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Significance of Subgrade Damping on Vehicle-Pavement Interactions

Chen-Ming Kuo[1], Chih-Chiang Lin[1+], Cheng-Hao Huang[1], Ting-Yi Tsai[1], and Yi-Cheng Lai[1]

Abstract: The falling weight deflectometer (FWD) produces pavement responses by a falling mass drop to evaluate structural parameters of the pavement. A reliable simulation model is helpful to calibrate the apparatus as well as the backcalculation programs. The key points in the finite element simulation of FWD tests were studied including model size, boundary conditions, and analysis time increment. The subgrade in the finite element model was formulated with springs of stiffness coefficient of subgrade and dashpots of the derived damping values. It was concluded that at least six times the radius of relative stiffness is required to avoid boundary cut-off error. A rigorous procedure is presented to derive subgrade reaction and damping for various types of subgrade soil. Subgrade damping plays an important role in simulations of the FWD test. Backcalculation programs without subgrade damping may over-predict structural condition of pavements.

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***Key words**: Dynamic analysis; Finite element analysis; FWD; Soil damping; Subgrade.*

Introduction

With quick testing and standardized procedures, the falling weight deflectometer (FWD) has been widely adopted by highway and airport agencies to evaluate the load transfer efficiency of pavement joints, or to backcalculate layer stiffness, both of which are critical information for pavement structural evaluation and rehabilitation design.

Pavement materials are characterized to be elastic, homogeneous, and isotropic, with full contact between layer interfaces. Most of these programs model the pavement structure with a layered elastic system and use an iteration scheme to find the set of layer elastic moduli that best matches the computed theoretical deflections with the measured pavement deflections. Several studies have shown that classical theories for concrete pavement analysis are not suitable for dynamic problems such as falling weight tests on concrete pavements [1].

In order to obtain the correct structural parameters of existing pavements, forward calculation of classical closed form solutions has been studied to assist calibration of backcalculation results [2]. Furthermore, dynamic analyses have been gradually adopted to resolve errors associated with static analysis in most existing backcalculation programs. Rigorous discussions of dynamic analyses are necessary in order to interpret dynamic behaviors correctly. For example, the time step of dynamic analysis, which is related to the characteristic frequencies of the pavement system, affects the resolution of the pavement responses. Besides, pavement mass, boundary effects, and material damping all contribute to dynamic behavior and are essential to ensure appropriate simulations of FWD tests.

A typical simulation of FWD tests generally applies the loading history of falling mass on pavement elements, instead of modeling the falling weight with physical elements [3, 4]. Slab mass and subgrade damping become significant in the structural dynamic responses [5]. In this study, a finite element model of concrete slab resting on continuous springs and dashpots was developed to simulate FWD tests by dynamic analysis. The determination of damping coefficient and the effects of subgrade damping on FWD simulation were studied to improve the adequacy of forward calculation.

Finite Element Falling Mass Model

The forward calculation tool was developed with a finite element package, ABAQUS, because there is no closed form solution for dynamic analysis of pavement models. As shown in Fig. 1, the falling weight was modeled with a 254mm×254mm×254mm (10in×10in×10in) C3D8R element, which is a cubic element with 8 nodes and reduced integration scheme [6]. The falling weight element was cushioned by four springs and four dashpots to simulate the rubber pad in practice. The model can be calibrated to ensure the simulated forces matching the field measurements by adjusting the constants of springs and dashpots. Man-made pavement layers were modeled with C3D20R quadratic elements. General meshing guidelines were followed to ensure accuracy of analyses [7]. For example, the aspect ratios of elements which in loaded region were kept below 1, and the others elements kept below 5, and steep change of element size was avoided. The standard model, shown as

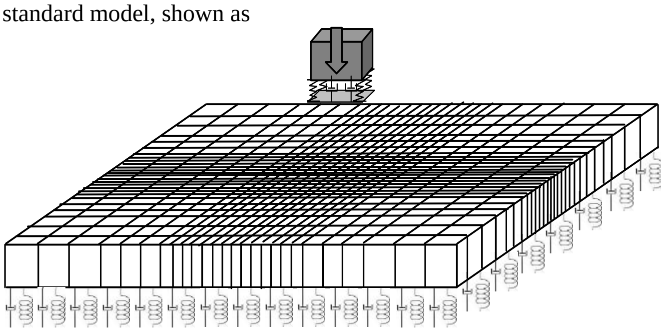

Fig. 1. Sketch of the Standard Model.

