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Evaluation of Prediction Models of Fatigue for Asphalt Mixes

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Abstract: To enhance the understanding of fatigue testing of asphalt mixes, especially at design stage, this study was initiated to investigate the possibility of evaluating fatigue life of asphalt mixes using simple fracture tests (i.e. Semi-Circular Notched Bending (SCNB) Test). Results have shown that a viable correlation exists between parameters determined by monotonic and cyclic SCNB, and based on these results three models were developed using statistical analysis. It was illustrated that monotonic strain energy had better prediction of fatigue life of asphalt mixes than other parameters. Furthermore, sensitivity analysis was carried out to evaluate these models. Tornado plots and extreme tail analyses showed that the strain energy parameter had the most contribution to fatigue life predictions among evaluated parameters. Finally, these models were developed not to replace traditional tests of fatigue life assessment, but to serve as a screening tool and help eliminate weak mixes before conducting more sophisticated and time consuming tests. Thus, pavement mix fatigue life of various mixes could be compared without the use of expensive and time consuming fatigue testing.

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***Key words**: Extreme tail analysis; Fatigue test; Hot mix asphalt; Sensitivity analysis; Strain energy; Tornado plot.*

Introduction

Modern engineering evaluation and design practices should be based on dynamic loading conditions, which simulate an actual condition. However, most engineering designs are based on experimental and field data in monotonic or quasi-monotonic loading conditions due to simplicity of test under dynamic conditions. Fatigue may be characterized as a progressive failure phenomenon that begins by the initiation and subsequent propagation of cracks to an unstable condition. Although, fatigue failure is the cause of most engineering failures especially in asphalt pavements, there is no complete agreement on the microscopic details of the initiation and propagation of cracks under fatigue mode.

To improve asphalt pavement design, simple and dependable tests, and validated procedures are needed to enhance the mix design process to prolong asphalt pavement life. Currently, the fatigue test that is used to estimate asphalt pavement life is considered expensive and time consuming [1]. Thus, fatigue testing may not be feasible for each project. Therefore, it is very important to develop effective and simple methods or predictive models to evaluate the fatigue resistance of asphalt pavements. In this study, simple and reliable models that predict fatigue resistance of asphalt mixes were developed and evaluated.

Theories and Prediction Models

When studying fatigue, some basic concepts should be understood. These simplified concepts are presented below, followed by various theories and models for predicting pavement fatigue life by using parameters including strength, material characteristic, strain energy concept, statistical approach, etc.

Stress spectra are required to assess fatigue life. The simplest fatigue stress spectrum is a zero-mean sinusoidal stress-time pattern of constant amplitude and fixed frequency, applied for a specified number of cycles.

In asphalt pavement, fracture occurs mainly due to concentration of tensile stresses in the bottom of the asphalt mix layer resulting from traffic loads. Many existing fatigue models in the area of asphalt pavement were developed using regression analysis of experimental data. Wöhler developed a relation that utilized tensile strains resulting from a cyclic load, change in temperature, and other conditions, to investigate pavement fracture as follows [2, 3]:

$$N_f = k_1\left(\frac{1}{\varepsilon}\right)^n \text{ or } N_f = k_1(\varepsilon_0)^{-k_2} \qquad (1)$$

where,

N_f : number of load applications to failure,

ε_o: tensile strain at the bottom of the asphalt layer, and

k_1, k_2, and n: factors that are functions of the composition and properties of the asphalt mix.

k_2 is the same as the parameter n of Paris's Law, ($\frac{da}{dN} = A(\Delta K)^n$).

Based on Eq. (1), the values of n have been estimated using regression analysis. The value of n was determined to be 5 by Bonnaure et al. [4] and 3.624 by Tayebali et al. [5]. Medani and Molenaar [3] simplified the estimation of fatigue characteristics of asphalt mixes. Their method was based on the mix properties and stiffness modulus master curve(s) which is the relationship between the loading time and the stiffness of asphalt mix. Other researchers [6-8] used a different approach, they utilized continuum damage models applied to asphalt concrete under monotonic loading and cyclic loading based on the work potential theory developed by Schapery [9]. The work potential theory was used to model the

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mechanical behavior of linear and nonlinear elastic media with changing structure and growing damage. The theory can be applied to describe the effects of viscoelasticity.

