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Reliability Calculation Considering Nonlinear Fatigue Damage in Asphalt Pavements

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Abstract: An asphalt pavement deteriorates due to repetitive application of vehicular loads over the service life. Due to materials deterioration, the accumulated fatigue damage increases nonlinearly with the number of load repetitions. This paper presents an approach for estimation of nonlinear fatigue damage, using traditional fatigue equation. The deterioration of asphalt stiffness has been utilised to measure the damage caused, specifically at intermediate conditions of the pavement section. In fact, the fatigue failure is considered at an intermediate pavement condition, while performing the pavement design based reliability requirement. The reliability calculation considering nonlinear fatigue damage accumulation has been illustrated through the numerical analysis.

***Key words**: Asphalt pavement; Fatigue failure; Nonlinear damage; Reliability.*

Introduction

Fatigue due to load repetitions is one of the important distress mechanisms for asphalt pavements. In the pavement design process, generally a parameter named as fatigue damage factor (D) is used to measure the damage caused. For given loading conditions, D may be defined as the ratio of total number of load repetitions (n) to the fatigue life (N) of the pavement section. For mixed traffic loading conditions, D can be evaluated using load equivalency factors [1-3] or Miner's hypothesis of linear damage accumulation [2, 4-5]. Deterministically, D = 1 (i.e. n = N) at the failure situation. Probabilistically, D value need not necessarily be one at the failure, and it depends on the reliability requirement. Thus, the estimation of D is essentially required for reliability based pavement design, where the design solution is obtained at an intermediate condition (or $n \neq N$) of the pavement structure.

For a given set of input data, the fatigue life (N) of the pavement section is constant, which is primarily obtained based on initial strain level. Thus, the damage parameter D (= n / N) becomes linear with n. That means, the fatigue damage caused by initial 'Δn' repetitions is same as that of last 'Δn' repetitions before failure, although the structural conditions are different due to material deterioration with n. Miner's [2] also postulated the damage based on initial condition of the structure, but with different load (or strain) levels. However, for the same loading intensity also, the strain level of an in-service pavement changes with the pavement age (or n), due to material degradation. Under such strain varying situation, D would be nonlinear with n. Thus, the objective of the present paper is to predict the nonlinear fatigue damage, based on material degradation characteristics. However, to accommodate the material degradation, Oliveira *et al*. (2008) [6] suggested a cumulative approach using traditional fatigue equation, and a new fatigue life (hereby referred as cumulative fatigue life) was predicted. The present work identifies some of the limitations of this approach, and accordingly, it suggests the necessary modifications. This modified cumulative approach has been explained through the reliability calculation, specifically at intermediate pavement conditions.

Estimation of Fatigue Life and Fatigue Damage

Fatigue of asphalt pavements occurs due to repetitions of vehicular load. To this effect, various energy/fracture based fatigue mechanisms are available in literature [7-10]. Fracture mechanics assumes that an initial crack exists, and thereafter, the number of load repetitions is empirically correlated with the propagated crack length. However, for an in-service pavement, most of the load repetitions till failure may pass before the cracks initiated [11]. Also, the fatigue failure for a pavement structure is specified by certain amount of surface cracks area and it is not specified by propagated crack length.

Knowing the fact that substantial complexity exists with fatigue mechanism of asphalt pavements [8, 10, 12-13], the fatigue equations used for pavement design are still phenomenological in nature. Primarily, the initial critical tensile strain (ε) at the bottom of the asphalt layer is used to predict the fatigue life (N). A simple form of traditional fatigue equation can be expresses as,

$$N = k_1 \times \varepsilon^{-k_2} \tag{1}$$

where k_1 and k_2 are two regression constants. Sometimes the stiffness, air voids and effective binder content of the asphalt mix are also included in the fatigue equation [2-4, 14-16]. A review on this aspect can be seen elsewhere [13, 17-21].

