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Characterizing the Low-Temperature Viscoelastic Behavior of Asphalt Mixtures: a comparative study

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Abstract: Thermal cracking is a critical pavement issues in the cold regions and has been of a major concern in asphalt pavements. The paper presents a viscoelastic modeling work using pure power law, generalized power law, and Prony series functions to characterize low temperature performance of asphalt mixtures. The bending beam rheometer (BBR) using asphalt mixture beams was employed to obtain creep compliance and further viscoelastic analyses were performed to determine relaxation moduli and thermal stresses of asphalt mixtures. The objective of this paper is to provide a better understanding on low temperature viscoelastic behavior of asphalt mixtures by quantifying creep compliance errors and differences among these representation models (pure power law, generalized power law, and Prony series functions) in the predictions of relaxation moduli and thermal stresses. Based on the comparison of viscoelastic analyses at low temperatures, the generalized power law function shows a better agreement with the experimental data and has the least creep compliance error as compared with other two representation functions.

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***Key words**: Asphalt mixtures; Bending Beam Rheometer; Low temperature; Thermal cracking; Viscoelastic analysis; Viscoelastic materials.*

Introduction

Thermal cracking is a critical distress issue in asphalt pavements within cold regions. A number of studies have been made in the past years with the intent to address the thermal-induced deterioration in asphalt pavements. As has been known, asphalt cement concrete is a composite material consisting of aggregate, binder, and air voids. When the loading conditions do not cause damage to asphalt mixtures, the response of asphalt mixtures can be defined as linear viscoelastic and expressed through the convolution integral [1]. Under small strain level, generally below 500 micro-strains, a viscoelastic material exhibits linear behavior such that stress and strain responses measured from a viscoelastic material are expressed as functions of time and loading rate. Thus, the state of stress and strain responses can be characterized by the theory of linear viscoelasticity. To obtain the low temperature properties of asphalt mixtures, creep compliance and relaxation modulus are two major response functions that can be used to represent the rheological properties of the materials. Based on the theory of linear viscoelasticity, creep compliance and relaxation modulus is correlated; one can be derived as long as another one is known [2]. The determination of creep compliance or relaxation modulus therefore is crucial to evaluate the time-dependent stress/strain responses in asphalt pavements [3-4].

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Direct measurement of relaxation modulus of asphalt mixtures requires robust instruments, which is not a favorable option in most laboratories. The measurement of creep compliance therefore becomes a commonly used method to evaluate the low temperature performance of asphalt mixtures. Creep compliance data are expressed as a time dependent function and can be obtained from tensile-related material testing methods such as Indirect Tensile Test (IDT), Bending Beam Rheometer (BBR), etc. To predict the thermal stresses of asphalt mixtures, creep compliance obtained needs to be converted to relaxation modulus through a series of Laplace transform, and then the time domain of the relaxation modulus converted is further switched to the temperature domain of that using the Time-Temperature Superposition Principle (TTSP). In order to generate required parameters for TTSP and response functions, one of methods is to shift multiple temperatures to form a creep compliance master curve that can represent the entire viscoelastic response of the asphalt material. Subsequently a nonlinear regression method must be used to generate desired parameters essential for viscoelastic representation functions; pure power law, generalized power law, and Prony series functions that will be discussed later on.

Power law and Prony series are two typical representation functions of creep compliance. Power law function has been widely used to fit experimental data and provide a smooth and stable representative function [4-5]. Research by Christensen [6] based on the IDT tests confirmed the accuracy of modeling the creep compliance of asphalt mixtures using a generalized power law function. In contrast, Prony series function denotes a mechanical model that can be regarded as a superposition of elementary processes to represent an analytical representation function for viscoelastic materials [1]. Kim et al. [4] employed a Prony series function to model complex modulus and obtained favorable results in fitting creep compliance data. In theory, both complex and relaxation modulus should derive same creep compliance as long as the time and frequency domains used are identical [4]. However, the nature of dynamic test and tensile test is different such that the use of representation functions to fit raw data and represent physical properties of viscoelastic materials can result in different engineering characteristics. Specifically, temperatures for the dynamic test are tested at intermediate levels (10℃~25℃) while temperatures of the tensile test are set at low levels (10℃~ -40℃). Thus, using representation functions of asphalt mixtures to fit experimental data from both tests should be thoroughly chosen. In comparison with previous research work in complex modulus, the goal of this study is to characterize low temperature properties of asphalt mixtures using power law and Prony series functions to quantify creep compliance errors and compare the differences of calculated thermal stresses between these two functions. The objective of the paper is to provide a better understanding on the low temperature viscoelastic behavior of asphalt mixtures and to compare differences of viscoelastic modeling between these three functions (e.g., pure power law, generalized power law, and Prony series functions).