[1] Department of Civil Engineering, National Cheng - Kung University, Tainan City, Taiwan.

[+] Corresponding Author: E-mail jason81121@hotmail.com

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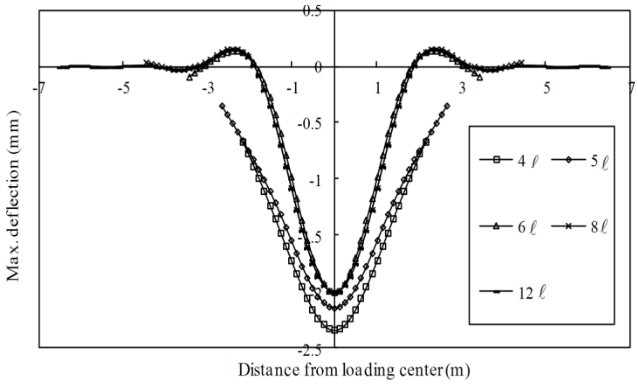

Fig. 2. Convergence of Maximum Deflections of FWD Simulations with Various Sizes of Pavement Models (□−4ℓ, ◇−5ℓ, △−6ℓ, ×−8ℓ, ━−12ℓ).

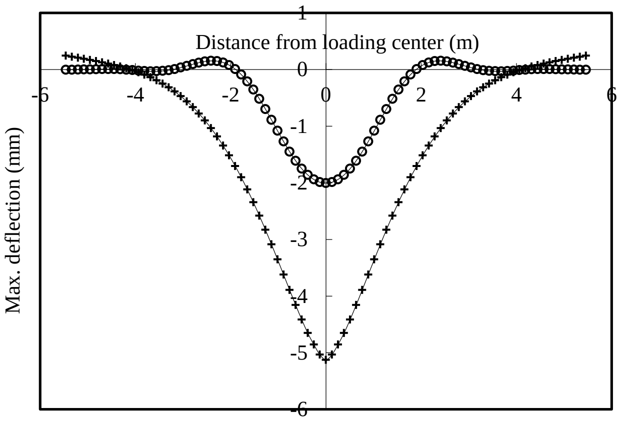

Fig. 3. Deflection Bowls of Static and Dynamic Analyses without Subgrade Damping (+ Static , o Dynamic).

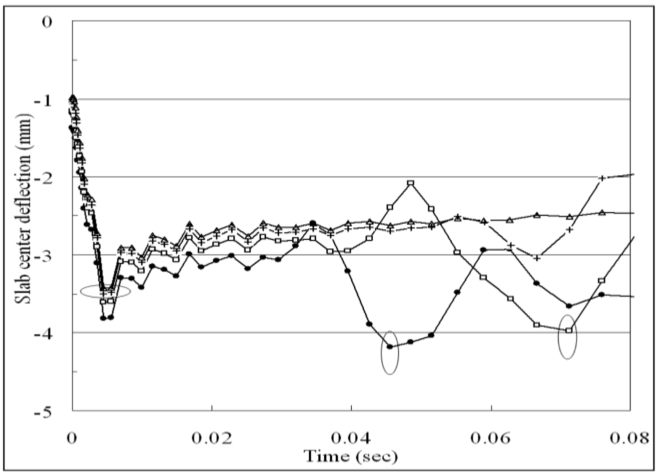

Fig. 4. Deflection Histories of Various Slab Sizes (No Subgrade Damping, No Slab Weight) (●−2ℓ, □−3ℓ, +−4ℓ, △−6ℓ).

Fig. 1, was built as a rectangular slab supported with the continuous springs and dashpots. Infinite elements were used along the edges of C3D20R slab elements to eliminate reflection of deformation waves at the model boundary.