Traditionally, Eq. (1) is developed based on initial strain value corresponding to initial stiffness of the asphalt material. However, the stiffness (E) of asphalt material for an in-service pavement decreases with increase in number of load repetitions (n). Various researchers [6, 22-24] investigated the reduction of E value with n. These studies attempted to correlate the E with n either from the laboratory study or from the in-service pavements using back calculation process. A stiffness reduction model may be expressed as [6],

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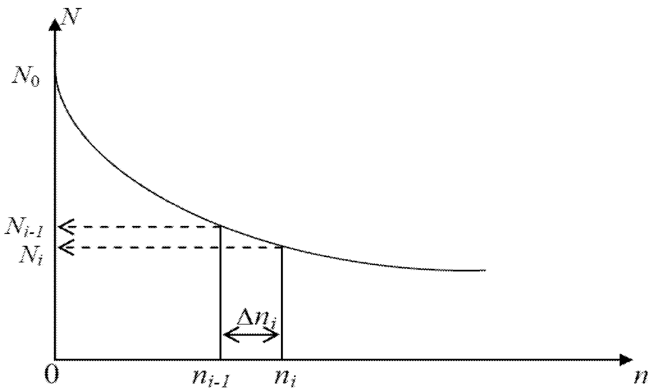

Fig. 1. A Schematic Diagram Showing the Variation of N with n.

Table 1. Properties of Pavement Materials Considered.

Material	Stiffness (MPa)	Poisson's Ration
Foundation Class 1	50	0.35
Foundation Class 2	100	0.35
Foundation Class 3	200	0.35
Grouted Macadam (GM)	8000 (= E_0)	0.25

$$E(n) = E_0 \times \left[-a \times \ln\left(\frac{n}{N_0}\right) + b \right] \tag{2}$$

where, $E(n)$ represents the stiffness of asphalt material after n repetitions; E_0 is the initial stiffness of asphalt material; N_0 is the traditional fatigue life based on initial strain (or E_0); and, a and b are two constants. Using the initial strain (ε_0) value corresponding to initial stiffness E_0, the N_0 value can be obtained from Eq. (1).

To find out the constants a and b, two boundary conditions may be applied into Eq. (2). These are (i) E = 50% of E_0 when $n = N_0$ (i.e. the failure condition), and (ii) $E = E_0$ when $n = 1$ (i.e. the initial condition). Thus, applying these two conditions the following two relationships can be established from Eq. (2).

At

$$n = N_0\,,\ \ E = 0.5E_0 \quad \Rightarrow 0.5E_0 = E_0 \times b \qquad \Rightarrow b = 0.5 \tag{3}$$

At

$$n = 1,\ \ E = E_0 \ \Rightarrow E_0 = E_0 \times [a \ln N_0 + b] \Rightarrow a = \frac{1-b}{\ln N_0} = \frac{0.5}{\ln N_0} \tag{4}$$

Thus, the a and b values can be obtained for known N_0 value of the section considered.

Eq. (1) estimates the fatigue life for different ε values. ε values may be different due to different layer(s) thicknesses, loading conditions, materials properties (say, stiffness E) etc. Therefore, for a given pavement section the parameter ε may be considered as function of E, which in turn depends on n (refer Eq. (2)). Thus, after suitable loading interval (Δn), the E value and the corresponding ε value can be determined. Accordingly, the remaining fatigue life (N) of the pavement section can be obtained using Eq. (1). The variation of N with n has been shown schematically in Fig. 1. It may be mentioned that the remaining fatigue life is evaluated based on reduction of asphalt stiffness with n. The stiffness reduction of lowering layers like granular base, subgrade etc may be considered negligible for fatigue consideration. Further, in the present work, the effect of temperature, subgrade moisture etc. have not been accounted separately.

