Technical Paper

ISSN 1997-1400 Int. J. Pavement Res. Technol. 6(6):745-754

An Algorithm to Estimate Rational Values of Phase Angles and Moduli of Asphalt Mixtures

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Abstract: The objective of this study was to develop and evaluate an algorithm based on Fast Fourier Transform (FFT) that can calculate rational values of phase angle (ϕ) and moduli of the variants of asphalt mixtures for the data obtained from the different frequency sweep tests. ϕ and moduli for ten different asphalt mixtures resulting in over 690 data points collected from both USA and Sweden were computed using FFT. Theoretical observations revealed that there were significant differences for ϕ between FFT and other methods to the order of 10-50%; however, there was no difference in moduli estimates for any mix and was independent of the test. Precisely, the FFT method produced rational ϕ for mixtures that deviate from conventional mixture properties. Furthermore, statistical comparisons corroborated the predicted ϕ estimates indicative of significant differences between the analysis techniques; but, the moduli were unaffected by the analysis methods. The study successfully illustrated the FFT technique, a user-friendly analytical procedure that can obviate the errors in the rational estimation of the acutely sensitive viscoelastic parameters.

DOI:10.6135/ijprt.org.tw/2013.6(6).745
Key words*: Asphalt mix; Fast Fourier Transform; Frequency-sweep test; Moduli; Phase angle.*

Introduction

Asphalt mixtures exhibit unique characteristics of both viscous and elastic properties, and hence are categorized as viscoelastic materials. Asphalt mixes possess time-dependent or rate sensitive stress-strain relations. In other words, the stress-strain relationship will change as the loading speed (or strain rate) changes. Understanding the viscoelastic properties of asphalt mixtures is therefore important to achieve performance-based structural design of bituminous layers [1, 2], which often includes design life prediction, aging effects, distress evaluation (cracking, rutting, *etc*.), and functional characteristics such as friction, texture, and noise.

Cyclic dynamic tests, such as dynamic complex modulus or dynamic shear modulus tests are used to obtain stiffness properties of asphalt mixtures such as dynamic modulus *(E*)*, and shear modulus *(G*)*. The tests are usually performed at several different frequencies (frequency sweep tests) and temperatures to capture the influence of loading rates and time-dependent properties. Thus, apart from the stiffness properties, one other important parameter is usually obtained that provides an understanding of the mix's viscoelastic properties, which is termed phase lag or angle (ϕ). Note that G^* and ϕ indicated in this study are the parameters used for the asphalt mixtures (based on shear modulus tests illustrated later on in the paper) and not of the bituminous binders.

Along with being used in the structural design of the bituminous layers, viscoelastic properties have been well-utilized to characterize the pavement noise behaviour of the various asphaltic materials [3, 4]. Recently, ϕ of asphalt mixtures was also used as an important quantity in estimating the dissipated energy during dynamic load testing which subsequently was utilized for estimating the fatigue resistance of the asphalt mixtures based on energy method [5].

A score of frequency sweep tests have been performed at VTI – Swedish National Road and Transport Research Institute, and other research institutes across the world to obtain stiffness or (dynamic) modulus *(E* or G*)*, and ϕ for the different asphalt mixtures. In the process of experimentation, huge amount of raw data is generated in the form of strain and stress signals which need to be manipulated to obtain the actual values of stiffness and ϕ. Admittedly, calculations of E^* or G^* is not very difficult owing to its prime relation with stress and strain. However, estimation of ϕ is usually cumbersome and error prone due to manual calculations and the intricacy of its estimation based on the various inherent estimators that are correlated with strains in the time (or frequency) domain.

Based on dynamic mechanical test data, in the past, when powerful computers and/or mathematical programs were unavailable, simple regression method was recommended to estimate viscoelastic properties as a more direct approach rather than using the Fast Fourier Transform (FFT) [6]. However, with the advent of advanced mathematical software tools (example: MATLAB, Mathcad, *etc*.), it is now possible for practitioners to easily apply the FFT method to understand the various frequency sweep test parameters.