Model size has been an important issue in finite element analysis to minimize cutoff errors. Eight times the radius of relative stiffness is widely accepted in the static analysis to approximate analytical solutions of infinite boundary [8]. To recommend the appropriate model size for dynamic analysis of FWD finite element model with finite boundary, models ranging from 4ℓ to 10ℓ in width were struck by the falling weight element of 50 kg (110 lb). As shown in Fig. 2, the deflection profiles show that the results converge well for the models larger than 6ℓ. Another set of analyses with 150 kg (330 lb) falling mass resulted in the same conclusion. The deflection basin of a FWD simulation was also found smaller than that of the static analysis with the same parameters, as shown in Fig. 3. In other words, the influence area of dynamic loading is smaller than that of static loading. Therefore, at least 6ℓ is recommended for a finite element model to minimize boundary cut-off errors in FWD simulations without considering slab weight and subgrade damping. As long as the slab weight or subgrade damping was included in finite element models, the peak pavement deflections are irrelevant to finite element model size. From the wave propagation point of view, the pavement edge may reflect deformation wave and interfere with the deflections at the slab center. The wave traveling time depends on the deformation wave speed of the concrete slab and the slab size [9]. It was also found that the deformation wave travels back to the slab center in milliseconds, which is faster than the occurrence of the peak deflection. In cases of constructive interference for reflective deformation wave and the primary wave, the peak deflection may lag behind the falling weight impact. The scenario was justified in the model 2 and the model 3. Thus, as shown in Fig. 4, the finite element models with various slab sizes result in different peak deflections.

However, the model size effect on the peak central deflection diminished if the damping effect associated with the body force of pavement slabs or subgrade are included in the model. The FWD simulations with slab mass or subgrade damping were found free of reflection wave interference at central deflections. The peak slab deflections are identical regardless of the extent of finite elements. It is concluded that the model size effect on slab deflections of dynamic pavement analysis is more complicated than that in static analysis. From the peak slab deflection point of view, a relatively rigorous finite element model which includes slab weight, subgrade damping, or infinite boundary should be free of disturbed central deflection by reflected deformation waves from the boundaries.

Arbitrary time increments in FWD simulations could lead to significantly different deflection histories, as shown in Fig. 5. The low resolution case with a 0.001 second time increment predicted peak pavement deflections only about half the size of those in the high resolution case of a 0.0001 second time increment. Referring to the field measurements and the deformation speed of concrete slabs, the pavement responses may change every 0.001 second. In addition, the typical resonant frequencies of concrete slabs ranges from 2 kHz to 10 kHz [10]. As a rule of thumb, time discretization (t) should be at least one-fourth of the natural vibration period of the system being analyzed. About $t = 1.25\times10^{-4}$ second is needed to ensure stable and accurate results, which is close to the time increment of high resolution case in this study, 0.0001 second. Stricter criteria, e.g., $t < T_n/20$, may produce high resolution on dynamic behavior at the expense of five times of computer runtime. Hence, 0.0001 second was chosen to be the time increment of dynamic analysis in this study.

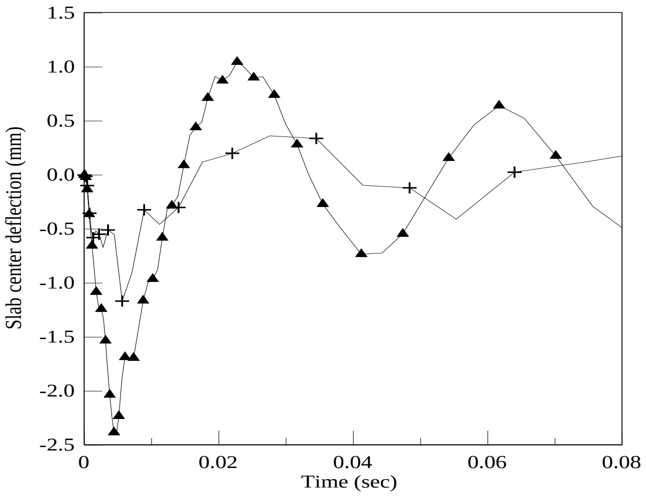

Fig. 5. Deflection Histories of Different Time Increment (+:ΔT =0.001Sec, ▲: ΔT =0.0001Sec).

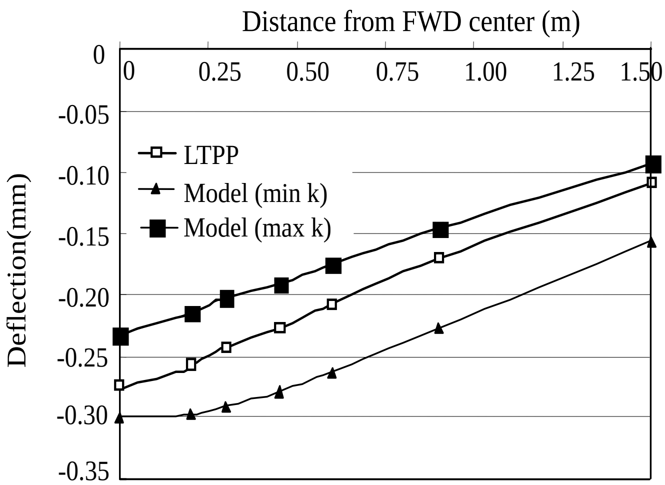

Fig. 6. Comparison of Simulated Deflections and Field Measurements.

