

Guidelines of Roadway Undulation Measurements with Straightedges

Chen-Ming Kuo¹⁺ and Ting-Yi Tsai¹

Abstract: Straightedge is still widely adopted as a simple and reliable tool in pavement smoothness measurements, although rolling profilers have become more and more popular. If measured and interpreted correctly, the data collected with a straightedge is able to provide not only undulation depth but also profile wavelength. This study examines the basic features of measurements of roadway undulations with straightedges as well as the critical vibration frequencies for riding comfort. A universal gain curve is proposed to adjust measurements with various straightedges. A vehicle-pavement coupled system is formulated to find the natural frequencies of a vehicle, wheel axle, and pavement. The system also determines the wavelengths critical to riding comfort on roadways. Analysis with the coupled system shows that the vibrations critical to riding comfort may be magnified and transmitted through the wheel axle. The research suggests the suitable straightedge for specific wavelengths. The 3 meter straightedge is recommended on freeways with speed limits over 100 km/h, whereas the 1 meter straightedge is more suitable for urban streets to assure correct amplitudes of the critical waviness to riding comfort. Measurement interval is critical to the resolution of profile wavelengths. It is suggested that the measurement interval not exceed half of the critical wavelength.

Key words: Gain curve, Riding comfort, Roadway undulation, Straightedge, Wavelength.

Introduction

While pavement studies have improved the structural design and service life of highway pavements in recent decades, riding quality still needs improvement and should be the next goal in enhancing pavement serviceability. Locating the undulation to ensure a smooth running surface, the 3 meter straightedge is one of the most popular and traditional apparatus with its minimum maintenance and training. Although the resolution of a pavement profile is limited because of the large interval of measurement points, it is still a major device in Taiwan used to determine the acceptance criteria of highways and streets because the measurements are straightforward and instinctively interpretable [1]. The 3 meter straightedge is also widely used by highway agencies worldwide according to a recent survey of smoothness measurements on flexible pavements, which show that 22% of U.S. highway agencies use the 3 meter straightedge [2, 3]. Thorough studies on the methods of measuring and assessing road evenness have been conducted through collaborations between several European highway research laboratories via theoretical and operational aspects [4]. Most efforts were devoted to data processing methods and the influences of errors on roughness indices as well as the correlations between different quantifiers [3].

A straightedge normally associates various gains for profiles with different wavelength. As shown in Fig. 1, the 62 ft straight chord may measure double the undulation amplitude on a 62 ft. wave and result in zero amplitude on a 31 ft wave. In other words, a straightedge can only find the actual amplitude for certain wavelengths [5]. The gain of a straightedge on a specific wavelength equals the ratio of the measured wave amplitude to the

actual wave amplitude. Since the gain curve of mid chord offset illustrates the geometric features of a cord and curves, it is applicable to both the rolling profilometer and physical straightedge.

The gain curve fluctuates dramatically in the wavelength range below 4 m, as shown in Fig. 2. For wavelengths ranging between 4 m and 10 m, the differences between the actual amplitudes and the measured amplitudes are bound between +75% and -75%. If the ride quality of a roadway is sensitive to undulations of smaller than a 4 m wavelength, the 3 meter straightedge may result in tremendous bias depending on measurement locations.

Before evaluating the adequacy of 3 meter straightedge, the critical wavelengths for vehicle occupants should be defined first. Ride quality is primarily dominated by vibration frequencies less than 20 Hz [6]. By taking into account the vibration amplification in the seat at frequencies of 3 Hz to 5 Hz, the most uncomfortable undulation wavelengths range between 1.7 m and 13 m for vehicle speeds between 30 km/h and 140 km/h [6]. The w_k filter in ISO 2631 considers the vertical vibration energy transfer into the body of a person standing or seated, such as a regular road user [7]. The greatest weight of the filter is applied to 4 Hz to 10 Hz, which are the most sensitive frequencies for human perception in vertical direction.