Daniel [10] presented a methodology based on a single testing condition that can be used to predict material response under different uniaxial tension test conditions. Utilizing the work potential theory and the continuum damage theory, a constitutive model for asphalt concrete was developed. Later, Daniel and Kim [7] found that a half of a cycle or a quarter of a cycle could be used for an approximation case; in this case a quarter of a cycle was used. From the pseudo strain and functional form of the characteristic curve, the stress response may be predicted. Lundstrom and Isacsson [11] further investigated the work of Daniel and Kim [7] to determine whether the monotonic test was able to predict fatigue characteristics of asphalt concrete mixtures. The work potential theory was a relatively simple approach for modeling material experiencing damage growth.

Lately, numerous studies have been investigating the correlation between strain energy and prediction of fatigue life and fracture in asphalt mixes. Shen and Carpenter [12] developed an asphalt fatigue model that is based on strain energy principles. Zhou et al. [13] presented another approach, which utilized overlay tester, to predict fatigue cracking initiation and propagations by determining fracture properties of asphalt mixes. Liu et al. [14] suggested that fatigue evolvement law in asphalt mixes can be based on the conception of energy consumption and that fatigue life of asphalt mixes was in good accordance with fracture resistance. Pi et al. [15] evaluated fatigue life of asphalt mixes using splitting creep test, and they indicated that there was a good relativity between creep damage time and fatigue damage time.

Further, Kim and Kim [16] argued that crack prorogation is controlled by the fundamental material properties of energy dissipated and energy threshold, they expected that their developed model would fill the gap between viscoelastic fracture and continuum damage mechanics, and provide better cracking predictions of asphalt mixes. Li et al. [17] proposed a fatigue prediction in asphalt mixes model based on fracture energy determined from indirect tension test results. Further, they argued that a fatigue model based on the ratio of dissipated energy change is independent of the material types and shows relatively high levels of prediction accuracy. Pérez-Jiménez et al. [18] indicated that fatigue characterization requires complex cyclic tests, thus, not usually taken into account when designing bituminous mixtures, and introduced a new fracture test, results showed that this new methodology could assess fatigue behavior of asphalt and the procedure could be used to relate stiffness and dissipated energy parameters obtained in the cracking process.

Sensitivity Analysis

Information flowing in and out of a model or equation is widely investigated and its relationships by using sensitivity analysis [19]. Many studies have been conducted on asphalt mixes design methods and distresses predictions [20-27]. In the past, sensitivity analysis was achieved by changing one variable in a model, while keeping the others fixed, however this may not lead to apprehend the effect of the interaction between variables on results [25]. Therefore, to achieve a better grasp on the relation between different parameters, a dynamic sensitivity analysis is required. Tornado Plots and Extreme Tail Analysis are found to be better methods of evaluating the sensitivity of variables in models since they take into consideration the effect of all variables collectively [28, 29].

Tornado Plots

Tornado plots are commonly used as a graphical solution, which illustrate the effect of all input parameters on the result of a model, it is generated using the Spearman's rank correlation coefficient (ρ) as,

$$\rho = 1 - \left(\frac{6\sum d_i^2}{n(n^2-1)}\right) \tag{2}$$

where,

ρ: Spearman's rank correlation coefficient,

d_i: difference in the ranks between input and output values in the same data pair, and

n: number of simulations.

When the absolute value of ρ is closer to one, the effect of the variable on the output of the model is maximum. When it is nearer to zero, the effect is minimal. In addition, a positive ρ value designates that the output of the evaluated model is proportionally related to the variable (i.e., when the variable value is high, the output will be high as well). When ρ has negative value, the output is inversely proportional to the variable [25].

Extreme Tail Analysis

Extreme tail analysis is used to indicate the uncertain critical variables in a model, which may contribute to extreme values in a model result. To pinpoint those parameters, the normalized variable (α) is calculated as described in Eq. (3) (method details can be found in reference [28]). The extreme values of variables in the group with $|\alpha| \geq 0.5$ are commonly considered to be significant and contribute to the extreme values of the predicted output. A positive significant value for an input indicates that extreme values of the input are one of the possible reasons for positive extreme output values and vice versa [25].

$$\alpha = \left(\frac{Median_{Group} - Median_{Total}}{\sigma_{Total}}\right) \tag{3}$$

where,

$Median_{Group}$: median of the input in the group,

$Median_{Total}$: median of the input in the total simulation, and

σ_{Total}: standard deviation of the input in the total simulations.