Table 1. LTPP Data and Model Inputs for Model Validation.

LTPP Section Number	29-5483-1
State	Missouri
Joint Spacing	18.75 m
Surface Layer	229 mm (9 inch JRCP)
Surface Strength	26.4×103 MPa (3.83×10^6 psi)
Geophone Arrangement	0 in, 8 in, 12in, 18in, 24in, 36in, 60in
Subgrade Type	SS
Subgrade Strength	16.7-50.5 MPa/m (61.89-186.41 psi/in)
Falling Mass	200 kg (440 lb)
Impact Speed (Model Input)	20 m/sec (78.7 in/sec)
contact Spring Constant (Model Input)	70.1×10^6 N/m (400000 lb/in)

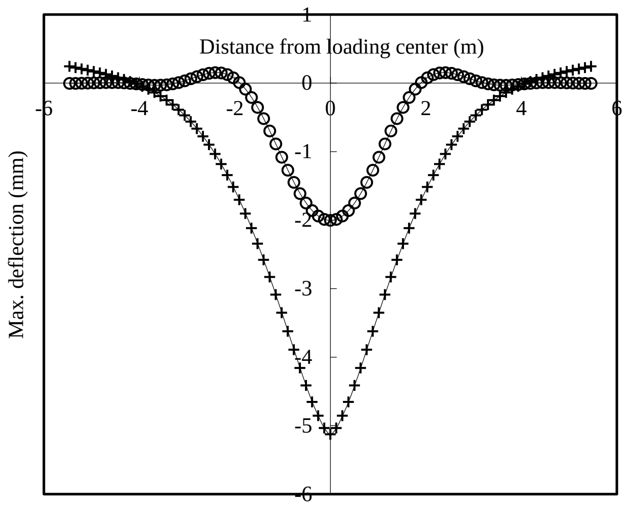

Fig. 7. Deflection Bowls of Static and Dynamic Analyses without Subgrade Damping (+ Static , o Dynamic).

The FWD forward calculation model was validated with the Long-Term Pavement Performance (LTPP) database, as shown in Table 1. The subgrade stiffness was input with the backcalculated modulus ranging between 16.7-50.5 MPa/m [11]. Fig. 6 illustrates that the field measured deflections were bounded by the simulated deflections with the upper bound and lower bound of subgrade moduli. This comparison justified that the proposed finite element model is capable of simulating field FWD tests. In addition, Fig. 7 also demonstrates that the dynamic deflection is less than half of the static deflection. Accordingly, the forward calculation methods based on static analysis normally predict larger deflection than the real FWD impacts and result in lower subgrade k-values. It is necessary to include subgrade damping and perform dynamic analysis to remedy the flaw.

The Determination of Subgrade Damping

Generally, damping can be categorized into two types: material damping and geometrical damping. The FWD test measures the deflections at the vicinity of impact; these deflections should be minimally correlated to geometrical damping. According to Lysmer's research on footings behavior in soils, the interaction of vertical vibration of a rigid circular foundation in soil was analog to a simple vibration model of single degree of freedom [12]. The equilibrium equation is shown as Eq. (1), in which m, z, and r_o are mass, displacement, and radius of foundation; G_s,v_s and ρ_s are shear modulus, Poisson's ratio, and density of soil; $p_o(t)$ is loading on foundation.

$$m\ddot{z}+\left(\frac{3.4}{1-v_s}r_0^2\sqrt{G_s\rho_s}\right)\dot{z}+\left(\frac{4G_sr_0}{1-v_s}\right)z=p_0(t) \tag{1}$$

It is straightforward to obtain the equivalent soil damping and resilient properties as Eqs. (2) and (3). The radius of foundation is assumed to be three times radius of relative stiffness of the pavement system as concluded in the previous section to adapt Lysmer's equation. By further substituting $r_0=3\cdot\sqrt[4]{\frac{E_ch^3}{12(1-v_c^2)k}}$

and $k = \frac{k_s}{\pi r_0^2}$ into Eq. (2), the soil damping can be calculated with Eq. (5), in which E_c, v_c, and h are Young's modulus, Poisson's ratio, and thickness of concrete pavement. It is clear that the analog soil