Although a few studies [7, 8] have attempted to analyze the outputs of the E^* raw test data, specifically ϕ using the various analytical procedures, none of them provides conclusive remarks about the best procedure to be used for analyses purposes. Additionally, the studies have used limited test data using only conventional dense graded mixtures and E^* tests (*i.e.*, did not make use of different varieties of asphalt mixtures and other mechanical test methods) to verify the procedures, which makes it uncertain if those analyses can be used for any type of mix and/or mechanical tests. Furthermore, although a few mechanical test equipment

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Note: Submitted November 10, 2012; Revised May 18, 2013; Accepted May 19, 2013.

available in the market have data acquisition systems with built-in analytical solutions using linear smoothing or FFT to analyse the raw data, there also exist some software data acquisition systems across the world that do not have the capabilities of providing analytical solutions to the raw data during the test and/or immediately after the test is completed. Thus, the user is bound to analyze the raw data using manual calculations, which are cumbersome and irrational.

To reduce the intensive manual work required to obtain ϕ and moduli of an asphalt mix from the outputs of any type of dynamic frequency-sweep test, a simple yet a robust program is needed that can automatically calculate the rational values of ϕ and moduli using raw data. Not only that, the use of high-sensitive estimators (such as ϕ) will also enhance the importance of viscoelastic parameters for further utilization as one of the major asphalt pavement materials' responses, which is often ignored by many across the world. Concurrently, rational moduli from laboratory tests could be well-utilized in the sound pavement design and sensible field performance prediction of the viscoelastic asphalt mixtures. Thus, the main objective of this study was to develop a robust computational algorithm that can estimate rational ϕ and moduli of the different types of asphalt mixtures for the data obtained from mechanical tests such as stiffness modulus, dynamic modulus, and shear modulus.

Background to Fast Fourier Transformation (FFT) Technique

The Fourier transform is a mathematical operation that decomposes a given function into its constituent frequencies known as its frequency spectrum. The composite waveform depends on time, and therefore is called the time domain representation. The frequency spectrum is a function of frequency and is called the frequency domain representation. Each value of the function is a complex number that contains information regarding the magnitude and phase of each frequency component. Mathematically, the Fourier transform of a function *f(x)* is given by [9]:

$$\hat{f}(\zeta)=\int_{-\infty}^{\infty} f(x)e^{-2\pi i x\zeta}\,dx \tag{1}$$

where: $\hat{f}$ denotes the Fourier transform of the function and ζ is the frequency.

Associated to the Fourier transform is the Cross Power Spectrum (CPS) of two signals, *g(t)* (stress signal) and *h(t)* (strain signal) which are defined as the Fourier transforms of the cross correlation functions of the two signals. The cross correlation, ρ_{gh}, of *g(t)* and *h(t)* is given by:

$$\rho_{gh}(\tau)=\int_{-\infty}^{\infty} g(t)h(t+\tau)\,dt \tag{2}$$

The CPS of *g(t)* and *h(t)* is thus, cross correlation theorem using the properties of Fourier transform:

$$Y_{gh}(\zeta)=\int_{-\infty}^{\infty}\rho_{gh}e^{-2\pi i x\zeta}\,dx=\frac{\hat{g}(\zeta)\overline{\hat{h}(\zeta)}}{N^2} \tag{3}$$

where $\hat{g}(\zeta)$ and $\overline{\hat{h}(\zeta)}$ are the Fourier transforms of *g(t)* and complex conjugate of the Fourier transform of *h(t)*, respectively; Y_{gh} is the CPS; and *N* is the sample size of the data.

The CPS shows the strength of the signal (energy) as a function of frequency. In other words, it shows frequencies at which the signal is strong or weak. As CPS is a complex valued function, it contains both magnitude $|Y_{gh}(\zeta)|$ and ϕ between *g(t)* and *h(t)*.

