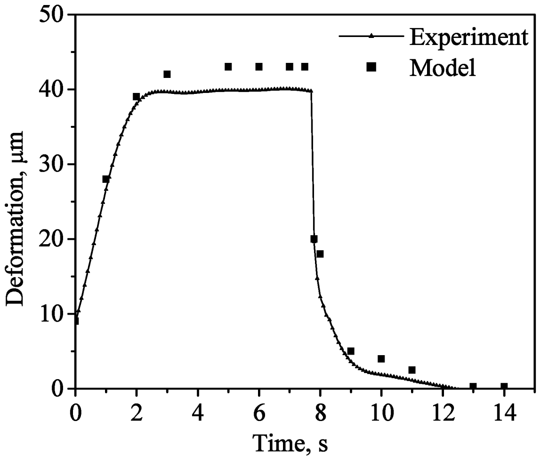

Fig. 6. Response at 100th Cycle for 8% Air Voids, 7*s* Loading/Rest Time, $\sigma_3 = 50kPa$ and $\sigma_1 = 250kPa$.

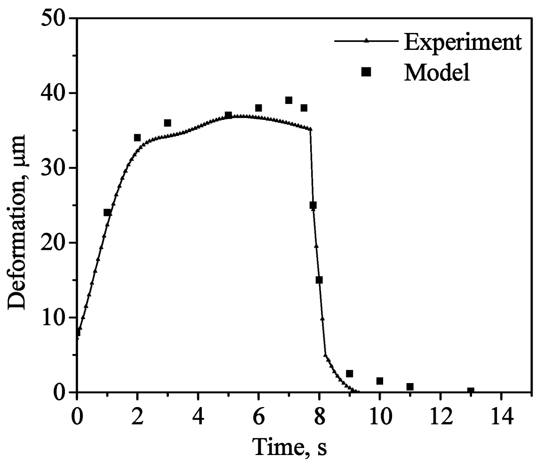

Fig. 7. Response at 101st Cycle for 8% Air Voids, 7*s* Loading/Rest Time, $\sigma_3 = 50kPa$ and $\sigma_1 = 250kPa$.

given by,

$$\frac{1}{2}\left(\frac{\partial B_{\kappa_{p(t)}zz}}{\partial t}+v_{\kappa_{p(t)}zz}\frac{\partial B_{\kappa_{p(t)}zz}}{\partial z}-2L_{\kappa_{p(t)}zz}B_{\kappa_{p(t)}zz}\right)=\frac{\mu_1(I_{\kappa_{p(t)}})}{\eta_1(\mathbf{B}_{\kappa_{p(t)}},\mathrm{tr}(\mathbf{T}))}\left[\frac{2B_{\kappa_{p(t)}zz}\left(B_{\kappa_{p(t)}rr}-B_{\kappa_{p(t)}zz}\right)}{2B_{\kappa_{p(t)}zz}-B_{\kappa_{p(t)}rr}}\right] \quad (51)$$

The initial conditions for this problem are determined from the fact that a sudden application of force elicits an instantaneous elastic response from the body and is given as $\Lambda(0)$ at time $t = 0$. Hence the initial conditions are given by,

$$B_{\kappa_{p(t)}zz}=\left(\Lambda(0)\right)^2 \quad (52)$$

$$B_{\kappa_{p(t)}rr}=B_{\kappa_{p(t)}\theta\theta}=\frac{1}{\Lambda(0)} \quad (53)$$

Ignoring the inertial terms, it is now necessary to simulate the Eqs. (49) and (51) for the initial conditions shown in Eqs. (52) and (53) to check whether the model corroborates well with the experimental observations. For this purpose, it is necessary to first assume appropriate forms for the shear modulus and viscosity. The following specific forms are used for this particular investigation.

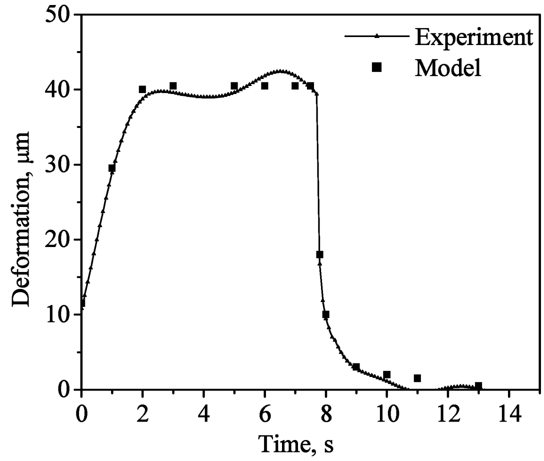

Fig. 8. Response at 200th Cycle for 8% Air Voids, 7*s* Loading/Rest Time, $\sigma_3 = 50kPa$ and $\sigma_1 = 250kPa$.

