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Accurate Determination of Equivalent Modulus of Nonlinear Anisotropic Granular Base Layer

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Abstract: A variety of mechanistic models have been proposed to characterize the resilient behavior of unbound granular materials as a function of applied stress states including confinement and shear effects. However, significant computation effort is needed to implement the nonlinear stress-dependent modulus in the pavement structure response analysis. In this study, an equivalent modulus of granular base layer was developed to represent the nonlinear anisotropic behavior of aggregate base layer using three-dimensional finite element modeling analysis. The analysis results indicate that the equivalent modulus is not a constant value but changes with temperature, vehicle speed, and load. The equivalent modulus increases as the temperature increases but decreases as the vehicle speed increases due to the viscoelastic nature of asphalt surface layer. The equivalent modulus results in comparable results in the prediction of critical pavement responses, compared to the responses predicted using the real nonlinear anisotropic model for the granular base. The developed equivalent modulus can be used as an alternative approach when the sophisticated model is not available or the analysis needs to be conducted in a quick manner.

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***Key words**: Equivalent modulus, Finite element modeling; Granular base; Nonlinear modulus.*

Introduction

The resilient modulus of granular base layer is a key mechanistic pavement analysis and design input for conventional flexible pavement. The conventional pavement design method treats the granular base layer as linear elastic material with a constant Poisson's ratio. However, the nonlinear anisotropic behaviour of the unbound base layer has been well documented. The modulus of the base layer varies depending on the stress transmitted into the base layer and the modulus is different in vertical and horizontal directions [1]. The orientation of aggregate is controlled by its shape, compaction methods, and loading conditions. A special type of anisotropy, known as cross-anisotropy, is commonly observed in pavement granular base layer due to compaction and the applied wheel loading in the vertical direction. Previous researchers have found that for a certain set of aggregate the horizontal stiffness in the granular layer is only 3 to 21% of the vertical stiffness, and the shear stiffness is 18 to 35% of the vertical stiffness. The nonlinear-anisotropic approach is shown to account effectively for the dilative behavior observed under the wheel load and the effects of compaction-induced residual stresses [2, 3].

As the pavement design method shifts from an empirical procedure to a mechanistic-empirical (M-E) method, it is critical to consider the nonlinear anisotropic granular behavior in the pavement response model [4-8]. However, significant computation time is needed in order to implement the stress-dependent model in the pavement structure analysis since iterations are required to determine the modulus corresponding to the stress state in the granular base layer. In addition, converge issue may be confronted if several other nonlinear factors are considered in the pavement model (such as the viscoelasticiy of asphalt material and plasticity of soil subgrade). Therefore, this study aims to develop an equivalent modulus of the nonlinear anisotropic granular base layer, which is linear isotropic modulus that can be used in the traditional multilayer elastic pavement analysis.

Two main methods can be used to derive the equivalent modulus of the granular base layer. The first approach is to subdivide the granular layer into a number of sublayers and determine the equivalent modulus for each stress point located at the mid-depth of each sublayer. The second method considers the granular layer as a single layer. In this case, the proper selection of the single stress point is critical and it is difficult for using a single stress point to represent the decreasing trend of modulus with depth. With the second method, it may not reproduce the same critical pavement responses as those obtained by using the real nonlinear stress-dependent modulus. Therefore, the first sublayer approach is used in this study.

It is noted that the author agrees that the nonlinear anisotropic behavior of granular baser layer should be considered in the pavement response and performance modeling if possible. The developed equivalent modulus should be used as an alternative approach when the sophisticated model is not available or the analysis needs to be conducted in a quick manner.