Although the offset between the midpoint of lath and road surface (so called double amplitude) is recorded in acceptance testing and routine evaluations in Taiwan, the practitioners generally do not acknowledge gains of measurements on various waviness [8]. This paper aims to find the most appropriate straightedge by considering its gain features, traveling speed, and vehicle suspensions. The purpose of this study is to reveal the critical undulation wavelengths for various roadway categories and to suggest the most appropriate straightedge to control pavement smoothness. Road authorities should be able to account for riding comfort by an overview of vehicle speed, spectrum of roadway undulation, features of straightedges, and vehicle suspension as a system.

Excitation Frequencies

¹ Department of Civil Engineering, National Chen Kung University, 1 University Road, Tainan City, Taiwan.

⁺ Corresponding Author: E-mail ckuo@mail.ncku.edu.tw

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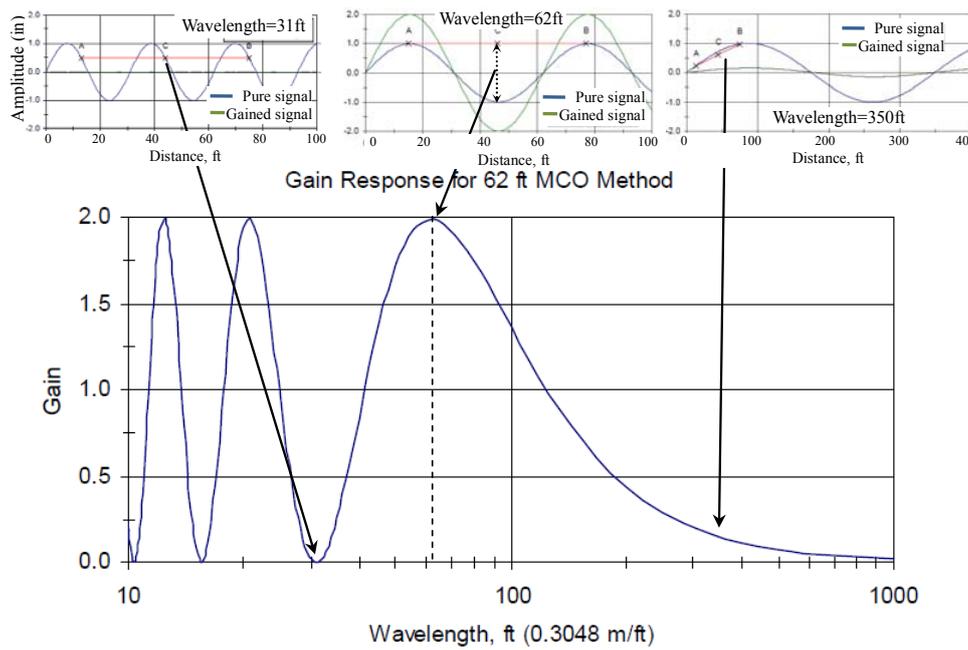


Fig. 1. Illustration of the Gain Curve of a 62ft Straightedge [12].

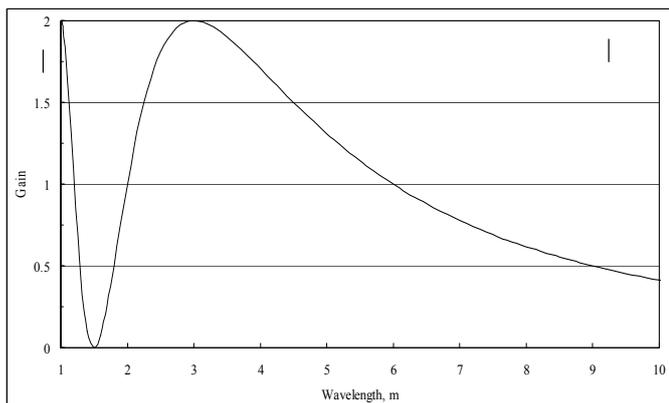


Fig. 2. Gain Curve of a 3 Meter Straightedge.