Now, applying the Miner's hypothesis, the cumulative fatigue damage may be evaluated as [2],

$$D^C(n_j) = \sum_{i=1}^{j} \frac{\Delta n_i}{N_{i-1}} \tag{5}$$

where, $D^C(n_j)$ represents the fatigue damage factor after n_j repetitions as per cumulative approach; Δn_i is the number of load repetitions during i-th loading interval; N_i is the remaining fatigue life after i-th loading interval; and, $n_j = \sum_{i=1}^{j} \Delta n_i$.

Considering $D^C = 1$ as failure criterion, the cumulative fatigue life (N^C) can be evaluated as,

$$N^C = \sum_{\forall i} \Delta n_i\,;\ \text{for}\ \ D^C = 1 \tag{6}$$

Similarly, using the traditional approach the accumulated fatigue damage can be written as,

$$D(n_j) = \sum_{i=1}^{j} \frac{\Delta n_i}{N_0} = \frac{1}{N_0}\sum_{i=1}^{j} \Delta n_i = \frac{n_j}{N_0} \tag{7}$$

where, $D(n_j)$ represents the fatigue damage factor after n_j repetitions as per traditional approach. $D = 1$ at the failure situation. It may be mentioned that $D^C(n_j) > D\ (n_j)$; $\forall j$. Also, the traditional damage factor D is a linear function of n, whereas cumulative damage factor D^C is a nonlinear function of n. It is expected that the respective values of traditional fatigue life N_0 and cumulative fatigue life N^C would be significantly different. This has been elaborated further through the numerical analysis.

Comparison of N_0 and N^C

For numerical comparison of N_0 and N^C values for a given pavement section, the following data are used.

Material properties for four different types of foundation class and grouted macadam (GM) asphalt layer of pavement sections are given in Table 1 [6]. GM layer thicknesses for different sections are taken as 150, 200 and 250 mm. k_1 and k_2 (Eq. (1)) values are 2.7×10^{-9} and 3.9718 respectively [6]. The present annual traffic (P) in terms of standard axle loads of 82 kN (dual wheel) is taken as 0.05 msa (million standard axles) with an annual growth rate (r) of 7.5%.

For a given pavement section, the initial strain (ε_0) value can be determined based on initial stiffness E_0. Strain can be obtained using multilayer elastic analysis [25]. Thus, the N_0 value of the section can be calculated using Eq. (1). For instance, N_0 value is obtained as 11.8 msa for the pavement section with Foundation class 1 and 150 mm GM layer thickness.

For the given traffic data, the traffic repetitions during the i-th year can be estimated as $P \times (1 + r/100)^{i-1} = 0.05 \times 1.075^{i-1}$ msa. That means, for yearly loading interval, $\Delta n_i = 0.05 \times 1.075^{i-1}$ msa

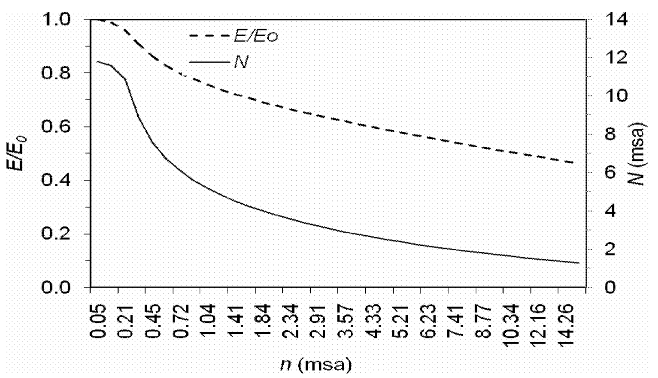

Fig. 2. Variation of E/E_0, and N with n (Foundation Class 1, 150 mm GM Layer).