Viscoelastic Asphalt Concrete Layer

Mechanistic analysis of pavement responses requires constitutive modeling of each pavement layer. While elastic theory may be a reasonable approximation for asphalt concrete in the conventional design of flexible pavements, the effect of time (or frequency) and temperature dependency of asphalt concrete modulus cannot be fully considered using this approach. The time-dependent nature of

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asphalt concrete modulus is characterized by the fact that the stress depends not only on the current state of strain but also on the full history of strain development. An integration model decomposed for the deviatoric and bulk stresses is usually used in the 3-D viscoelastic theory (Eqs. (1) and (2)) [9]. In these equations, the relaxation modulus can be modeled as a generalized Maxwell solid model in terms of a Prony series, Eqs. (3) and (4) [10].

$$s=\int_{-\infty}^{t}2G(t-\tau)\frac{de}{d\tau}d\tau \tag{1}$$

$$p=\int_{-\infty}^{t}K(t-\tau)\frac{d(tr[\varepsilon])}{d\tau}d\tau \tag{2}$$

$$G(t)=G_0(1-\sum_{i=1}^{n}G_i(1-e^{-t/\tau_i})) \tag{3}$$

$$K(t)=K_0(1-\sum_{i=1}^{n}K_i(1-e^{-t/\tau_i})) \tag{4}$$

where s is deviatoric stress; e is deviatoric strain; p is volumetric stress; $tr[\varepsilon]$ is trace of volumetric strain; G is shear modulus; K is bulk modulus; t is relaxation time; G_0 and K_0 are instantaneous shear and volumetric elastic moduli; and G_i, K_i; and τ_i are Prony series parameters.

The temperature dependency of asphalt concrete modulus is characterized by time-temperature superposition principle because the asphalt concrete has been proved as a thermorheologically simple (TRS) material. Therefore, the effect of temperature on the asphalt concrete modulus can be considered using the reduced time (or frequency). This behavior allows for the horizontal shifting (along time or frequency axis) of the material property to form a single characteristic master curve as a function of reduced time (or frequency) at a desired reference temperature (Eqs. (5) and (6)). The amount of horizontal shift is decided by the time-temperature shift factor. The relationship between the shift factor and the temperature can be approximated by the Williams-Landell-Ferry (WLF) function (Eq. (7)) [10]. When combined with the master curve, the time-temperature shift factor allows for the prediction of the viscoelastic behavior over a wide range of conditions.

$$E(t,T)=E(\xi) \tag{5}$$

$$\xi=t/a_T \tag{6}$$

where t is time before shifting for a given temperature, T; ξ is reduced time at reference temperature; and a_T is shift factor for temperature T.

$$log(a_T)=-\frac{C_1(T-T_0)}{C_2+(T-T_0)} \tag{7}$$

where T_0 is reference temperature; T is actual temperature corresponding to the shift factor; and C_1, C_2 are regression parameters.

The viscoelastic property of asphalt concrete can be predicted using the time-dependent creep compliance test, or the frequency-dependent complex modulus test. The relaxation modulus E(t) is the ratio of stress response to a constant strain input, while the creep compliance, D(t), is the ratio of the strain response to a constant stress input. For a purely elastic material, E(t) and D(t) are reciprocals. However, for the viscoelastic material, this is only true in the Laplace transform domain. There is an exact relationship between the creep compliance and relaxation modulus using the convolution integral in Eq. (8) [9].

$$\int_0^t E(t-\tau)D(\tau)d\tau=t \qquad \text{for } t>0 \tag{8}$$

An approximate method can be used to convert from creep compliance to relaxation modulus if both the creep compliance and relaxation modulus are modeled using a power law analytical form, as shown in Eq. (9). Practically, the laboratory-determined creep compliance is not exactly represented by power law function. In this case, the local slope of the power model can be determined using Eq. (10) [11]. The bulk (K) and shear (G) relaxation modulus are calculated from relaxation modulus (E) assuming a constant Poisson's ratio and fitted into the Prony series as a generalized Maxwell solid model (Eqs. (3) and (4)).

$$E(t)D(t)=\frac{\sin n\pi}{n\pi} \tag{9}$$

$$n=\left|\frac{d\,log D(t)}{d\,log t}\right| \tag{10}$$

where $E(t)=E_1t^{-n}$ is relaxation modulus; and $D(t)=D_1t^{n}$ is creep compliance.