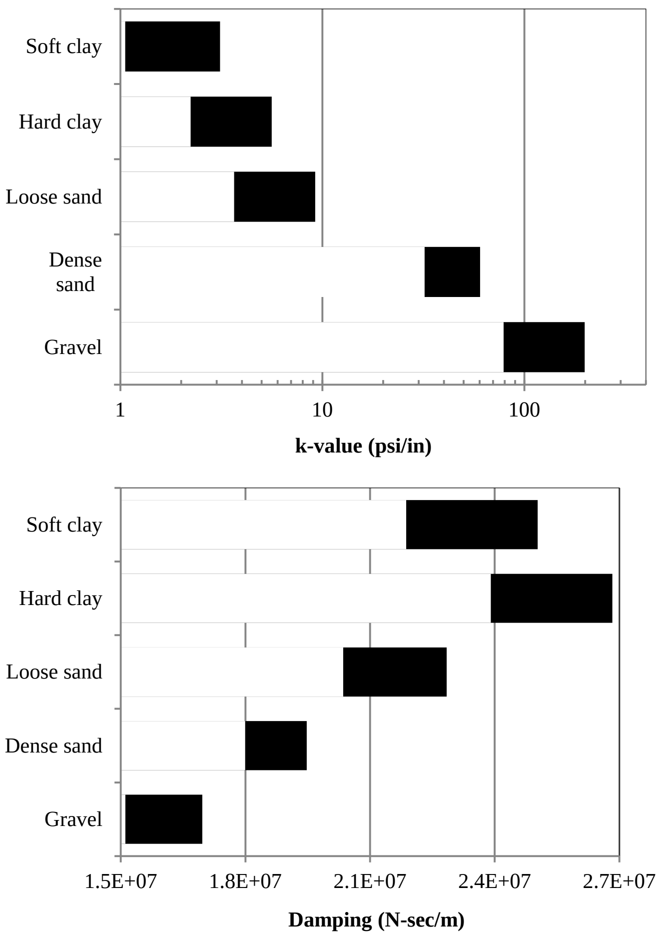

Fig. 8. Ranges of k and c of Various Subgrade Soil (Ec = 4×10^6 psi, c = 0.15, h = 12 inch).

Table 2. Typical Poisson's Ratio and Shear Modulus of Soils [14].

Subgrade Soil	Poisson's Ratio	Shear Modulus, *G* (MPa)	Density (kg/m^3)
Granite	0.15~0.24	1240~2480	1650
Lime	0.16~0.23	827~2070	1600
Sandstone	0.17	552~1172	1500
Gravel	0.35	38~76	1900
Dense Sand	0.35	19~31	2000
Loose Sand	0.35	3.8~7.6	1600
Hard Clay	0.4	2.2~5.0	1800
Soft Clay	0.4	0.4~1.0	1300

Table 3. Nominal Damping of Soils [12].

Subgrade Soil	Damping (N·s/m)
Gravel	1.585×10^7
Dense Sand	1.863×10^7
Loose Sand	2.135×10^7
Hard Clay	2.508×10^7
Soft Clay	2.309×10^7

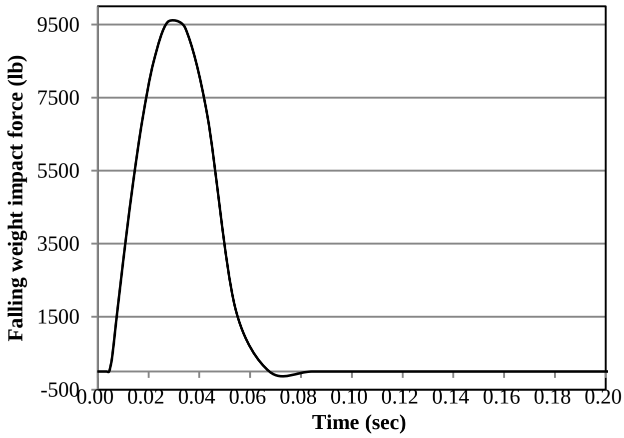

Fig. 9. Time History of Impact Loading Simulated with Finite Element Falling Mass.

damping depends not only on material parameters of the soil but also those of the structure on it. The equations provide a systematic alternative to obtain the subgrade reaction coefficient and damping via basic soil parameters, in addition to the expensive plate load tests and empirical conversions.