Materials

A wide range of commonly used mixes in the State of Idaho were selected to construct a mix matrix. To evaluate aggregate structure effect on fatigue life of asphalt mixes, four different aggregate structures (including Mix 1, Mix 2, Coarse Mix, and Fine Mix) were used, as shown in Fig. 1. In addition, to evaluate binder effects; 3 binder contents (optimum asphalt content and ±0.5% from optimum content for Mix 1 and Mix 2), and eight binder grades were used

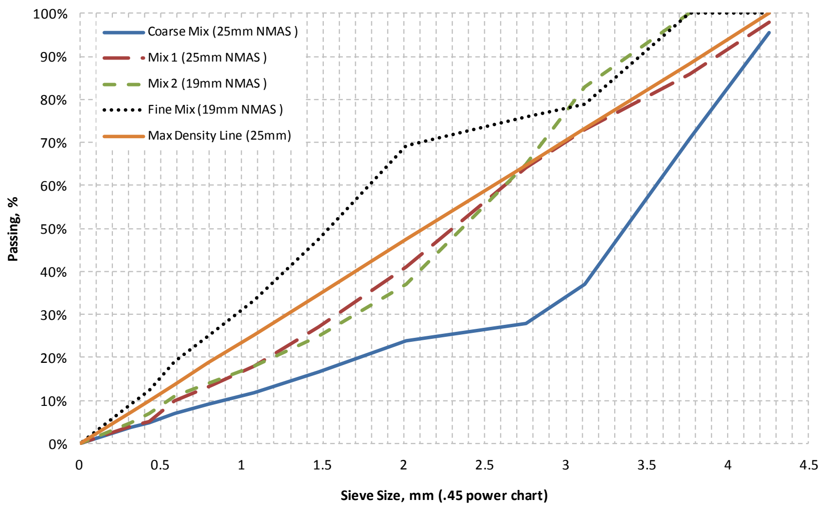


Fig. 1. Aggregate Gradation (After Abu Abdo et al. [30]).

(PG 70-34, PG 70-28, PG 70-22, PG 64-34, PG 64-28, PG 64-22, PG 58-34, and PG 58-28), as shown in Table 1. Additional seven field mixes were also obtained for additional testing and evaluation (Table 2).

Sample Preparation and Tests Setup

Sample Preparation

All specimens were compacted under controlled lab conditions. Specimens were compacted using the Servopac Gyratory Compactor to a number of gyrations to produce specimens with 4±1% air voids. Compacted cylindrical specimens were sliced into 4 semi-circle specimens, that is each specimen became a half circle beam as shown in Fig. 2. Samples were notched with 12.7, 25.4 and 31.8 mm notch depths to evaluate the different notch depths on crack propagation and their effect on fatigue evaluation of tested samples.

To obtain the dynamic modulus for tested mixes, specimens were compacted to achieve 175 mm high specimens with a total 9% air voids. Then cored and sawed to obtain 100 mm diameter and 150 mm high specimens with 7% air voids.

Test Setups

Earlier studies [1, 31-34] have shown that the Semi-Circular Notched Bending (SCNB) test setup can be used efficiently to measure the fracture and fatigue properties. In addition, with implementation of Superpave mix design method, the utilization of compacted samples that are produced by the Superpave Gyratory Compactor (SGC) was preferred by the pavement industry, since the gyratory compacted samples are easier to prepare, more consistent, and the compaction method was developed to simulate actual compaction on site. Thus, it was decided to use semi-circular beams in this study.

Static Semi-Circular Notched Bending (SCNB) Test

Semi-circular beams were tested in a 3 point test setup with spacing between the two roller supports of 120mm. An MTS 810 machine and Flex Test SE Version 5.0 C 2299 system were used for applying loads to the specimen. A ramp load with a constant vertical deformation rate of 0.075mm/minute was used [1, 35].

Cyclic Semi-Circular Notched Bending (SCNB) Test

Fatigue testing for assessing relationships between cyclic and Static tests consisted of cyclic loading at a loading frequency of 1 Hz, and loading rate of 0.075mm/minute. The applied load ranged from approximately 30 to 60% of the static maximum strength. The

Table 1. Matrix of Lab Mixes.