Nonlinear Anisotropic Granular Base

The resilient modulus of unbound material is defined as the ratio of the deviatoric stress to the recoverable part of the axial strain from the triaxial load tests, as shown in Eq. (11). Many nonlinear models have been proposed over the years to incorporate the effect of stress level on the resilient modulus. The most commonly used nonlinear elastic model is the k-θ model or the two-parameter bulk stress model. Uzan (1992) introduced the effect of octahedral shear stress to the k-θ model and added atmospheric pressure as a normalizing factor [1]. The octahedral shear-stress term is believed to account for the dilation effect that takes place when a pavement element is subjected to a large principal stress ratio directly under a wheel load.

$$M_r=\frac{\sigma_d}{\varepsilon_r} \tag{11}$$

where σ_d is deviatoric stress; and ε_r is recoverable strain.

An isotropic model has the same resilient modulus in all directions, while a cross-anisotropic model has different material properties (i.e., resilient modulus and Poisson's ratio) in the horizontal and vertical directions. Previous research studies have proved that granular base layers in the pavement exhibit cross-anisotropic behavior due to compaction and the wheel loading applied in the vertical direction [2, 3]. Assuming the 1-2 plane (horizontal plane) to be the plane of isotropy, the constitutive stress-strain relation for cross anisotropy can be expressed as in Eq. (12) [13]. Therefore, five material parameters ($E_1, E_3, G_{13}, \nu_{12}$, and ν_{31}) are needed to define a cross-anisotropic elastic material.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\ & D_{2222} & D_{2233} & 0 & 0 & 0 \\ & & D_{3333} & 0 & 0 & 0 \\ & & & D_{1212} & 0 & 0 \\ & sym & & & D_{1313} & 0 \\ & & & & & D_{2323} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix} \quad (12)$$

with

$$\begin{aligned} D_{1111} &= D_{2222} = E_1(1-\nu_{13}\nu_{31})\lambda \\ D_{3333} &= E_3(1-\nu_{12}\nu_{12})\lambda \\ D_{1122} &= E_1(\nu_{12}+\nu_{31}\nu_{13})\lambda \\ D_{1133} &= D_{2233} = E_1(\nu_{31}+\nu_{12}\nu_{31})\lambda \\ D_{1212} &= G_{12} = E_1/(2(1+\nu_{12})) \\ D_{1313} &= D_{2323} = G_{13} \\ \lambda &= 1/(1-\nu_{12}^2-2\nu_{13}\nu_{31}-2\nu_{12}\nu_{13}\nu_{31}) \end{aligned}$$

where E_1 is modulus in the plane of isotropy; E_3 is modulus normal to the plane of isotropy; ν_{12} is Poisson's ratio for strain in direction 2 due to stress in direction 1; ν_{13} is Poisson's ratio for strain in direction 3 due to stress in direction 1; ν_{31} is Poisson's ratio for strain in direction 1 due to stress in direction 3; G_{13} is shear modulus in 1-3 plane; and $E_1/E_3 = \nu_{13}/\nu_{31}$.

In this study, the granular base layer is modeled as a cross anisotropic material and the vertical, horizontal, and shear moduli are described using the generalized model adopted in the proposed AASHTO MEPDG, Eqs. (13), (14), and (15), respectively [13]. In this model, the first stress invariant or bulk stress term considers the hardening effect, while the octahedral shear stress term considers the softening effect. The stress dependency of Poisson's ratios is not considered in this study and the in-plane and out-of-plane Poisson's ratios are assumed constant.

$$M_r^v = k_1 p_a \left(\frac{\theta}{p_a}\right)^{k_2} \left(\frac{\tau_{oct}}{p_a}+1\right)^{k_3} \quad (13)$$

$$M_r^h = k_4 p_a \left(\frac{\theta}{p_a}\right)^{k_5} \left(\frac{\tau_{oct}}{p_a}+1\right)^{k_6} \quad (14)$$

$$G_r = k_7 p_a \left(\frac{\theta}{p_a}\right)^{k_8} \left(\frac{\tau_{oct}}{p_a}+1\right)^{k_9} \quad (15)$$

where M_r^v is vertical resilient modulus (kPa); M_r^h is horizontal resilient modulus (kPa); G_r is shear resilient modulus (kPa); θ is bulk stress (kPa); τ_{oct} is octahedral shear stress (kPa); k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9 are exponent parameters; and p_a is atmospheric pressure (100kPa).