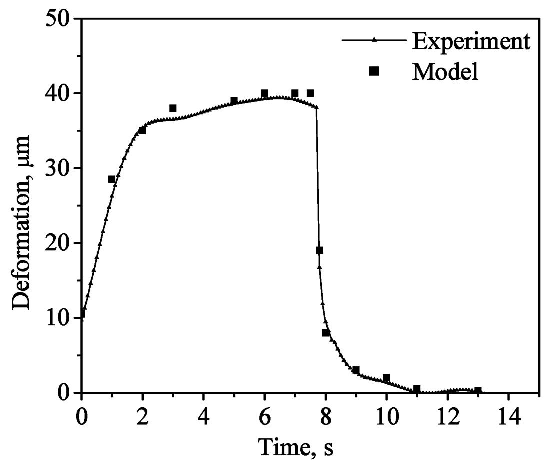

Fig. 9. Response at 201st Cycle for 8% Air Voids, 7*s* Loading/Rest Time, $\sigma_3 = 50kPa$ and $\sigma_1 = 250kPa$.

$$\mu_1(I_{\kappa_{p(t)}})=\overline{\mu}_1\left(1+\frac{1}{1+\exp(-a(I_{\kappa_{p(t)}}-3-b))}\right) \quad (54)$$

$$\frac{\mu_1}{\eta_1}=c\left(1+\frac{d}{n}(I_{\kappa_{p(t)}}-3)\right)^{n-1}\left(\frac{g(I_{\kappa_{p(t)}}-3)^f}{\mathrm{tr}(\mathbf{T})}\right)^m \quad (55)$$

where $\overline{\mu}_1$, a, and b are material constants related to the shear modulus of the material and c, d, f, g, n, and m are material constants related to the relaxation time of the material. Also $\frac{\mu_1}{\eta_1}$ is related to the relaxation time of the material. Eq. (51) is the evolution equation for the left Cauchy-Green stretch tensor (defined in Eq. (17)) and this equation is used to arrive at the values of stretch in the Z direction (Λ(t)) for the motion given in Eq. (41). The calculated stretch (current length to the original length) is used to calculate deformation given in Figs. (6) to (9). The methodology

Table 2. Material Constants Used in Modelling Creep Experiments.

Material Constant	Value
$\overline{\mu}_1$	899.37*MPa*
a	$1\text{x}10^{-4}$
b	$2\text{x}10^{-4}$
c	$7.6\text{x}10^{-5}s^{-1}$
d	$6\text{x}10^{4}$
n	6
f	0.7047
g	2577.22*MPa*
m	4

used for calibrating and validating the material parameters essentially follows the usage of least squares as the maximum likelihood estimator. It is required to pick appropriate values for all these parameters such that the difference in the sum of the squares between predicted and measured values are minimized. This optimization problem is solved using appropriate routines available in software packages such as MATLAB.

The predicted model values along with the experimental parameters are shown here. Figs. (6) to (9) show the model predicted values for cycles 100, 101, 200, and 201 respectively and the material constants used in modelling are shown in Table 2. It is seen that the experimental observations closely match the model predictions.

Summary

The main idea used in the model development stems from the fact that deforming a material like asphalt concrete from its initial configuration (for instance, immediately after placement and compaction) changes the microstructure in such a manner that the subsequent response of the material to loading depends on the current microstructure rather than on the original microstructure. Classical elastic theories correspond to responses wherein the material microstructure does not change during and/or after deformation. Hence, the internal structure to be dealt with is with reference to the reference configuration. However, as discussed above, for materials such as asphalt mixtures, the response changes for every loading/unloading condition and the challenge is to take this into account in the modelling stage. If the experimental investigations reported in this work are carefully observed, it is seen that the response of the material after one hour rest period is markedly different from the response before the rest period. Also, it is seen from the experimental data that this change does not take place infinitely, in the sense that there is an upper bound for the beneficial effects. This is understandable intuitively as one cannot have material properties changing infinitely for ever.

In this investigation, healing is characterised by means of material functions that depend on the difference in the elastic response before and after the rest periods and also on the mapping between the natural configurations of the material before and after the rest periods. The results obtained in this investigation and the framework used in predicting the experimental results can be used in building an appropriate nonlinear viscoelastic constitutive model for asphalt mixtures that takes into account all the complex response characteristics.

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