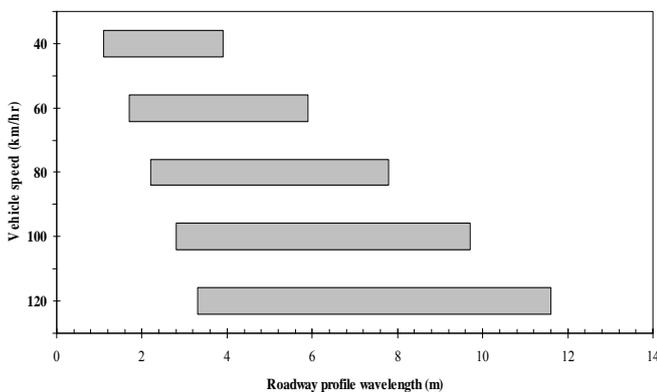


Fig. 3. Critical Wavelength Ranges Causing Unpleasant Riding Comfort.

A vehicle passing a specific roadway undulation with various speeds causes different excitation frequencies at the vehicle-pavement contact area. For example, a $v=10$ m/s moving wheel passes a

complete cycle of a sinusoidal undulation of $L=1$ m wavelength in 0.1 s. In other words, the wheel experiences a full displacement cycle in 0.1 s, (e.g. an $f=10$ Hz excitation) as shown in Eq. (1). Referring to the most unpleasant vertical frequencies mentioned above, Fig. 3 illustrates that wavelengths around 1 m to 4 m may result in 10 Hz to 4 Hz excitation for 40 km/h vehicles, whereas long wavelengths around 3 m to 11 m become critical for high speed vehicles.

$$f = \frac{v}{L} \tag{1}$$

The bouncing frequencies experienced by passengers in a suspended vehicle might not be the same as excitation frequencies resulting from road undulations because of the filtering effect of vehicle suspensions. The dynamic magnification factor (D) through wheels can be calculated with Eq. (2) [9]. p_0 represents the wheel load on pavement. The ratio between wheel amplitude ρ and the static deformation of the tire is related to the ratio of the vehicle-pavement interaction force frequency to the natural frequency of wheel (β). In the quarter-car model, only tire stiffness k is considered. The damping effect of the tire, η , is neglected and assumed to be 0. Fig. 4 illustrates the variation of dynamic magnification factor with a frequency ratio. Vehicle-pavement contact forces with frequency ratios below $\sqrt{2}$ may be magnified through wheel suspensions. This implies that the interaction forces above $\sqrt{2}$ times natural frequency of wheel axle will be filtered out of the car body. On the other hand, the interaction forces below $\sqrt{2}$ times natural frequency of the wheel axle may be amplified and transmitted to the car body.

$$D = \frac{\rho}{p_0/k} = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\eta\beta)^2}} \tag{2}$$

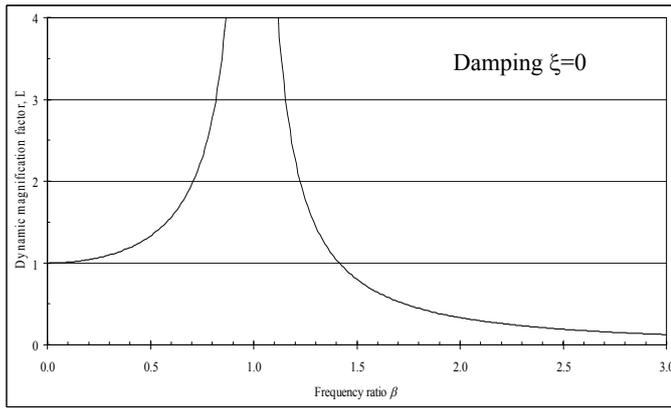


Fig. 4. Variation of Dynamic Magnification Factor with Frequency (Undamped System).

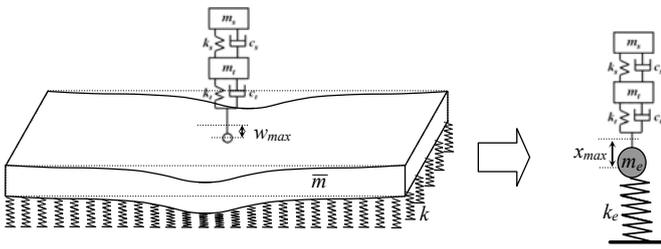


Fig. 5. Coupled Model of Quarter-car and Equivalent S.D.O.F. Pavement.