Binder Grade	PG 70-34			PG 70-28			PG 70-22	
Coarse Mix					4.9%			
Mix 1		4.9%		4.4%	4.9%	5.4%		4.9%
Mix 2		4.4%			4.4%			
Fine Mix					4.9%			
Binder Grade	PG 64-34			PG 64-28			PG 64-22	
Mix 1					4.9%			
Mix 2	3.9%	4.4%	4.9%		4.9%			4.4%
Binder Grade	PG 58-34			PG 58-28				
Mix 1					4.9%			
Mix 2		4.4%						

Table 2. Properties of Field Mixes.

Mix ID	1	2	3	4	5	6	7
Class	SP4	SP3	SP3	SP2	SP3	SP3	SP3
ESALs	> $30x10^6$	3 - $30x10^6$	3 - $30x10^6$	0.3 - $3x10^6$	3 - $30x10^6$	3 - $30x10^6$	3 - $30x10^6$
N-design	125	100	100	75	100	100	100
Mix Properties							
G_{mm}	2.449	2.424	2.568	2.480	2.448	2.581	2.460
AV%	4%	4%	4%	4%	4%	3.5%	3.5%
Sample wt., gr	4700	4600	4800	4600	4750	-	-
Binder Properties							
PG	70-28	64-34	70-28	58-34	70-28	70-28	70-28
P_b%	4.90%	4.35%	5.40%	6.20%	5.12%	5.90%	5.10%
G_b	1.021	1.025	1.034	1.009	1.021	1.036	1.036
Mix Temp., ℃	166	168	162	162	166	164	164
Comp. Temp., ℃	152	153	145	141	152	152	152
Aggregates Properties							
G_{sb}	2.586	2.558	2.771	2.731	2.589	2.822	2.822
G_{se}	2.639	2.568	2.808	2.744	2.648	-	-
Absorption	1.3%	1.5%	1.9%	1.7%	1.9%	-	-
Fine Agg. Angularity	49%	52%	51%	47%	46%	44%	44%
Flat and Elongation	1%	6%	1%	0%	1%	-	-

Note: PG: Binder Grade,
G_{mm}: Maximum Theoretical Specific Gravity of Mix,
AV%: Air Voids,
P_b: Binder Content,
G_b: Binder Specific Gravity,
G_{sb}: bulk Specific Gravity of Aggregates, and
G_{se}: Effective Specific Gravity of Aggregates

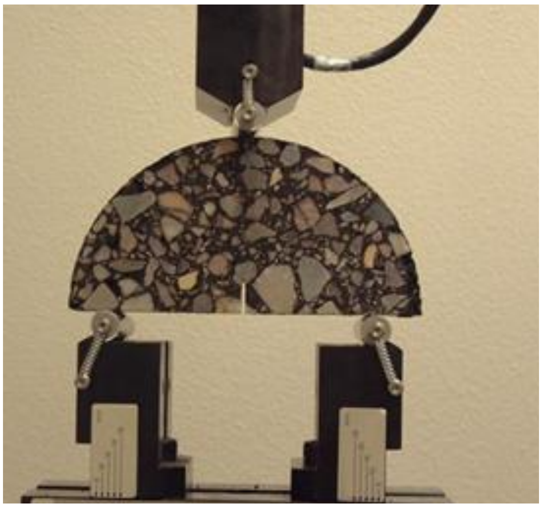


Fig. 2. Three Point Load Testing Set-up on the MTS Machine.

failure of specimens could be defined by identifying a transition point between elastic and plastic behavior of tested samples, at which an uneven peak interval between maximum and minimum loading wave could be observed [34].

In this study, testing was conducted at room temperature to minimize the effect of temperature, both tests were conducted at room temperature (21℃) using the test set-up shown in Fig. 2. This temperature was chosen as a representative temperature for all mixes, since Superpave mix design, determines the fatigue resistance at the mid temperature range of binder grades.

Dynamic Modulus (|E|) Tests*

The test was conducted as per AASHTO TP 62-03 [36], which consists of applying a uniaxial sinusoidal compressive stress to an unconfined HMA cylindrical test specimen and measure the corresponding strain using 3 LVDTs mounted on the middle of the specimen. The dynamic modulus test was conducted with a temperature of 21℃ and loading frequency of 1 Hz, to match the loading condition of the cyclic SCNB test.