$$c_s = \frac{3.4}{1-v} r_0^2 \sqrt{G\rho} \tag{2}$$

$$k_s = \frac{4Gr_0}{1-v} \tag{3}$$

$$k = \frac{4G}{\pi r_0 (1-v)} \tag{4}$$

$$c_s = \frac{10.34 \cdot \sqrt{\rho_s}}{\sqrt[3]{1-v_s} \cdot \sqrt[6]{G_s}} \cdot \left[\frac{E_c \cdot h^3}{1-v_c^2} \right]^{2/3} \tag{5}$$

Coefficients and damping values of various soils were calculated as shown in Fig. 8 for a 12-inch concrete pavement with the typical soil parameters listed in Table 2. The k-value of gravel and dense sand subgrade are located in general range of subgrade reaction coefficient, whereas the other types of soils are analog lower than 10 psi/in. Since k-value is not a basic physical parameter of soil and depends on loading characteristics [13], the values in Fig. 8 provide valuable perspectives for selecting k-value for the Winkler foundation because those are rigorously derived from basic material parameters. However, it should be noted that it is not appropriate to use the k-value derived with the above equations in pavement design guides, which were basically developed based on static theories and analysis, whereas Lysmer's analog equations were based on dynamic theories.

Finite element analysis of falling weight tests on various subgrade soils were conducted with the calculated soil damping to reveal the significance of subgrade damping in FWD simulations. The falling mass was modeled with a 110 lb steel block hitting concrete slab with 6 in/sec. The size of concrete slab varies from 24 ft to 68 ft to meet the slab size requirement of three times radius of relative stiffness. The simulated impact force is shown in Fig. 9. It was