Research Approach

Sample Preparation and Material Testing

The Bending Beam Rheometer (BBR) instrument was selected to obtain creep compliance data to characterize the low temperature performance of asphalt mixtures. Recent research work by Marasteanu et al. [7] and Zofka et al. [8-11] have shown the advantage of replacing a standard asphalt binder beam with an asphalt mixture beam to effectively evaluate thermal-induced stiffness of asphalt mixtures. The comparison between standard asphalt binder beams and asphalt mixture beams is shown in Fig. 1. This study follows work by Marasteanu and Zofka et al. and aims at providing viscoelastic modeling efforts to characterize the low temperature performance of asphalt mixtures using the BBR instrument. Using such a small beam in the BBR to represent the global thermal properties of the asphalt material has been a concern in the asphalt industry. However, as of today, a significant number of projects using asphalt mixture beams in the BBR have been studied and the results have been in support of using the BBR to evaluate the low temperature aspects of the asphalt material. For instance, to further assess the concept of representative volume element (RVE) in the BBR, Velásquez et al [12] performed three point bending creep tests on beams of different sizes (6.25 ×12.5 ×100 mm, 12.5 ×25 ×200 mm, and 18.75 ×37.5 ×300 mm) at high temperature, intermediate temperature, and low temperature level to compare creep stiffness of asphalt mixtures in an attempt to evaluate the effect of aggregates on the global properties of asphalt mixtures. Based on the test results, they concluded that "…as the temperatures decreases, the mismatch between the stiffness of aggregates and of the binder (mastic) diminishes and the creep stiffness of asphalt mixture becomes less dependent on the size and distribution of aggregate particles." Research projects by Ho and Romero [13] and Romero et al. [14] were to evaluate the applicability of using asphalt mixture beams in the BBR for use in quality control during asphalt pavement construction. These are evidence in support of using small asphalt mixture beams in the BBR for the prediction of low

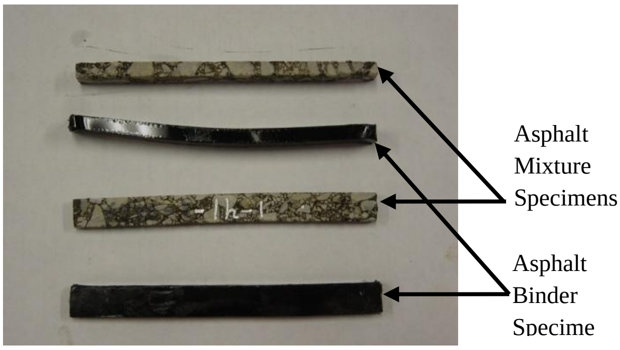


Fig. 1. Preparation of Asphalt Mixture Beam.

temperature performance of the asphalt material.

In collaboration with the Utah Department of Transportation (DOT), USA, asphalt materials were mixed with the binder PG 64-34 using a nominal maximum aggregate size (NMAS) of 12.5 mm. This mix is a typical design used in asphalt production and construction in Salt Lake City, Utah. It should be noted that varying asphalt binders and NMAS selections may be used to gather more information on thermal cracking. However, that was not the scope of the paper and the desired mix design used by the Utah DOT. Asphalt mixture beams with the dimensions of 12.7 mm x 6.35 mm x 127 mm (width x thickness x length) were prepared in the Materials Laboratory at the University of Utah. Asphalt mixture beams produced were placed on the platform in the bath of the BBR and imposed with 450 gram (4413 micro Newton ± 50 micro Newton) loading conditions. The testing temperature was set at -24℃ in compliance with the AASHTO T 313 standard (10 degrees higher than the lowest binder range). Specimens were tested at three temperatures, -18℃, -24℃, and -30℃ (-24℃ ±6℃) allowing a 6-degree range in an effort to obtain three creep compliance curves for the development of a master creep compliance curve.

Development of Mater Creep Compliance Curve

To obtain a creep compliance master curve, the Time-Temperature Superposition Principle (TTSP) must be used in association with nonlinear regression methods. The process begins with shifting individual curve measured at three temperatures to partly overlap together so as to develop a mater creep compliance curve that can represent the entire set of creep compliance data. The process of developing a master curve was performed in the following steps [15-18]:

1. Select a reference temperature (T_R): In this study -24℃ was selected.
2. Determine the shift factor for each creep compliance curve: Schwarzl and Staverman [19] stated that the effect of temperature change on the viscoelastic properties of materials is equivalent to a shift on the log time scale expressed below:

$$\xi = \frac{T}{a_T(T)} \tag{1}$$

where ξ =reduced time, a_T (T) =shift factor, and T= temperature

(1) Determine a shifting function: There are three major shifting functions (i.e., sigmoidal function, Williams-Landel-Ferry (WLF)

function, and Arrhenious function) available to be used to determine proper shifting factors for each creep curve [20-22]. Franck [23] recommended that at low temperatures the Arrhenious function (Eq. (2)) appears to be appropriate than other functions in the prediction of shift factors:

$$log[a_T(T)] = 2.303\frac{E_a}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_R}\right) \tag{2}$$

where,
E_a = the activation energy for flow below T_R, 261 kJ/mol.
R = the ideal gas constant, 8.34J/mol-K
T_R = reference temperature, ℃ or K
T = selected temperature, ℃ or K

Creep compliance curves at three temperatures obtained from the BBR tests are shown in Fig. 2(a). After using the Arhennious function associated with the TTSP, a mater creep compliance curve was developed (Fig. 2(b)). The mathematical relation between shift factors and temperatures is presented in Fig. 2(c).