Finite Element Model

A three-dimensional (3-D) FE model of flexible pavement was developed using the general-purpose FE software ABAQUS. In the FE model, the eight-node, linear brick elements with reduced integration were used in the finite domains, whereas infinite elements were used to reduce a large number of far-field elements without significant loss of accuracy and to create a "silent" boundary for the dynamic analysis. A sensitivity analysis was performed to define the infinite boundaries at horizontal sides, as well as the bottom of FE mesh. After comparing the maximum tensile and shear strains in the asphalt layer, the locations of the infinite boundary were determined. The FE mesh is refined around the loading area along the wheel path; a relatively coarse mesh is used far away from the loading area. To check the mesh and boundary conditions, the FE solutions were compared with an analytical solution through a layered elastic theory based on simple assumptions (e.g., static loading, fully-bonded interface conditions, uniform circular contact stress, and linear elastic material behavior). Good agreements were achieved between the predicted stresses and strains using the two aforementioned approaches. This indicates that the mesh refinement and boundary conditions of the FE model are appropriate. More details on the developed FE model can be found in the literature published by the author [7, 8, 14].

Table 1. Viscoelastic Parameters of Asphalt Concrete in Thin Asphalt Pavement at 25°C.

I	Surface Course		WLF	
	G_i or K_i	τ_i		
1	6.31E-01	2.06E-02	C_1	18.1
2	2.51E-01	1.73E-01		
3	8.47E-02	1.29E+00		
4	2.67E-02	5.35E+00	C_2	164.7
5	6.66E-03	1.06E+02		

The model is capable to predict pavement responses under moving vehicle loading and considers the asphalt surface layer as a viscoelastic material and the granular base as a nonlinear anisotropic material, respectively. The modeled pavement structure is composed of a 76-mm asphalt layer and a 305-mm granular base layer placed on weak subgrade at a California Bearing Ratio (CBR) of 4% [7, 8]. The creep compliance of the asphalt material is tested in the laboratory at various temperatures and then converted to the relaxation modulus that was used as inputs in the FE model. The subgrade material is modeled as linear elastic material so the nonlinear effect of pavement structure only comes from the granular base layer. Table 1 shows the viscoelastic material parameters for the asphalt surface layer expressed as the Prony series.

A user material subroutine (UMAT) was developed to calculate the anisotropic modulus of the granular material corresponding to the stress state at each iteration. The UMAT allows users to implement general constitutive models other than the default models in ABAQUS. The UMAT program requires nine exponent parameters (k_1 - k_9) and two Poisson's ratios (ν_{12} and ν_{13}) for calculating the nonlinear anisotropic modulus. In addition, the initial vertical stress is calculated as the overburden stress that results from the density (ρ) and thickness of the material above the point of interest. The initial horizontal stress depends on the material properties, over-consolidation history, and the residual stress caused by compaction. A coefficient of horizontal stress (k_0 = 1.0) is defined as the ratio of horizontal stress to overburden stress. To prevent unreasonable values, cutoff values are used for the minimum resilient modulus at low stress levels.

ABAQUS/Standard uses the iterative Newton-Raphson method to solve nonlinear equations. The applied load is augmented incrementally, and at each increment the program solves a system of equations through iterations. The iterations continue on the basis of the previous solutions until it reaches a reasonable convergence. Because the modulus of the granular material is a function of the total stress state, a modified Newton-Raphson approach with secant stiffness was developed and used in this study [8]. The nonlinear anisotropic material properties used in the model is shown in Table 2, which was obtained from a previous study [15].

The equivalent modulus of granular base layer is determined from the pavement model under a stationary impulsive loading. The applied loading amplitude is a time-dependent haversine-shape load with 30ms duration. The computation time can be significantly reduced by using the relative simple stationary impulsive loading assumption. To check the accuracy of the equivalent modulus for pavement performance prediction, the pavement responses calculated using the equivalent modulus is compared with the pavement responses calculated using the nonlinear anisotropic model for the granular base layer. In this case, the loading is simulated as a moving load with both the loading area and amplitude change as the tire is rolling on pavement surface. In both the stationary and moving loading cases, the tire loading area and amplitude is represented using the measured tire-pavement contact stress and contact area that were obtained from a previous study conducted by the author [7, 8].