Table 1. Input Values for Coupled model [11].

Quarter-car	
m_t/m_s	0.15
c_s/m_s	$6s^{-1}$
c_t/m_s	0
k_s/m_s	$62.3s^{-2}$
k_t/m_s	$653s^{-2}$
Pavement	
Slab Thickness (h)	0.2 m
Young's Modulus of Concrete (E)	2.64×10^{10} Pa
Poisson's Ratio of Concrete (μ)	0.15
Unit Weight of Concrete slab (u_c)	2300 kg/m^3
Coefficient of Subgrade Reaction (k)	$2.7 \times 10^7 \text{ N/m}^3$

In order to find the frequency of a wheel axle, a mass-spring-series model was developed by coupling a quarter-car model and an equivalent pavement model of single degree of freedom, as shown in Fig. 5 [10]. The equilibrium equations of the system are shown as Eqs. (3) - (6), in which m_s , k_s , c_s , and y_s are mass, suspension spring stiffness, suspension damping, and displacement of the car body, respectively; m_t , and y_t are mass and displacement of the wheel axle, respectively; and k_t and c_t are spring stiffness and damping of the quarter-car wheel, respectively. The vehicle-pavement interaction force can be determined by responses to the wheel spring and damping, as shown in Eq. (3). Equations (4) and (5) represent the equilibrium states of the suspended car body and unsprung wheel axle, respectively. The pavement motion is described as Eq. (6). The equations were further expressed in matrix form, transformed by Fourier transformation and then solved in frequency domain. The characteristic frequencies of the system were determined by

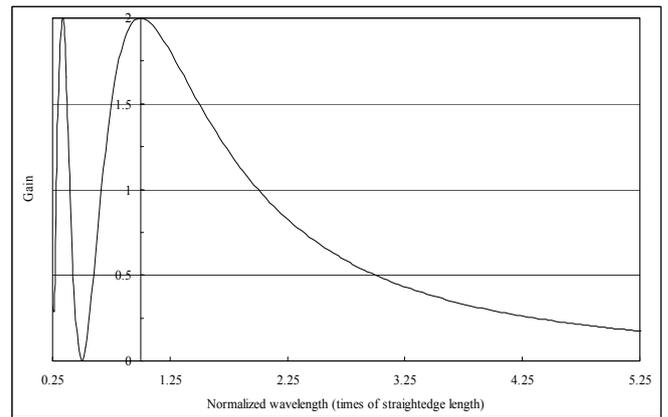


Fig. 6. Gain Curve for All Lengths of Straightedge.

deriving the eigenvectors of the equations above in matrix forms. The typical quarter-car parameters, as shown in Table 1, were used to evaluate typical natural frequencies of the coupled system. The resonance frequencies are 1.2 Hz for the suspended carbody, 10.8 Hz for the wheel axle, and 26.4 Hz for the 1 d.o.f. pavement model.

$$p = c_t(\dot{y}_t - \dot{\xi} - \dot{w}) + k_t(y_t - \xi - w) \tag{3}$$

$$m_s \ddot{z}_s + m_s \ddot{z}_t + m_s \ddot{w} + c_s \dot{z}_s + k_s z_s = -m_s \ddot{\xi} - m_s g \tag{4}$$

$$m_t \ddot{z}_t + m_t \ddot{z}_e - c_s \dot{z}_s + c_t \dot{z}_t - k_s z_s + k_t z_t = -m_t \ddot{\xi} - m_t g \tag{5}$$

$$m_e \ddot{w} - c_t \dot{z}_t - k_t z_t + k_e w = -m_e g \tag{6}$$

where $z_s = y_s - y_t$; $z_t = y_t - \xi - w$; ξ =surface roughness, w = vertical displacement of the equivalent pavement model, g = acceleration of gravity, m_e = pavement mass of the equivalent pavement model, k_e = subgrade stiffness of the equivalent pavement model.