Analytical Representation Functions of Viscoelastic Modeling

To accurately characterize the responses of asphalt mixtures from the BBR tests, it is essential to employ analytical representation functions to fit exported creep compliance data over a broad time range. Three functions were used in this research: power law (pure power law and generalized power law) and Prony series functions.

Power Law Function

The power law contains two types of functions: pure power law (PPL) function and generalized power law (GPL) function. The difference between the pure power law and the generalized power law functions is that the pure power law neglects the portion of elastic compliance. Both functions are expressed as:

$$D_{PPL}(t) = D_1\cdot t^n \qquad \text{(Pure power law)} \tag{3}$$

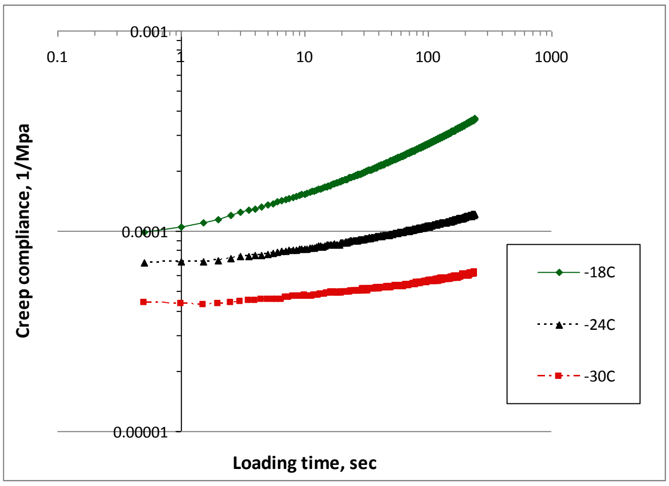
where D_1 refers to the value of creep compliance curve at time t and n denotes the power parameter.

$$D_{GPL}(t) = D_0 + D_1\cdot t^n \qquad \text{(Generalized power law)} \tag{4}$$

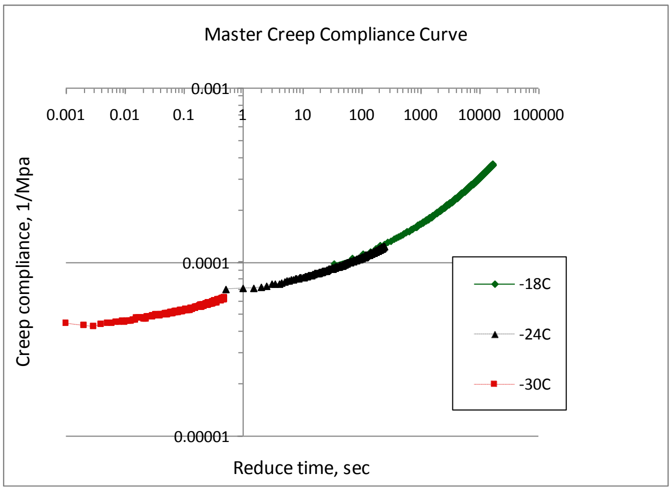
where D_0 represents the value of elastic creep compliance

Park and Kim [24] indicated that the pure power law function has its inability to represent the data in regions other than the transient zone. Generally, the generalized power law function has more reliability of fitting raw creep compliance data better than the pure power law's at short-time ranges [1]. In the nature of linear viscoelasticity, relaxation modulus can be derived from creep compliance through Laplace transformation. Taking the Laplace transform of Eq. (3) gives:

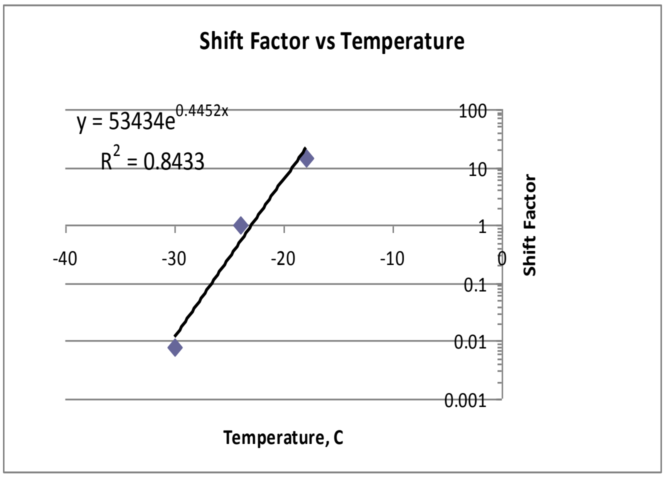
$$\hat{D}_{PPL}(s) = D_1\frac{\Gamma(n+1)}{s^{n+1}} \tag{5}$$