Results and Analysis

Equivalent Modulus from Nonlinear Analysis

Fig. 1 shows the calculated equivalent modulus for each sublayer of the granular base layer, respectively, at 25^{o}C and 47^{o}C. The loading applied is a 35.5-KN dual tire loading at 8kph.The granular base layer was divided into six sublayers with the depth ranging from 76mm to 381mm. The depth shown in the figures is the depth at the mid-point of each sublayer. Although the model outputs include vertical, horizontal, and shear modulus of each sublayer due to the nonlinear anisotropic effect, only vertical modulus is used to derive the equivalent modulus. The equivalent modulus for each sublayer of granular base was calculated as the distance-averaged vertical modulus along tire contact width within each sublayer when the nonlinear anisotropic model was used for the granular base. No horizontal modulus was used in the derivation of equivalent modulus considering their relative small values compared to vertical modulus.

As expected, the modulus decreases as the depth increases due to the reduced load effect. An important finding is that the equivalent modulus increases as the temperature increases from 25℃ to 47℃. This is because as the temperature increases, the asphalt surface layer becomes softer and more stress is carried by the granular base layer. The increasing percentage is about 11-27% with the highest increase at the upper part of the granular base layer. On the other hand, it was found that the modulus difference between each

Table 2. Nonlinear Anisotropic Model Parameters for Granular Base Layer (Eqs. (130 – (15)).

Modulus	k_1 / k_4 / k_7	k_2 / k_5 / k_8	k_3 / k_6 / k_9	ν_{12}	ν_{31}
Vertical	1010	0.791	-0.477		
Horizontal	227	1.071	-1.332	0.35	0.35
Shear	321	0.857	-0.681		

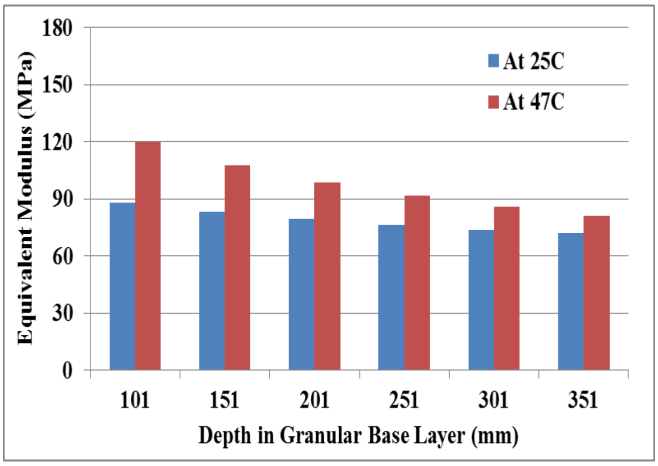


Fig. 1. Equivalent Modulus of Each Sublayer in the Granular Base at 8kph and 35.5kN.

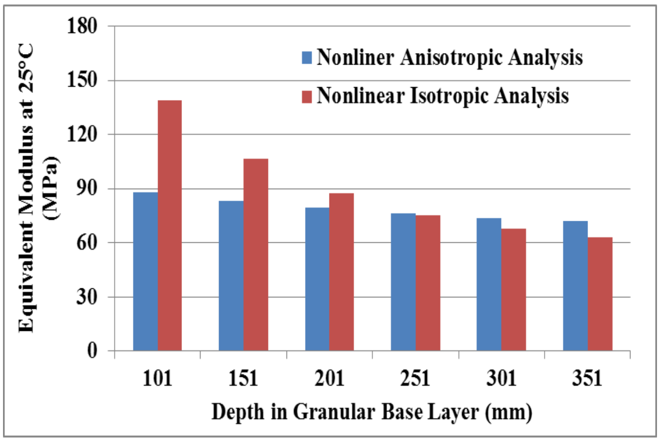
sublayer becomes more significant as the temperature increases. These findings indicate that the temperature has a significant effect on the equivalent modulus of the granular base layer.