Data Analysis

Fatigue Models

A total number of 426 tests, both monotonic and fatigue were performed as part of this study. Of these 426 tests, 212 were monotonic and 214 were fatigue tests. The test results were averaged for each testing condition to represent the actual result; these results showed that fracture (monotonic) results followed a similar trend as fatigue (cyclic) results. Therefore, using static SCNB test, which is a simple and short test, and by plugging its results in a model we can rank mixes and decide which will be better to use. To assess fatigue life based on static fracture test results, statistical analysis was utilized to develop fatigue models.

Three models relating monotonic fracture test results to fatigue life were developed and one existing model was assessed using the data from this study. The purpose of these models was not to replace

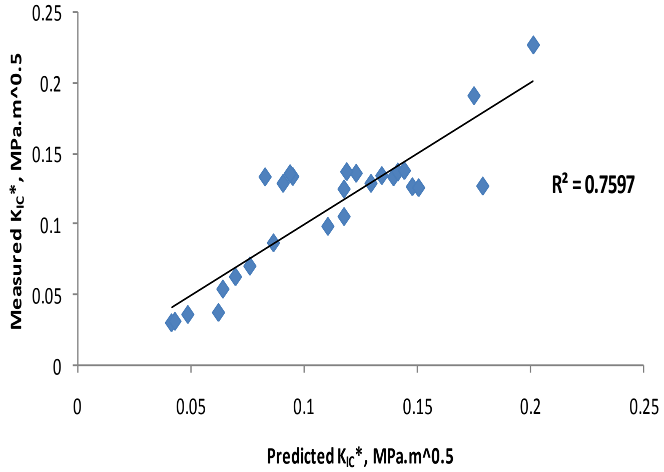


Fig. 3. Measured K_{IC}* Versus Predicted K_{IC}* (Model I).

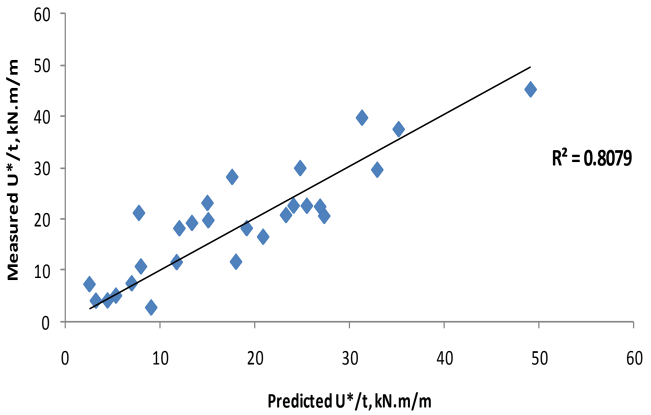


Fig. 4. Measured U*/t versus Predicted U*/t (Model II).

the assessment of fatigue life as estimated using the mechanistic empirical pavement design guide, but to serve as an initial assessment of pavement mix fatigue life. Thus, pavement mix fatigue life of various mixes can be compared without the use of expensive and time consuming fatigue testing.

Model I: Cyclic Stress Intensity Factor (K_{IC})*

The widely used Asphalt Institute model (Eq. (4)) was used as a base to develop a model to estimate the cyclic stress intensity factor (K_{IC}*) that is calculated from the fatigue test. Using a sensitivity analysis, it was found that the ultimate static (monotonic) load (P_{max}) and the dynamic modulus (|E*|) at the same load application frequency to the cyclic test could be utilized effectively to estimate K_{IC}*. The model is described in Eq. (5) with an R^2 of 0.7597 as shown in Fig. 3, and Se/Sy equals to 0.4902.

$$N_f = 0.0796(\varepsilon_t)^{-3.291}\left(|E^*|\right)^{-0.854} \tag{4}$$

where,
N_f: number of load application to failure,
ε_t: tensile strain at the bottom of the asphalt mix layer, and
|E*|: asphalt mix dynamic modulus.