(a) Creep compliance curves at three temperatures



(b) Creep compliance curves were shifted



(c) Mathematical relation between shift factors and temperatures

Fig. 2. Development of Master Creep Compliance Curve.

where a caret (^) over the symbols shows that the quantity is now a function of Laplace transform and s is a Laplace transform parameter; and Γ is a gamma function and can be described as:

$$\Gamma(n) = \int_0^\infty t^{n-1}e^{-t}dt \tag{6}$$

Similarly, performing the Laplace transform of Eq. (4) becomes:

$$\hat{D}_{GPL}(s)=\frac{D_0}{s}+D_1\frac{n!}{s^{n+1}}=\frac{D_0}{s}+D_1\frac{\Gamma(n+1)}{s^{n+1}} \tag{7}$$

Based on the theory of viscoelasticity, creep compliance and relaxation modulus in the Laplace domain can be correlated by the following interconversion relation:

$$\hat{D}(s)\hat{E}(s)=\frac{1}{s^2} \tag{8}$$

Thus, the relaxation modulus of the pure power law in the Laplace domain is given by:

$$\hat{E}_{PPL}(s)=\frac{1}{\hat{D}_{PPL}(s)s^2}=\frac{1}{D_1\Gamma(n+1)s^{1-n}} \tag{9}$$

The relaxation modulus of the generalized power law in the Laplace domain is shown as:

$$\hat{E}_{GPL}(s)=\frac{1}{\hat{D}_{GPL}(s)s^2}=\frac{1}{sD_0+D_1\Gamma(n+1)s^{1-n}} \tag{10}$$

Relaxation moduli of both pure power law and generalized power law functions can be derived by directly inversing the Laplace transform functions in Eqs. (9) and (10). Therefore, the inverse of Laplace transform of Eq. (9) yields:

$$E_{PPL}(t)=\frac{1}{D_1\Gamma(n+1)\Gamma(n-1)t^n} \tag{11}$$

Unfortunately, the direct reverse of Eq. (10) cannot be solved. However, an approximate method [25, 26] can be applied to arrive in an approximate solution. The detail of the calculation is ignored, thus the relaxation modulus of the generalized power law is then expressed as:

$$E_{GPL}(t)=\frac{1}{D_0+D_1\Gamma(n+1)(1.786t)^n} \tag{12}$$

Prony Series Function

In addition to power law functions, Prony series function has also been broadly used as an analytical representation function in viscoelastic modeling. Prony series consists of one Maxwell model (a spring and a dashpot connected in series) and several Kelvin elements (a spring and a dashpot connected in parallel) [27] that can be mathematically expressed as:

$$D_{Prony}(t)=D_0+\frac{1}{\eta}t+\sum_{i=1}^{N}D_i\left(1-e^{-t/\tau_i}\right) \tag{13}$$

where D_0, D_i, and η = Prony series parameters; and τ_i =retardation times.

Performing the Laplace transform of the Prony series function (Eq. (13)) associated with the interconversion relation (Eq. (8)) obtains:

$$\hat{E}_{Prony}(s)=\frac{1}{D_0\times s+\sum_{i=1}^{N}D_i\left(\frac{s/\tau_i}{s+1/\tau_i}\right)+\frac{1}{\eta\times s}} \tag{14}$$

Eq. (14) can be rearranged by a ratio of two polynomials [2, 27]:

$$\hat{E}_{Prony}(s)=\frac{a_is^i+a_{i-1}s^{i-1}+\ldots a_0}{b_is^i+b_{i-1}s^{i-1}+\ldots b_0} \tag{15}$$

Eq. (15) is then further simplified by using a partial fraction expansion:

$$\hat{E}_{Prony}(s)=\frac{C_1}{s+r_1}+\frac{C_2}{s+r_2}+\frac{C_3}{s+r_3}+\ldots\frac{C_j}{s+r_j} \tag{16}$$

where r_1, r_2, r_3, etc. represent the roots of the denominator and C_1, C_2, C_3, etc. are coefficients of numerators.

The simplified formation of Eq. (16) gives an effective way to perform the direct inverse of Laplace transform of Eq. (14) presented as:

$$E_{Prony}(t)=C_1e^{-\gamma_1}+C_2e^{-\gamma_2}+C_3e^{-\gamma_3}+\ldots+C_je^{-\gamma_j}=\sum_{j=1}^{N}C_je^{-\gamma_j} \tag{17}$$

where C_j = Prony regression coefficient, $\gamma_j = \frac{t}{\lambda_j}$ in the viscoelastic analysis, t = creep compliance times, and τ_j= regression coefficient

Eqs. (11), (12), and (17) represent analytical representation functions of asphalt mixtures in the process of viscoelastic modeling derived from the pure power law, generalized power law, and Prony series functions, respectively.