Fig. 2 compares the equivalent modulus in the granular base layer calculated from the nonlinear anisotropic and the nonlinear isotropic model, respectively. The results indicate that the equivalent modulus derived from the nonlinear isotropic model is greater than those derived from the nonlinear anisotropic model. This is because the anisotropic model has smaller modulus in the horizontal direction and results in less confinement stress, compared to the isotropic model. Since the granular material is modeled as a stress-hardening material, less confinement would cause smaller modulus. As expected, the highest modulus difference between the anisotropic and isotropic models was observed at the upper part of granular base layer where the loading effect is more significant. As the depth increases, the equivalent modulus predicted from both models become close to each other.

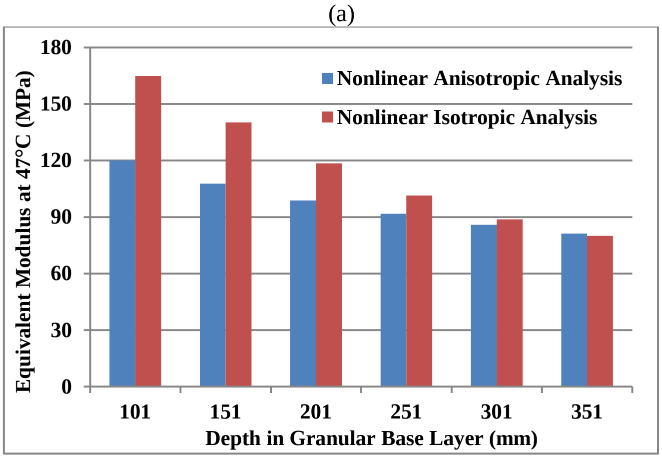
After the equivalent modulus of each sublayer has been calculated, it can be directly used in the multi-layer elastic model to calculate critical pavement responses by modeling the granular base layer as multiple sublayers with linear elastic properties. On the other hand, the sublayers with different equivalent modulus can be transformed into one single layer with one modulus but equivalent thickness using the Odemark method.

Effect of Load and Temperature on Equivalent Modulus

Due to the viscoelastic behavior of the asphalt surface layer, the modulus distribution in the granular base layer is not only affected by wheel load and pavement structure, but also the temperature and vehicle speed. A series of analysis was conducted to determine the



(a)



(b)

Fig. 2. Comparison of Equivalent Modulus from Anisotropic and Isotropic Analysis at (a) 25℃ and (b) 47℃.

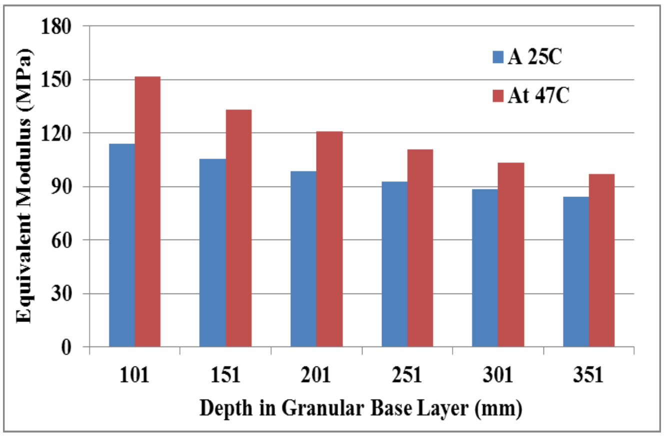


Fig. 3. Equivalent Modulus of Each Sublayer in the Granular Base at 8kph and 53.3Kn.

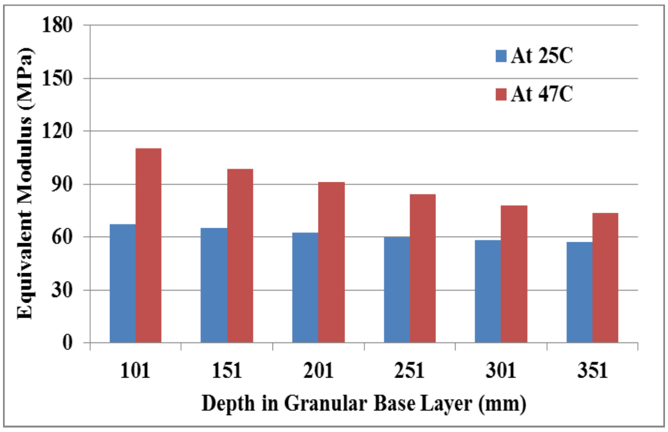


Fig. 4. Equivalent Modulus of Each Sublayer in the Granular Base at 64 kph and 35.5 kN.

equivalent modulus of the granular base layer at different loading and environmental conditions.