Consequently, the excitation force above $\sqrt{2} \times 10.8 = 15.3$ Hz on the wheel axle may be prevented from extending into the car body. However, the aforementioned vertical frequency for human perception in a car (4-10 Hz) is below 15.3 Hz. That means the uncomfortable excitations may transmit into the car body through wheel axles. As long as the pavement undulation is removed to avoid 4 Hz to 10 Hz of excitation frequency for a specific speed on a matching wavelength, the riding comfort of vehicle occupants may be assured.

Features of Straightedges

The gain curve of a straightedge can be calculated and plotted with Eq. (7), where L is the length of the straightedge and λ is the undulation wavelength. The curves of various straightedge lengths are similar and can be plotted as the universal curve, as shown in Fig. 6, by introducing the normalized wavelength, λ/L .

$$g_L(\lambda) = 1 - \cos\left(\frac{\pi L}{\lambda}\right) \tag{7}$$

Although bias is inevitable in profile measurements with

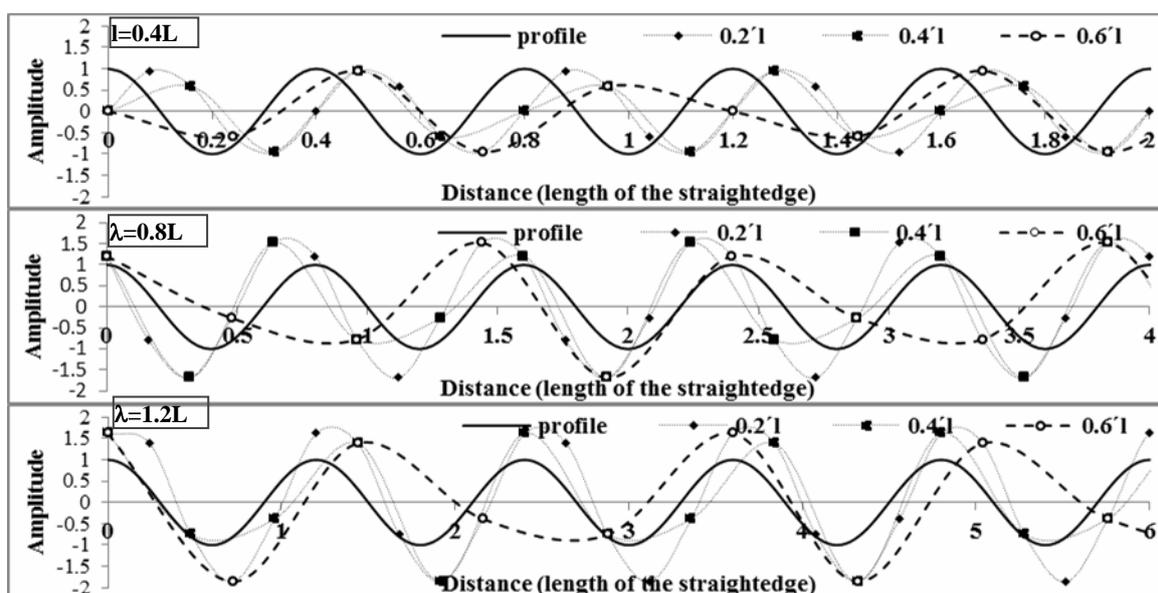


Fig. 7. Wavelength Identification on Various Profiles with Various Straightedges.