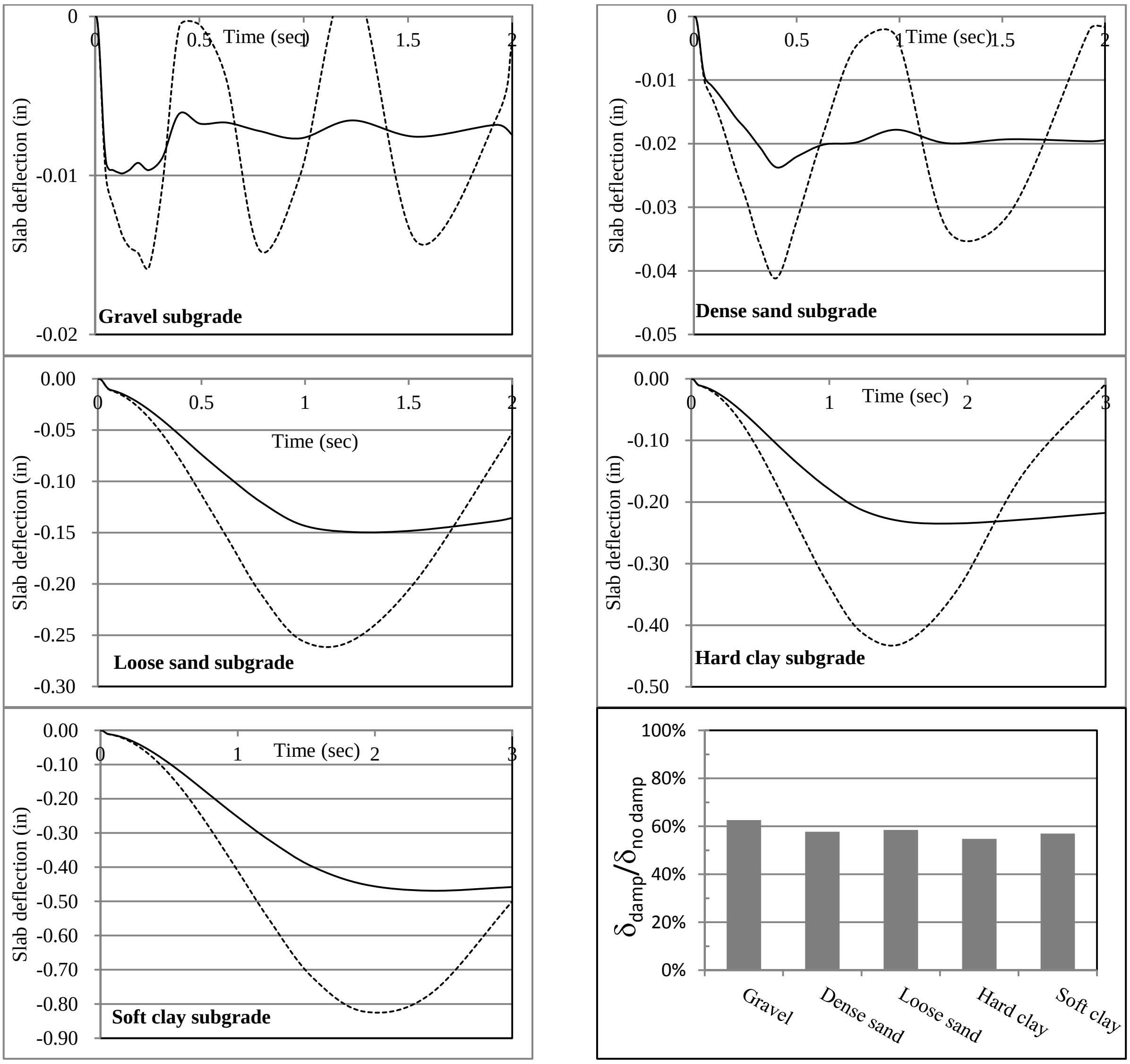

Fig. 10. Peak Deflection Comparisons of Models with and without Subgrade Damping for Various Subgrade Soils (—— with Subgrade Damping, - - - - No Subgrade Damping).

found that the magnitude and time history of all models are exactly identical regardless of the stiffness and damping of subgrade. The parameters of the falling mass and the contact settings between the falling mass and the slab dominate the shape of impact force. In this analysis, the peak impact force is 9,865 lb and the duration is about 70 ms. The nominal damping of soils is listed in Table 3. The calculated deflections of the models with and without subgrade damping are compared in Fig. 10. The deflections vary in a wide range because the analog k-value ranges from 2 psi/in for soft clay to 113 psi/in for gravel subgrade. The ratio of peak deflections of damping model and no damping model are fairly consistent for all kinds of subgrade and averaged at 58%. In other words, neglecting subgrade damping may over-predict pavement deflections by 70%. Consequently, a very stiff subgrade is needed in a forward calculating model, which neglects subgrade damping to match the small deflection measured in field where damping actually exists. Hence, this may explain why the backcalculation results are generally much higher. This point is helpful to evaluations of backcalculation programs involving FWD simulations.

Conclusion

Forward calculation is important in development and validation of pavement backcalculation techniques. This study examined several factors of dynamic finite element analysis for a concrete slab resting on subgrade. Experiences and findings are provided to enhance reliable utilization of dynamic analysis in pavement engineering. The major conclusions are summarized as follows.

The deformation wave reflected from the pavement boundaries of finite element models may interfere with the peak deflection at slab center. Infinite boundary, slab weight, and subgrade damping are effective to avoid or damp out the reflection wave.

The length and width of FWD simulation models should be at least six times the radius of relative stiffness to avoid boundary cut-off errors if slab weight and subgrade damping are neglected in the analyses. The model size effect on the peak deflection becomes less significant as long as infinite boundary, slab weight, or subgrade damping is modeled.

The time increment of 0.0001 second in the simulations of FWD tests on concrete pavement has been found adequate to achieve sufficient resolution. It is suggested to follow the general criteria of time increment to ensure sufficient resolution of finding accurate peak deflections.

The discrepancies between the pavement models with and without subgrade damping were found significant. The ratios of peak deflections of damped case and undamped case are fairly consistent for various soils. The finite element analysis with the Winkler foundation resulted in the peak deflections 70% higher than those with Kelvin foundation. The backcalculation programs based on the Winkler foundation tends to over-predict the subgrade stiffness, which is actually the combination of stiffness and damping in field.

Although analyses of slabs on the Kerr model, Pasternak model, or even elastic half-space models may have the chance to match measured deflections better than the Winkler model, the majority of pavement analysis programs are still based on the Winkler foundation due to the advantages of its simplicity in formulation and computation. A systematic and theoretically rigorous approach is presented to obtain the subgrade reaction coefficient and damping based on the analog foundation spring, ks, and foundation damping, cs, derived by Lysmer.

The k and c of subgrade derived from the analog ks and cs are applicable to pavement dynamic analysis. The k and c not only depend on soil types but also the rigidity of concrete slabs. Gravel has the highest k-value and lowest c-value. Significant differences were observed among various soil types in deflection magnitude as well as deformation frequency.

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