In essence, FFT technique uses a sound mathematical function explained earlier with a robust and well-established numerical algorithm. Additionally, FFT has been an appropriate choice particularly, in the analysis of cyclic or periodic functions. So, the analysis makes it possible to easily distinguish between the frequencies contained within a given set of periodic or sinusoidal data. With the many merits of the technique, FFT has been used in various applications where mathematical sensitive parameters are required to be analyzed after eliminating "noise" such as signal processing, x-ray crystallography, sound spectrograms, *etc*.

Methodology / Algorithm to Obtain Rational ϕ and Moduli using FFT Technique

This section provides a detailed procedure in the form of an algorithm adopted to obtain rational ϕ and *E** (or *G**). Prior to performing FFT of the raw data obtained from any mechanical test, the linear trend in the stress and strain signals must be removed. This step is necessary for the reason that FFT is performed on discretized signal similar to a Discrete Fourier Transform (DFT), which is the discrete version of the integral Fourier transform, except that FFT employs a fast algorithm.

Thus, FFT treats the time series data as periodic. Therefore, if there is a large difference or discontinuity between the starting and the ending points, it is impossible to accommodate the frequencies in a band limited signal without introducing resonance. However, the discontinuities in the stress and strain signals do not exist in the original signal. They are created when the signals are approximated or discretized. Furthermore, this problem is likely to mask other frequencies of interest as the magnitude of the resonance (the value at zero frequency) is significantly large. Similarly, if the signal exhibits a slow variation in time, then the spectrum will be polluted by low frequency components. Therefore, it is necessary to remove trends in the signal to make the frequencies of interest more detectable. With this important note, the following describes the step-wise algorithm used to obtain rational ϕ for the raw data of a frequency sweep test:

- First, the trends in the stress and strain signals need to be removed using the method of least-squares to fit a straight line to the raw data of the mechanical test used in the study. Essentially, the straight lines obtained using least-squares model the linear trend in the data and are removed by subtracting them from the original raw data.
- Second, the resulting trend-free stress and strain signals are then transformed from time domain to frequency domain using

FFT and the CPS; concurrently, Y_{gh} of the two signals are computed. When transforming the data using FFT, it is recommended to transform the data for each cycle separately. This can be done using the cycle tag which is recorded by frequency–sweep testing equipment. This helps to easily identify the frequencies of interest.

- Finally, ϕ is calculated from the ratio of the real and imaginary parts of the CPS of the transformed data. Mathematically, ϕ is given by:

$$\phi = tan^{-1}\left(\frac{imag(Y_{gh})}{real(Y_{gh})}\right) \tag{4}$$

Likewise, the algorithm used to obtain E^* or G^* using FFT approach is described as follows:

- First, the modulus is determined from the maximum (normal or shear) stress and strain, or dynamic stress (σ) and dynamic strain (ε), whichever is applicable. σ and ε are the amplitudes of stress and strain signals, and are calculated from the magnitudes of the transformed data using FFT.
- Second, the result of the FFT transformation is double sided spectrum; therefore, it is necessary to convert the FFT output into a single sided amplitude spectrum by multiplying the FFT magnitude by two and discarding the other half of the results. This is mainly done as FFT results are symmetric about the zero frequency amplitude. Moreover, a scaling by the sampling period T is needed to account for the effect of sampling. Therefore,

$$\sigma = 2*T*max(|\hat{g}(\zeta)|) \quad and \quad \varepsilon = 2*T*max(|\hat{h}(\zeta)|) \tag{5}$$

where:

$max(\hat{g}(\zeta))$ and $max(\hat{h}(\zeta))$ denote maximum values of the FFT stress and strain signals,

T is the sampling period.