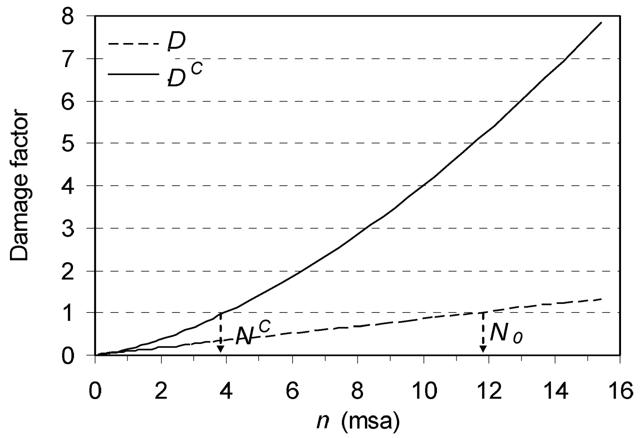

Fig. 3. Variation of D^C and D with n (Foundation Class 1, 150 mm GM Layer).

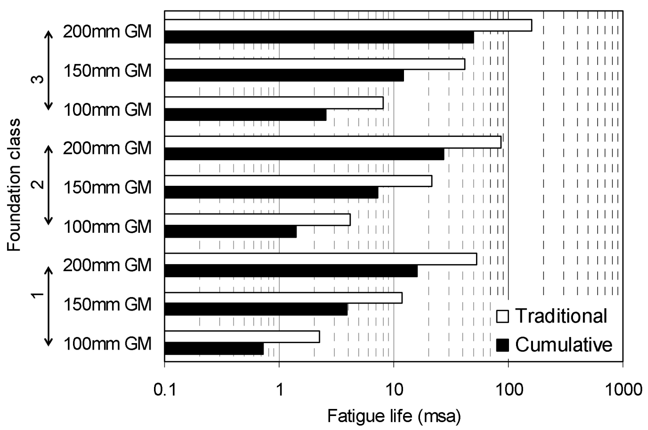

Fig. 4. Comparison of N_0 and N^C for Different Pavement Sections.

and the total load repetitions till i-th year, $n_i = P \times \left[(1 + r/100)^i - 1\right]/(r/100) = 0.667 \times \left[1.075^i - 1\right]$ msa. Thus, using Eq. (2) the year wise E value can be predicted for each n value. Accordingly, the corresponding strain (ε) value and the remaining fatigue life N (Eq. (1)) can be computed. Fig. 2 shows the variation of E and N with n, for the case of Foundation class 1 with 150 mm of GM layer thickness.

At the end of every Δn_i repetitions, the D^C and D values can be estimated using Eq. (5) and Eq. (7) respectively. The variation of D^C and D with n is shown in Fig. 3 (for Foundation class 1 with 150 mm GM layer thickness). Finally, the value of N^C can be obtained using Eq. (6). The N^C and N_0 values are also depicted in the figure (Fig. 3), for the case of Foundation class 1 with 150 mm of GM layer thickness.

In a similar way, the N_0 and N^C values can be evaluated for all the pavement sections. Fig. 4 shows a comparison between N_0 and N^C (in logarithmic scale) values for different sections. It is seen that N_0 and N^C values are significantly different for each pavement section. The N^C / N_0 value varies from 0.28 to 0.32 for the sections considered. This is being discussed further in the next section. It may be mentioned that for a given pavement section, the value of N^C / N_0 would remain same irrespective of the traditional fatigue equation used.

Discussion on Cumulative Fatigue Approach

At this stage, the cumulative fatigue approach as proposed by Oliveira *et al.* (2008) [6] has been elaborated and is compared with the traditional approach. This cumulative approach has certain limitations as discussed below.

a) The field fatigue life and laboratory fatigue life for the same strain level are different. Field life can even vary from 5 to 700 times of the laboratory life [26]. Such significant mismatch between the field and laboratory fatigue life is primarily due to the differences in failure criteria and loading conditions. Therefore, to calculate the fatigue life (N_0 or N^C) of any pavement structure, the traditional fatigue equation (i.e. Eq. (1) in present case) needs to be a field fatigue equation. In case a laboratory fatigue equation is used [6], the computed N^C will represent another laboratory fatigue life (under the strain varying situation), which would always be lower than N_0. However, the actual life of pavement structure is normally much higher than the laboratory life. As such, while using a laboratory fatigue equation, the parameter N^C (or the cumulative approach) does not provide any useful information for practical / field purpose. Also, the consideration of strain varying situation (or N^C calculation) arises under the field condition (normally being a constant strain amplitude test under the laboratory condition).