Fig. 3 shows the equivalent modulus of each sublayer in the granular base when a 53.3-kN dual tire loading is applied at 8 kph. Compared to the equivalent modulus at Fig. 1, it was found that the equivalent modulus increase by 20-40% as the load increases from 35.5 kN to 53.3 kN. This is reasonable because the higher load will cause the greater stress in the granular base layer; thus increase the modulus.

Fig. 4 shows the equivalent modulus of each sublayer in the granular base when a 35.5-kN dual tire loading is applied at 40 mph. Compared to the equivalent modulus at Fig. 1, it was found that the equivalent modulus decrease by 10-25% as the speed increases from 8 kph to 64 kph. As the vehicle speed increases, the loading frequency increases. Because the asphalt mixture is a viscoelastic material, the asphalt surface layer will have the greater modulus under the higher loading frequency. In this case, more loading is carried by the asphalt surface layer and less loading is transferred to the granular base layer. So the stress-dependent modulus of the granular base layer decreases. The results indicate that the interaction between the asphalt surface layer and base layer in the pavement system should be considered under the vehicle loading. It suggests that the viscoelastic effect of the asphalt surface layer is important for calculating the accurate modulus of the granular base layer. In addition, the data in Fig. 4 show that the equivalent modulus at each sublayer become close to each other at 25℃ when the vehicle speed is relatively high.

Comparison of Pavement Responses with Nonlinear Model and Equivalent Modulus

In this part, the derived equivalent modulus was checked to see if it can produce the accurate results in the prediction of critical pavement responses. Table 3 shows the comparison of pavement responses using the equivalent modulus and the real nonlinear anisotropic model for the granular base layer, respectively. In both cases, an 8-kip dual tire loading is applied at 5mph. Two types of critical pavement responses were compared; tensile strains at the bottom of the asphalt layer and shear strains at the shallow depth of the asphalt layer. The results indicate that the equivalent modulus produces the results that are comparable to the results using the real nonlinear anisotropic model for the granular base layer. The differences for the tensile strains are around 1-6% and the differences for the shear strains range from 6 to 20%. This suggests that the equivalent modulus could produce more accurate results in the prediction of tensile strains but the relatively higher error in the prediction of shear strains.

Conclusions

Table 3. Comparison of Pavement Responses Predicted Using Equivalent Modulus and the Nonlinear Anisotropic Model for Granular Base Layer.

Temperature	Locations	Pavement Responses	Using Nonlinear Anisotropic Model	Using Equivalent Modulus
25℃	Bottom of Asphalt Layer	Long. Tensile Strain (Micro)	375	373
		Trans. Tensile Strain (Micro)	288	306
	Shallow Depth of Asphalt Layer	Shear Stress (kPa)	311	278
		Shear Strain (Micro)	467	439
47℃	Bottom of Asphalt Layer	Long. Tensile Strain (Micro)	1139	1065
		Trans. Tensile Strain (Micro)	1162	1132
	Shallow Depth of Asphalt Layer	Shear Stress (kPa)	1573	1272
		Shear Strain (Micro)	303	270

In this study, an equivalent modulus approach was developed to represent the nonlinear anisotropic behavior of the granular base layer using three-dimensional finite element modeling analysis. The equivalent modulus was expressed as the modulus of each sublayer to avoid the difficulty of selecting a single stress point for the whole base layer. The analysis results indicate that the equivalent modulus is not a constant value but changes with temperature, vehicle speed, and load. The equivalent modulus increases as the temperature increases but decreases as the vehicle speed increases due to the viscoelastic nature of the asphalt surface layer. The equivalent modulus can produce comparable results in the prediction of critical pavement responses, compared to the responses predicted using the real nonlinear anisotropic model for the granular base layer. Further study will be conducted to quantify the relationship between the equivalent modulus and other variables (such as the subgrade modulus, temperature, speed, and load). This will provide practical dimension to pavement designers using the multilayer elastic analysis.

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