- Finally, E* (or G*) is given by:

$$E^*(\,or\,G^*) = \frac{\sigma}{\varepsilon} \tag{6}$$

It is important to note that the FFT algorithm requires the number of data points N to be a power of 2 (2^m, for any positive integer m), and it is necessary to use a sampling rate ($1/\Delta t$) at least twice the highest frequency component in the signals in order to fully reconstruct the signal. The highest frequency that can be reconstructed from a given data with sampling frequency ($1/\Delta t$), referred to as Nyquist frequency (f_N), is given by:

$$f_N = \frac{1}{2\Delta t} \tag{7}$$

where: f_N is the Nyquist frequency, and Δt is the time increment in seconds.

The selection of N and sampling rate should not be a problem for the calculation of ϕ and/or E^* and G^* because the stress and strain data from frequency-sweep tests is usually collected at a very high sampling frequency. Furthermore, the data is nearly pure cyclic loading except for the small “noise”. Therefore, it is possible to fully reconstruct the stress and strain signals. The sampling rate for the data used in this study is 500 Hz which is more than the magnitude of the frequencies of interest. The whole process of obtaining ϕ or *moduli* using FFT approach is summarized in the flowchart as shown in Fig. 1.

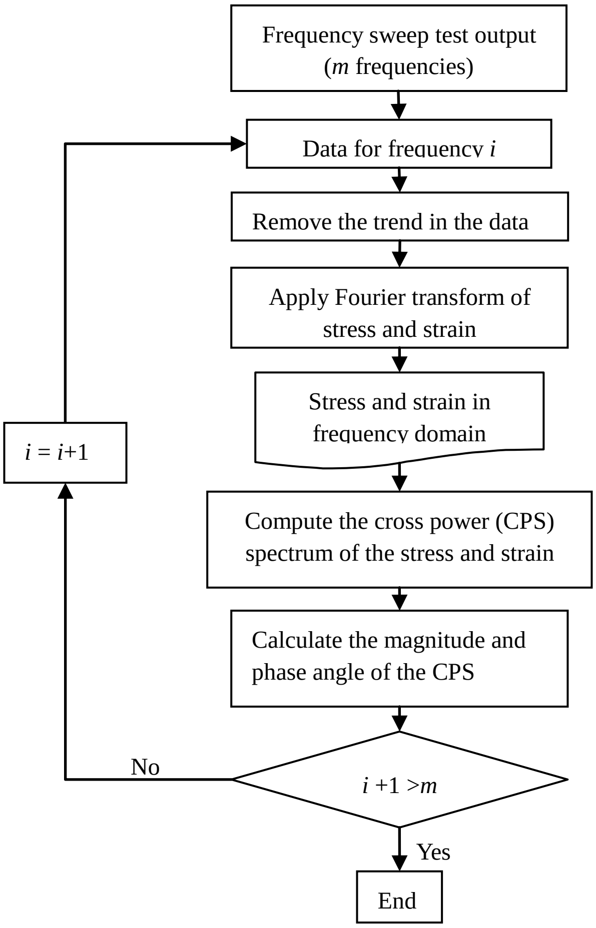


Fig. 1. Algorithm of Phase Angle Calculation Using Fast Fourier Transform (FFT) Technique.

The study scope of the effort included the following major tasks:

- Collect raw data from frequency sweep tests such as stiffness modulus, dynamic modulus and shear modulus tests for conventional dense graded (CONV), modified mixes such as open graded asphalt rubber friction course (ARFC), gap graded asphalt rubber (ARAC), fiber-reinforced asphalt concrete (FRAC), and polymer modified asphalt mixtures.
- Calculate the ϕ and moduli values of the different asphalt mixtures for the data obtained from the various mechanical tests using the Fast Fourier Transform (FFT) method developed in this study.
- Compare the ϕ and moduli results obtained using the FFT method and the already existing manual methods based on both theoretical and statistical analyses.
- Develop an algorithm to estimate the rational ϕ and moduli using the FFT methodology that basically provides a reasonably accurate measure of viscoelastic properties of the

asphalt mixtures. This algorithm must be based on fundamental mathematical functions and robust computational tools that is both user-friendly and rational.