straightedge, it is remediable with the gain curve as long as the wavelength is correctly identified. In order to justify that a straightedge can correctly detect a profile with a wavelength shorter than the straightedge, two profiles with wavelengths shorter than the straightedge ($0.4L$ and $0.8L$) and one profile with a wavelength longer than the straightedge ($1.2L$) were analyzed. On the other hand, measurement spacing of 0.2λ and 0.4λ , which are shorter than half of the profile, and 0.6λ were tried to justify the threshold of measurement interval for detecting correct wavelengths of a roadway profile. The spacing of measurement positions dominates the wavelength resolution, as shown in Fig. 7. The 0.6λ sampling interval resulted in a profile of false wavelength compared to the real profile (dashed line versus solid line as shown in Fig. 7), whereas the 0.2λ and 0.4λ reproduced profiles with the same wavelength. It was concluded that a 3 meter straightedge is able to identify profile wavelength correctly as long as the measurement interval is smaller than 0.5λ . The results justify the Nyquist sampling theorem –“the sampling frequency must be greater than twice the bandwidth of the input signal in order to be able to reconstruct the original perfectly from the sampled version.” Consequently, the amplitude can be adjusted with corresponding gain functions.

The pavement smoothness in Taiwan is measured with a 3 meter straightedge at 1.5 m interval. Accordingly, the data should be interpreted for undulations over 3 m wavelength, which fit the target wavelength for a national highway over 100km/h according to Fig. 3. In other words, the present specifications in Taiwan may not garner the correct amplitudes for wavelengths below 3 m, which is critical for riding comfort on streets and local arterial roads.

Selection of Straightedge

Concluding from the work described above, an arbitrary straightedge may not measure amplitudes correctly, especially for sections with critical wavelengths. For example, the critical

wavelengths for riding comfort on streets with 40 km/h speed limit ranges between 1 m and 4 m. If a 3 meter straightedge is adopted to evaluate the street smoothness, it may either under-measure undulations shorter than 2 m or over-measure those longer than 2 m, according to Fig. 2. The curve suggests that straightedge measurements on pavements with stochastic undulations usually deviate from real amplitudes.

However, appropriate straightedges that give relatively proper measurements for roadways with varying characteristics are still needed. Fig. 6 illustrates that the gains are limited within $\pm 50\%$ for pavement profile wavelengths between $1.5L$ and $3L$. Hence, a 1 meter straightedge is good for 1.5 m to 3 m waves, and a 3 meter straightedge is suitable for 4.5 m to 9 m waves. For a 40 km/h street with critical undulations of 1 m to 4 m wavelength, a 1 meter straightedge is more appropriate than a 3 meter straightedge.

Taking into consideration human perception, vehicle-pavement interaction, and straightedge gains, Fig. 8 illustrates the average gains of 1 m, 1.5 m, 2 m, 3 m, 4 m, 5 m, and 6 meter straightedges on roadways with speed limit ranging between 20 km/h and 120 km/h. Only the gains of critical wavelengths for the specific speed were averaged. The shaded areas show that the indicated straightedge results in less than 40% deviation from the real amplitudes. For example, with a feeder road with a speed limit around 60 km/h to 80 km/h, a 1.5 meter straightedge performs better than the others for the critical wavelengths for riding comfort (i.e. 0.4 m to 5.6 m according to Eq. (1)). It was also found that the average gain of a 1 meter straightedge for streets with speed limits between 40 km/h and 50 km/h (critical wavelength = 1.1 m - 3.5 m) is less than 40%. This result matches the rule of thumb mentioned in the previous paragraph.

Fig. 8 suggests that a 2-meter straightedge is most suitable for arterial highways with speed limits between 70 km/h and 100 km/h. A 3 meter straightedge is advised for freeways with speed limits over 100km/h, because the speed is sensitive to long wave undulations. Straightedges longer than 3 m are not only