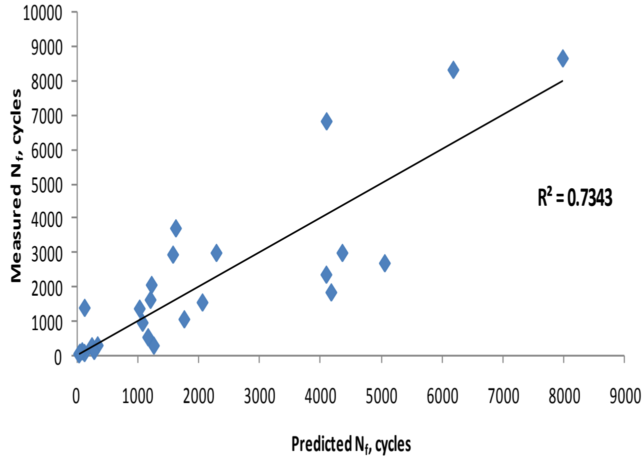


Fig. 5. Measured N_f versus Predicted N_f (Model III).

$$K_{IC}^* = 2.11\times 10^6 (P_{max})^{0.584339}\left(|E^*|\right)^{-4.8} \tag{5}$$

where,
K_{IC}*: stress intensity factor for fatigue (cyclic) testing (MPa.m$^{0.5}$),
P_{max}: ultimate monotonic (fracture) load (N), and
|E*|: asphalt mix dynamic modulus at 1Hz (MPa).

Model II: Cyclic Strain Energy (U/t)*

Following the same approach as Model I, the Asphalt Institute model (Eq. (4)) was used as a base to estimate the cyclic strain energy per unit thickness (U*/t) that was determined using the fatigue test. Using a sensitivity analysis, it was found that strain energy per unit thickness (U/t) determined using the monotonic test, dynamic modulus (|E*|) at the same load application frequency to the cyclic test, and sample' air voids could be utilized effectively to estimate U*/t with an R^2 of 0.8079 (Fig. 4) and Se/Sy equals to 0.4383. The model is presented in Eq. (6).

$$U^*/t = 1.74\times 10-11(U/t)^{0.5974}\left(|E^*|\right)^{0.2610}(AV)^{-7.4404} \tag{6}$$

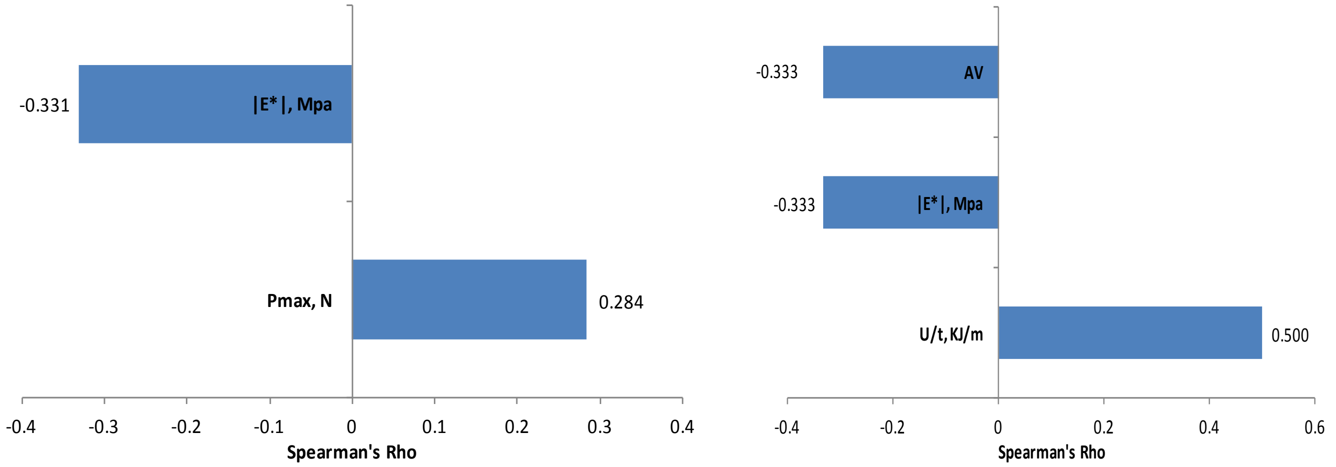
where,
U*/t: cyclic strain energy per unit thickness (N.m/m),
U/t: monotonic strain energy per unit thickness (N.m/m),
|E*|: asphalt mix dynamic modulus at 1Hz (MPa), and
AV: asphalt mix sample's air voids.