b) N_0 value obtained using field fatigue equation represents a fatigue life under field situation. It is assumed that a field equation accommodates the effect of various field factors including strain variation due to material degradation, through the calibration process. Therefore, the value of cumulative fatigue life N^C should be closed to N_0 for the section considered. But, it is not seen so (refer Fig. 4). The value of N^C is observed in between 28% to 32% of N_0 in the present analysis.

The above discussed cumulative approach (refer to Eq. (5) or Eq. (6)) can take into account of nonlinear fatigue damage (refer Fig. 3). However, while calculating the nonlinear fatigue damage, the traditional fatigue equation (Eq. (1)) is being used repetitively at various intermediate conditions of the pavement section. In fact, Eq. (1) being a field fatigue equation, it correlates the field failure repetitions with the initial strain value of the pavement section.

Therefore, the effect of strain variation caused by the material degradation during the process of field failure has already been accounted while developing Eq. (1). Thus, the traditional fatigue life N_0 shall be equal to the cumulative fatigue life N^C. This is possible through normalization of the cumulative parameter N^C or D^C so that D^C = 1 when D = 1. At the same time, the cumulative damage approach would separate the effect of material degradation (or strain variation) with the pavement age or n. Further, to separate the effect of material degradation, it may not be possible to develop the different field fatigue equations for different intermediate pavement conditions. Keeping these aspects in mind, a modified approach has been proposed and is presented in the next section.

Proposed Approach

Consideration of nonlinear fatigue damage is useful when a pavement design is performed at intermediate condition (i.e. $D \neq 1$), such as reliability based pavement design. Fatigue reliability (R) depends upon probability distribution of the damage parameter D. $R \approx 0.5$ when D = 1 (i.e. the failure criterion used in deterministic design process). $R > 0.5$ when $D < 1$ and vice versa. Normally, a reliability based pavement design is performed with $R > 0.5$ i.e. $D < 1$ as the failure criterion [27-28]. Moreover, $0 < D < 1$ does not indicate any specific amount of fatigue cracking. Under such intermediate pavement conditions, no visible cracks may even exist at the pavement surface. Moreover, certain damage has taken place due to which the material is deteriorated. Therefore, for such intermediate pavement conditions, the material degradation (or stiffness reduction) parameter may be utilized to measure the damage accumulated. At the same time, this shall not affect on the predicted fatigue life of the section, irrespective of the method used. That means, the N_0 and N^C shall approach to the same value. This is possible through normalization of the traditional fatigue equation that can be adopted for estimation of remaining pavement life at various intermediate pavement conditions. Considering these aspects, a modified equation for remaining fatigue life (N) is proposed as follows:

$$N = k_1 \times \varepsilon^{-k_2} \times \left(\frac{N_0}{N^C} \right) \tag{8}$$

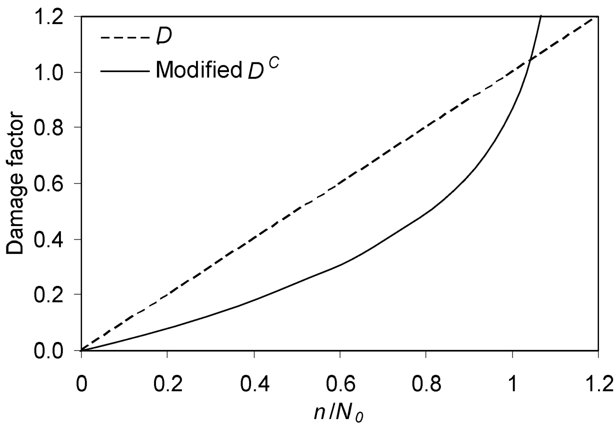

Fig. 5. Variation of Modified D^C and D with n/N_0 (Foundation Class 1, 150 mm GM Layer).