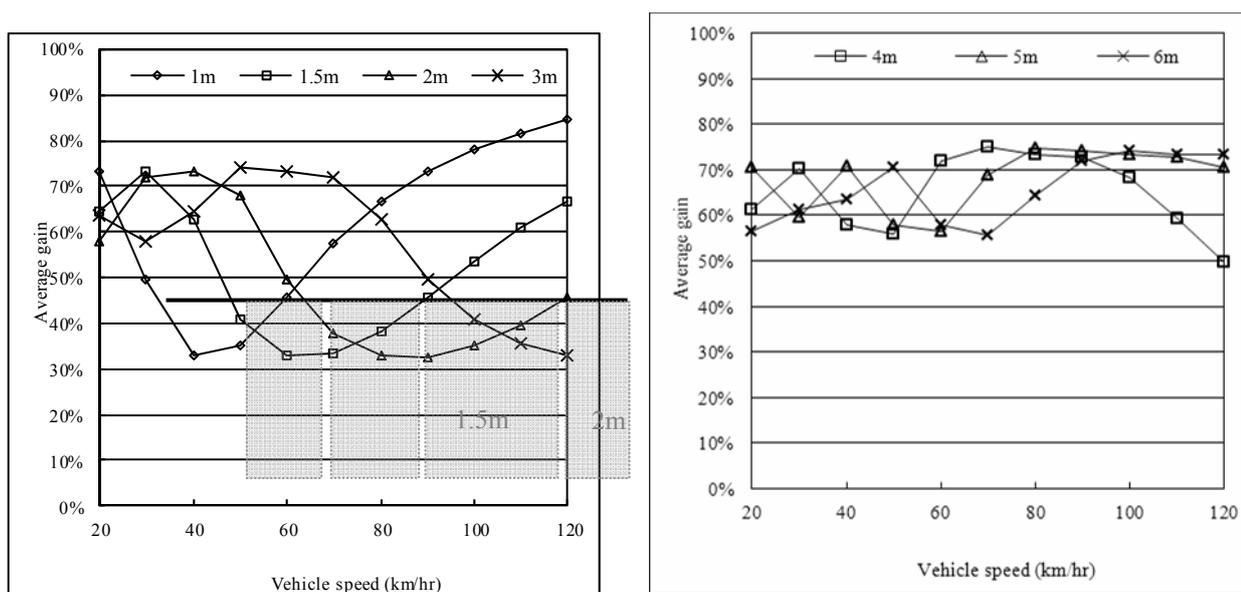


Fig. 8. Average Gains of Various Straightedges for Target Wavelengths of Various Speeds.

inconvenient to carry and operate but are also prone to more deviations on highways of general vehicle speeds.

Conclusions

This study examines the basic features of measurements of roadway undulations with straightedges. Pavement maintenance will be more efficient and effective if resurfacing is focused on the selected wavelengths critical to riding comfort as perceived by passengers. A vehicle-pavement coupled system was formulated, and the theory of structural dynamics was reviewed to determine the critical wavelengths on roadways for riding comfort in vehicles. Several conclusions are summarized:

1. The excitation forces caused by vehicle-pavement contact may be magnified at frequencies below $\sqrt{2}$ times the natural frequency of the wheel axle. For the example in this study, the uncomfortable frequencies (4 Hz - 10 Hz) are located in the magnification range, which are below $\sqrt{2} \times 10.8 = 15.3$ Hz, in which 10.8 Hz is the resonance frequency of the wheel axle.
2. A 1 meter straightedge is more efficient and accurate than long straightedges for roadways with speed limits of 40-60 km/h in detecting the waves that range between 1 m and 4 m, which are most critical to riding comfort with the traveling speed. For freeways with speed limits over 100 km/h, a 3 meter straightedge is relatively efficient and accurate in identifying the vertical profile wavelengths that range between 3 m and 11 m.
3. The gains of measurements are normalized and presented as a universal gain curve for all wavelengths of roadway vertical profiles measured with a straightedge. The gain value only depends on a single dimensionless factor—ratio of straightedge length to profile wavelength. The measured amplitudes should be adjusted according to the gain curve to compensate the gains.
4. Measurement interval dominates wavelength resolution of

roadway undulations. Roadway profile measurements can only identify undulation wavelengths longer than 2 times the sampling interval. For urban streets with 40-60 km/h speed limits, it is important to eliminate the surface waves with 1 m to 4 m wavelengths. Therefore, it is necessary to measure the undulation amplitude at intervals less than 0.5 m to ensure the correct amplitudes for short waves (1 m). The present specifications in Taiwan requiring a 1.5 m sampling interval is suitable for all kinds of roadway profile surveys.

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