Model III: Number of Cycles to Failure (N_f)

This model was developed using an approach presented by Van Dijk et al. [37]. They developed a model that predicted the number of cycles to failure in a fatigue test of bitumen and asphalt–aggregate mixtures based on the dissipated energy (W_{fat}) concepts as follows,

$$W_{fat} = AN_f^z \tag{7}$$

Assuming that the total strain energy in a monotonic test is mostly dissipated, Van Dijk's model was modified to match that assumption as shown in Eq. (8). This model has an R^2 of 0.7343 as



a) Model I　　b) Model II

Fig. 6. Tornado Plots for Model I and II.

Table 3. Extreme Tail Analysis of Model I, II, and III

K_{IC}* (Model I)	α, Left Tail	α, Right Tail
Ultimate Load, P_{max}	-0.0134	-0.0199
Dynamic Modulus, |*E**|	1.9839	-1.954
U/t* (Model II)	α, Left Tail	α, Right Tail
Monotonic Strain Energy per Unit Thickness, *U/t*	-1.5188	1.0393
Dynamic Modulus, |*E**|	-0.2737	0.3435
Air Voids, AV	0.7273	-1.7178
N_f (Model III)	α, Left Tail	α, Right Tail
Monotonic Strain Energy per Unit Thickness, *U/t*	-1.5506	2.1658

shown in Fig. 5, and Se/Sy equals to 0.5155.

$$N_f = 16.835(U/t)^{1.871} \qquad (8)$$

Sensitivity Analyses

The first step of any sensitivity analysis is to determine the probability distributions of the model variables. The distributions of the variables in this study were determined using the mean and the standard deviation of the determining variables. To generate the tornado plots for the parameters (Eqs. (5) and (6)). Model III was excluded since the model has one variable, which would show an absolute value of ρ equal to 1, thus tornado cannot be used. Based on Model I and II means and standard deviations, 50000 data points were generated using Monte Carlo Simulation, and then the Spearman's rank coefficients were determined using Eq. (2), as shown in Fig. 6.

Results illustrated (Fig. 6) that among the developed Models, Model II showed the best sensitivity to its prediction (ρ values 0.333 and 0.5). Further, it was found that the dynamic modulus (|*E**|) and air voids have negative relations (negative ρ) between them and *U*/t* (Model II), where *U/t* and P_{max} have a positive relation (positive ρ). A negative relation between a model variable and a predicted value means the higher the variable the lower the predicted value would be and vice versa.

Using the same set of data points generated earlier, the extreme tail analysis was conducted on the three models. The predicted values were arranged in descending order, for a reliable output, left and right tail of the distributions were evaluated. The lower and higher 5% of the predicted values were taken as the left and right tails, respectively. Then α for all variables were determined using Eq. (3), and the results are shown in Table 3.

Unlike the tornado plots results (Fig. 6), it was found that in model I, |*E**| has a high contribution to the extreme values of K_{IC}* (Model I) with α ≥ |0.5|. However, |*E**| has no contribution to the extreme values of *U*/t* (Model II) with α ≤ |0.5|. In addition, it was found that P_{max} has no contribution to the extreme values of K_{IC}* (Model I) with a low α. In addition, *U/t* in Model II and Model III, was the most effecting contributor to the prediction of *U*/t* and N_f. Overall results have shown that the strain energy approach was the better approach to predict fatigue property of asphalt mixes when utilizing monotonic testing.

Summary

The purpose of this study was to investigate the possibility of evaluating fatigue life of asphalt mixes using simple fracture tests (i.e. Semi-Circular Notched Bending (SCNB) Test). A total number of 426 tests, both monotonic (fracture) and cyclic (fatigue) were performed as part of this study. Of these 426 tests, 212 were monotonic and 214 were fatigue tests. Results showed that fracture (monotonic) results followed a similar trend as fatigue (cyclic) results. Based on these finding, three models were developed using traditional statistical analysis. It was found that models that are based on strain energy have better prediction of fatigue life of asphalt mixes. Further, sensitivity analysis was conducted to further evaluate the developed models. Tornado plots and extreme tail analysis showed that fatigue life predictions were extremely sensitive to the strain energy parameter obtained by monotonic SCNB test. These models were developed not to replace the assessment of fatigue life as estimated using traditional methods, but to serve as an initial evaluation of pavement mix fatigue life. Thus, pavement mix fatigue life of various mixes could be compared without the use of expensive and time consuming fatigue testing.

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