Fitting Experimental Creep Compliance Data

To run viscoelastic modeling, the unknown parameters (D_0, D_1, D_2, etc.) in Eqs. (11), (12), and (17) must be solved. One of approaches is to fit experimental data to generate the parameters, so the representation functions can be formulated. The fundamental of fitting experimental data presented in this paper is based on nonlinear regression techniques by minimizing the sum of squared errors between the raw data and fitted values. This approach is stated as:

$$\text{Minimize}\sum\left|D_p(\xi)-D(\xi)\right|^2 \tag{18}$$

where

$D_p(\xi)$ = fitted representation function at reduced time, ξ

$D(\xi)$ = raw experimental data at reduced time, ξ

It has been identified that using the Prony series function to fit raw experimental data can potentially generate negative values in the Prony regression coefficients. However, a method has been available to address this issue. Recent studies by Park and Kim [24] and Chehab and Kim [1] provided an algorithm called "pre-smoothing" that can be used to smooth scattered raw data thus allowing such "smoothed" data to be fitted. After completion of data pre-smoothing, the Prony series function is presented to fit these "pre-smoothed" experimental data so as to avoid any negativity during fitting data analyses. Kim et al. [27] also showed a process of fitting creep and complex compliance data using the pure power law function. The principle of fitting "pre-smoothed" experimental data using the Prony series function is given as:

$$\begin{bmatrix} D_{power\_1} \\ D_{power\_2} \\ D_{power\_3} \\ \cdot \\ \cdot \\ \cdot \\ D_{power\_N} \end{bmatrix} = [D_0] + \begin{bmatrix} \frac{t_1}{\eta} + \sum_{i=1}^{n} D_i \left(1 - e^{-t_1/\tau_1}\right) \\ \frac{t_2}{\eta} + \sum_{i=1}^{n} D_i \left(1 - e^{-t_2/\tau_2}\right) \\ \frac{t_3}{\eta} + \sum_{i=1}^{n} D_i \left(1 - e^{-t_3/\tau_3}\right) \\ \cdot \\ \cdot \\ \cdot \\ \frac{t_N}{\eta} + \sum_{i=1}^{n} D_i \left(1 - e^{-t_N/\tau_N}\right) \end{bmatrix} \tag{19}$$

where D_{power_1}, $D_{power\_2,\ldots,}$ D_{power_N} are fitted compliance creep values (pre-smoothed data) obtained using the generalized power law model.

Retardation times, τ_j are assumed with decade increments (i.e., 1/100, 1/10, 1…etc). In this work, the Prony series model was composed by one Maxwell model connected with six Kelvin Elements. By using nonlinear regression techniques, Prony parameters (D_0, D_1, D_2, D_3, D_4, D_5, D_6, and η) in Eq. (19) can be generated and the Prony series function in Eq. (17) was therefore established. The parameters of three representation functions obtained from nonlinear regression methods are depicted in Table 1.

Viscoelastic Modeling Results and Discussion

Results of Fitting Experimental Data

After solving all unknown parameters, it is of interest to evaluate the differences of fitted results among the three functions. Fitted results are sometime shown in a log scale. However, this study found out that if an axis is shown in a log scale or normal scale, the fitting results can present a significant difference among these three representation functions. The paper used four scales to show the differences of fitted results: log $D(t)$ versus time; $D(t)$ versus log time; log $D(t)$ versus log time; and $D(t)$ versus time. The comparisons of fitting experimental data among the three representation functions using a log scale or normal scale are shown in Fig. 3(a) through Fig. 3(d).

Clearly, if the fitted results are displayed in a log creep compliance scale versus a normal time scale as shown in Fig. 3(a), all of representation functions show good agreement with experimental data. However, if fitted results are presented with different scales, the creep compliance curves resulting from three representation models are significantly different. Based on comparisons in Fig. 3, the generalized power law function admits a good option to fit the experimental creep compliance data. To further quantify the differences between the three functions, the creep compliance error (D error) was computed as follow [34]:

$$\text{D error} = \left[\frac{1}{m-1} \sum_{m} \left(D_{raw\,data} - D_{fitted} \right)^2 \right]^{1/2} \tag{21}$$

where

m = the number of data points, $D_{raw\ data}$ = experimental creep compliance data, and