N_0 / N^C is the normalization factor for estimation of remaining fatigue life. In a similar way, Eq. (8) can be used to predict the modified N value at different strain (ε) levels based on stiffness reduction (Eq. (2)) of the asphalt material. Thus, the modified D^C value can be obtained using Eq. (3). Fig. 5 presents the comparison between the modified D^C and D for the case of Foundation class 1 with 150 mm GM layer thickness. Similar plots are also observed for the other cases.

From Fig. 5, it is seen that modified D^C value is approached to unity when n / N_0 = 1 (or D = 1). That means, the fatigue life predicted through this procedure is approached to N_0 value. At the same time, this procedure facilitates the prediction of nonlinear fatigue damage with n (or pavement age). This has been further illustrated through reliability calculation.

Reliability Calculation

For reliability calculation, one needs to know the probability distribution of damage parameter. This can be derived for any known distribution of traffic and fatigue life parameter [28]. In this work, it is considered that the damage parameter (D or D^C) follows a lognormal distribution with coefficient of variation (COV) equal to 40%. Traffic data are taken as mentioned earlier. Pavement section with Foundation class 1 (refer Table 1) and 150 mm thickness of GM layer is considered.

For lognormal distribution of D or D^C (i.e. normal distribution of ln(D) or ln(D^C)), the fatigue reliability (R) at any intermediate conditions (i.e. $D < 1$ or $D^C < 1$) of pavement can be evaluated as [28],

$$\begin{aligned} R &= 0.5 + \mathbf{N}\left(-\frac{\ln D}{sd_{\ln D}} \right); \text{for linear damage} \\ &= 0.5 + \mathbf{N}\left(-\frac{\ln D^C}{sd_{\ln D^C}} \right); \text{for nonlinear damage} \end{aligned} \tag{9}$$

where, $\mathbf{N}(z)$ indicates the probability for standard normal deviate z ($\mathbf{N}(0) = 0$); and sd_X represents the standard deviation of variable X. $sd^2_{\ln X} = \ln\left(1 + \{sd_X / X\}^2\right) = \ln(1 + COV_X^2)$. Therefore, in this case $sd_{\ln D} = sd_{\ln D^C} = \sqrt{\ln(1 + 0.4^2)} = 0.385$. Thus, using Eq. (9) the different R values can be estimated for different values of D or D^C. For the case of Foundation class 1 with 150 mm of GM layer thickness, the estimated R values are tabulated in Table 2 for different values of load repetitions (n). It is observed that for the same value of n, the estimated R values based on D and modified D^C are significantly different. Thus, it may be concluded that for a

Table 2. R values based on D and Modified D^C Parameter (N_0 = 11.8 msa).

Parameter	20th year	25th year	30th year
Repetitions n (msa)	4.330	6.798	10.34
D value	0.37	0.58	0.88
Modified D^C value	0.16	0.28	0.68
R value (based on D)	0.99	0.92	0.63
R value (based on D^C)	1.00	1.00	0.84

given design reliability level, the design solutions would be different based on D and modified D^C parameter. It may also be mentioned that the similar differences on estimated R values are observed for the other cases.

Conclusions

This paper presents an iterative approach for nonlinear fatigue damage evaluation in asphalt pavements. The accumulated fatigue damage increases at increasing rate with the number of load repetitions (n). At the same time, the deterministic value of fatigue life (i.e. modified N^C) remains same as that of traditional fatigue life (N_0) of the pavement section. Traditional field fatigue equation and the asphalt stiffness reduction model are used simultaneously, while evaluating the fatigue damage. This approach is general enough, which can accommodate any traditional fatigue equations and any stiffness reduction models. This methodology may be useful in reliability based pavement design process, where the design solution is obtained at an intermediate pavement condition, i.e. $n \neq N$ at the failure condition.

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