D_{fitted} = fitted value by a representation function

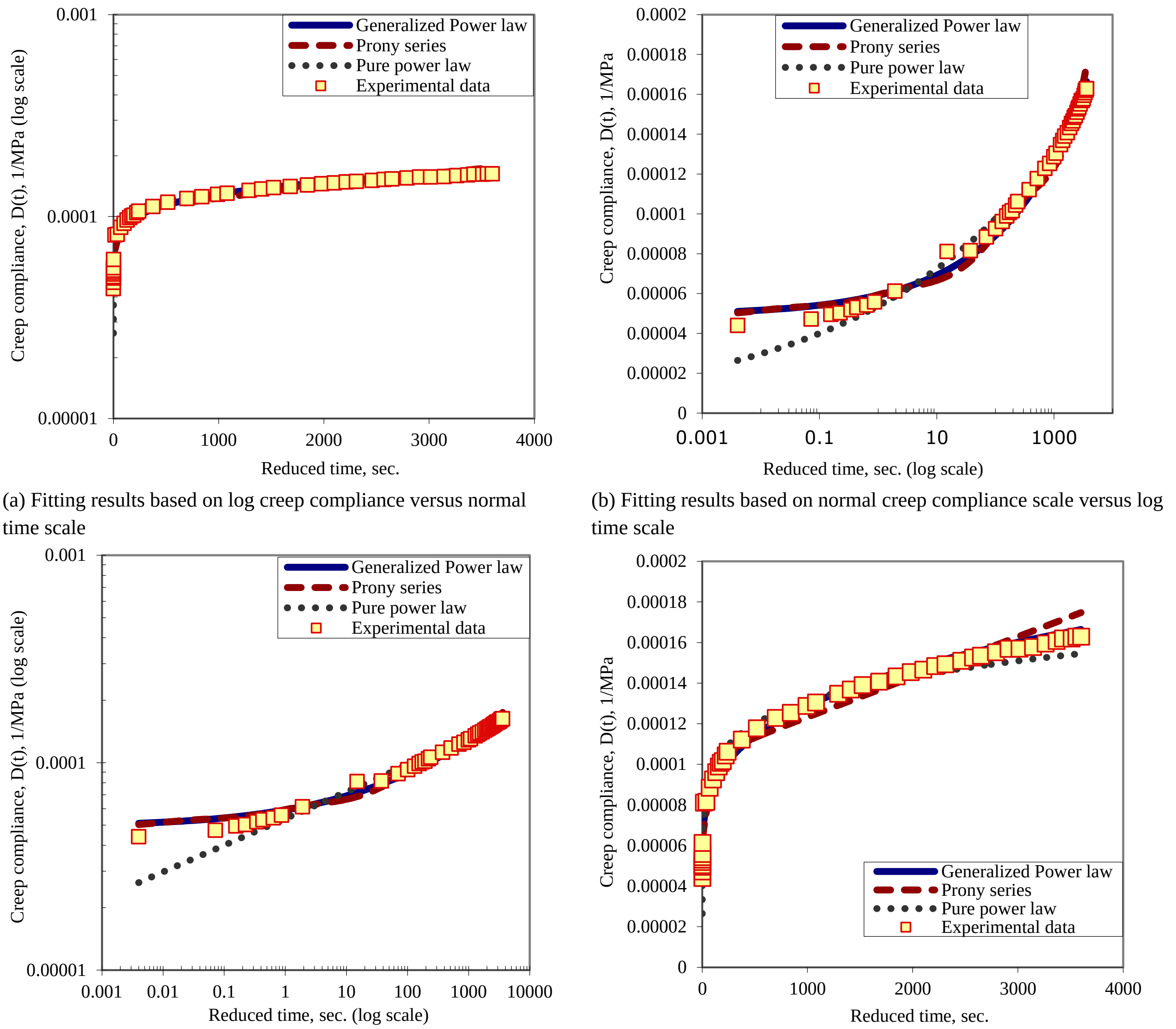
The D error computation was divided into 6 segments in a 600-second increment basis. The comparison is depicted in Table 2. Apparently, the Prony series function has the greatest D error values than other two power law functions. In contract, the generalized power law function shows good agreement with the least D errors as compared to the rest of two functions (Table 2). Comparisons from Fig. 3 and Table 2 indicate that the generalized power law functions can be a good resource of representation function used to fit experimental data at low temperatures. This finding using the generalized power law function to fit creep compliance from the BBR tests is also supportive by the work from the IDT tests as described by Christensen [6].

Prediction of Relaxation Modulus of Asphalt Mixtures

Since all parameters required for each function are determined, viscoelastic analysis can be performed. Inserting given parameters in Eqs. (11), (12), and (17), the relaxation modulus of asphalt mixtures from each function was calculated. The viscoelastic modeling results of asphalt mixtures were shown in Fig. 4. It can be seen that the pure power law shows a good trend along with the Prony series functions in the prediction of relaxation modulus as compared with the generalized power law function. Approximately a 0.2 MPa difference between the generalized power law and the Prony series functions is estimated after the loading passed 500 seconds. However, such results cannot provide evidence of which

Table 1. Parameters of Representation Functions.

Representation Functions	D_0	D_1	D_2	D_3	D_4	D_5	D_6	n	η
Pure Power Law		5.3810E-05						0.12884	
Generalized Power Law	4.9130E-05	1.0057E-05						0.29996	
Prony Series	5.0588E-05	1.1204E-06	2.9164E-06	5.0875E-06	1.1039E-05	2.0765E-05	4.5375E-05		117832677



(a) Fitting results based on log creep compliance versus normal time scale

(b) Fitting results based on normal creep compliance scale versus log time scale

(c) Fitting results based on log creep compliance scale versus log time scale

(d) Fitting results based on normal creep compliance scale versus normal time scale

Fig. 3. Comparisons of Fitting Experimental Data among Three Representation Functions.

Table 2. Comparison of Creep Compliance Error between Three Representation Functions.

Creep Compliance Times, sec	0-600	600-1,200	1,200-1,800	1,800-2,400	2,400-3,000	3,000-3,600	Total Duration
	Creep Compliance Error, 1/MPa						
Generalized Power Law	1.152E-04	2.696E-06	5.749E-06	1.405E-05	1.886E-05	4.973E-05	2.063E-04
Pure Power Law	1.424E-04	9.506E-06	3.128E-06	1.720E-05	3.520E-05	1.305E-04	3.380E-04
Prony Series	2.543E-04	1.318E-05	7.479E-06	3.273E-05	5.424E-05	1.775E-04	5.394E-04

function can better represent the viscoelastic behavior of the asphalt material unless a direct measurement of relaxation modulus is conducted. Thus, a further analysis is needed to facilitate the evaluation. The prediction of thermal-induced stress was performed to evaluate the low temperature properties of asphalt mixtures using the generalized power law and the Prony series functions; due to the inability of representation in the transit zone, the pure power law was not used to calculate the thermal stresses.

Prediction of Thermal Stresses of Asphalt Mixtures Using Power Law and Prony Series

Hills and Brien [28] proposed a pseudo-elastic beam analysis mechanism to calculate the thermal-induced stress:

$$\sigma(T) = \alpha \cdot E(T) \cdot \Delta T \tag{20}$$

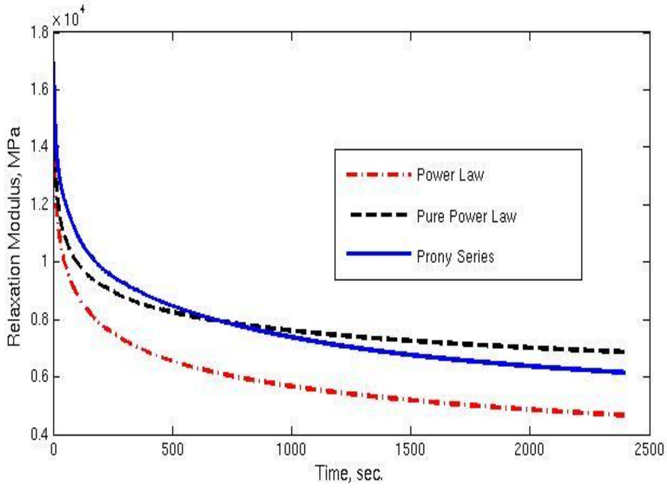


Fig. 4. Comparison of Relaxation Moduli between Three Representation Functions.

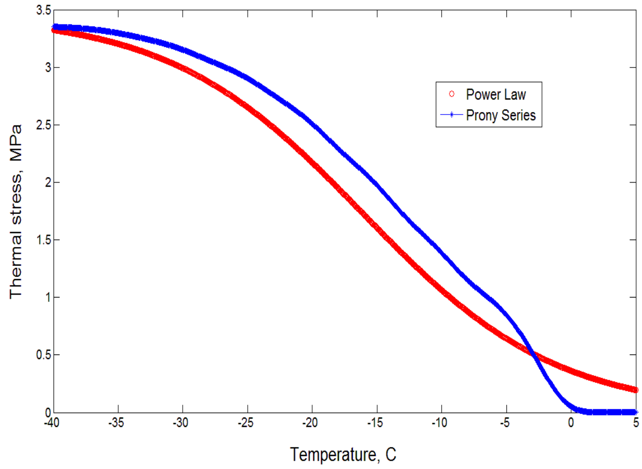


Fig. 5. Comparison of Thermal-induced Stresses between the Generalized Power Law and the Prony Series Functions.

The accuracy of thermal stresses depends on the primary two input parameters selected or assumed in Eq. (20); the coefficient of thermal contraction, α, and the relaxation modulus of asphalt mixtures, *E(T)*. Vinson et al. [29] indicated that thermal-induced stresses that cause low temperature cracking are directly related to the relaxation modulus of the asphalt material obtained by calculations [15], Christensen [26], and Bouldin et al. [30], indirect measurements [31-32], or direct measurements [33]. As mentioned previously, the accurate prediction of relaxation moduli of asphalt mixtures is based on the selection of viscoelastic representation functions. Thus, it is important to quantify creep compliance errors and compare the predicted thermal stresses between the power law and the Prony series functions.

Research by Bouldin et al. [30] recommended that the coefficient of thermal contraction (α) for asphalt concrete can be selected as $1.7\text{x}10^{-4}$ mm/mm/°C, and the 1°C per hour of the temperature increment (ΔT) can be assumed in freeze regions. Given that the calculated relaxation moduli (Eqs. (12) and (17)) are time dependent, *E(t)*, so they are in need of being converted to a temperature domain *E(T)*. This can be done by using the relationship between the shift factor and the temperature given in Eq. (1) and Fig. 2(c). Using such mathematical relations, relaxation moduli of asphalt mixtures *E(t)* derived from Eqs. (12) and (17) can be converted to a function of temperature *E(T)*.

Substituting the recommended values of α (coefficient of thermal contraction) and ΔT (temperature increment) along with the determined relaxation modulus into Eq. (20), the thermal stresses of asphalt mixtures using the power law and the Prony series functions were computed. The comparison of the thermal stresses between these two models was displayed in Fig. 5. It is observed that the curve of the Porny series model is not as smoothed as that of the generalized power law. The possible reason will be discussed later. To further predict the thermal cracking resistance, the determination of the critical cracking temperatures from the two functions was performed.

The strength of asphalt mixtures of 3.0 MPa was tested from the IDT by Bouldin et al. [33]. Applying their work to this study, the critical thermal cracking temperature for the power law model is

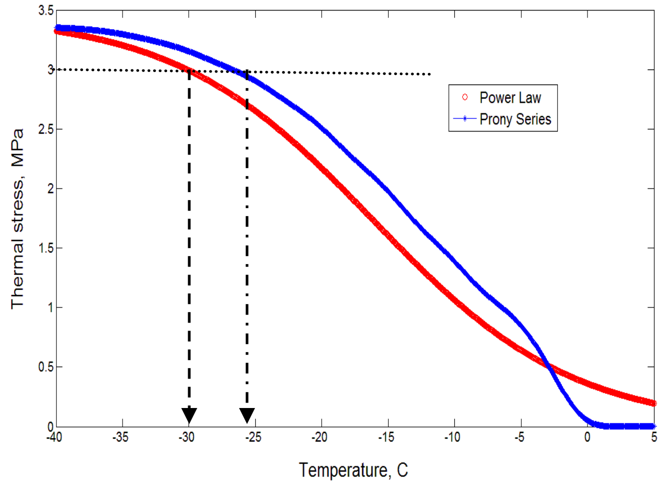


Fig. 6. Critical Thermal Cracking Temperatures.

determined approximately at -30°C as the critical thermal cracking temperature for the Prony series model is located at -26°C (Fig. 6). Both representation functions provide a significant 4°C difference in the critical cracking temperature. Since asphalt specimens were mixed with binder PG 64-34, the prediction of thermal stresses using the power law model (generalized power law function) presents a reasonable result than the Prony series model in terms of thermal resistance to low temperature changes.

The difference of thermal stress predictions between these two models can be attributed to the fact that the generation of Prony series parameters is based on a pre-smoothing technique using the power law model. Thus, the parameters used to compute relaxation moduli and thermal stresses are not directly obtained from BBR tests. Such 'indirect" process of obtaining Prony series parameters may create inconsistent results between the formulation of representation functions, and computations of relaxation moduli and thermal stresses.

Conclusions

Thermal cracking is a critical pavement distress in cold regions. The BBR test using asphalt mixture beams has been recommended to be an appropriate material testing method to characterize low temperature properties of asphalt mixtures. While there are three major representation functions that have been used to compute relaxation moduli and thermal stresses of asphalt mixtures, a comparative study was implemented to provide informative results for the selection of a proper representation function. Through linear viscoelastic analyses, this paper has the follow conclusions:

1. Creep compliance data were obtained from the BBR tests. The pure power law, the generalized power law, and the Prony series representation functions were used to fit these experimental data. The generalized power law model shows good agreement with experimental data and has the lowest creep compliance error values as compared with other two functions. This finding confirms with the previous work by IDT tests.
2. The generalized power law results in a reasonable prediction of thermal stresses with the determination of critical cracking temperature at -30℃ which is within the designed low temperature binder range of -34℃.
3. Through a series of linear viscoelastic analyses associated with nonlinear regression methods, the generalized power law appears to be a good resource of viscoelastic representation function used to characterize the low temperature performance of asphalt mixtures. When performing the BBR test or the IDT test, the generalized power law model presented can be considered as a representation function for modeling the low temperature viscoelastic behavior of asphalt mixtures.

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