Data Collection

Viscoelastic Parameters from Frequency Sweep Tests

Frequency sweep or sweep-frequency tests are used to quickly determine the broadband (range of frequencies) deformation or mechanical responses of asphalt mixtures that otherwise would require a number of separate tests for each frequency. Various frequency sweeps may be used to characterize asphalt mixtures' material responses during a mechanical test. Sweep-frequency techniques are applicable for dynamic modulus tests or shear modulus tests which are mainly utilized to characterize the permanent deformation and fatigue cracking performance behaviour of viscoelastic materials. These tests are usually performed at several test temperatures in such a way that each specimen is tested in an order of increasing test temperature, and for each temperature the specimens are tested in an order of decreasing test frequency. This temperature-frequency sequence is carried out to cause minimum damage to the specimen before the next sequential test. The outputs of frequency sweep tests are usually presented in the form of a master curve (for E^*, G^* and/or ϕ) which is a plot of the material property in question versus frequency (or time) at a standard temperature. Thus E^*, G^* and ϕ at any combination of test temperature and loading frequency can be obtained from the respective master curves.

The following subsections provide a brief documentation of the commonly used frequency sweep tests, which are currently utilized by the pavement community around the world to obtain viscoelastic properties of the different asphalt mixtures. The ensuing section on data collection provides the mixtures data used in this study that have been procured from the mentioned mechanical tests.

E Dynamic Modulus Test*

Frequency sweep E^* test was recommended as a simple performance test to complement the mixture design process under the National Cooperative Highway Research Program (NCHRP) Project 9-19 of the United States [10]. For linear viscoelastic materials such as asphalt mixtures, the stress–strain relationship under a continuous uniaxial sinusoidal loading is defined by a complex number called the complex modulus E^* [10, 11]. Fig. 2(a) presents an actual test setup for the E^* test. E^* tests are usually conducted on unconfined cylindrical specimens having a height to diameter ratio of 1.5 and uses a uniaxially applied sinusoidal load [12]. The E^* testing program involves testing at five different temperatures (-10, 4.4, 21.1, 37.8, and 54.4^{o}C) and six loading frequencies (25, 10, 5, 1, 0.5 and 0.1 Hz). The complex form of E^* is the ratio of the peak stress (σ_o) to peak recoverable strain (ε_o). The ratio of the energy lost ($E'' = E^* \sin \phi$) to the energy stored ($E' = E^* \cos \phi$) in a cyclic deformation is referred to as loss tangent, tan (ϕ). The phase angle, ϕ is simply the angle at which ε_o lags σ_o, and is an indicator of the viscous (or elastic) properties of the material under consideration. It is given by:

$$\phi = 360 \times f \times t_{lag} \tag{8}$$

where t_{lag} is the average time lag (in seconds) between stress and strain and f is the loading frequency (Hz). In general, for a purely elastic material, $\phi = 0^{o}$; and for a purely viscous material, $\phi = 90^{o}$. However, it is extremely rare to observe a value of ϕ closer to 90^{o} in the case of asphalt mixtures as there is a preponderant effect of an elastic response due to aggregates at high temperatures; nonetheless, the asphalt binders can have a ϕ very close to 90^{o} at higher temperatures.

Cyclic Stiffness Modulus Test (IDT)

The stiffness modulus test or the cyclic indirect tensile test is considered as a simple and cost effective non-destructive laboratory test method for measuring the stiffness modulus of bituminous mixtures [13, 14]. The test consists of applying a certain number of cyclic loading along the vertical plane of a specimen to achieve peak horizontal strain. The procedure is repeated by rotating the specimen through 90° and applying a second loading. The testing program includes testing at four different temperatures (-10, 0, 15, and 30^{o}C), and eight loading frequencies (20, 10, 5, 2, 1, 0.5, 0.2, and 0.1 Hz). Stiffness modulus / cyclic indirect tensile actual test setup is shown in Fig. 2(b). Similar to the E^* test, the outputs of the Cyclic IDT tests are E^* and ϕ.

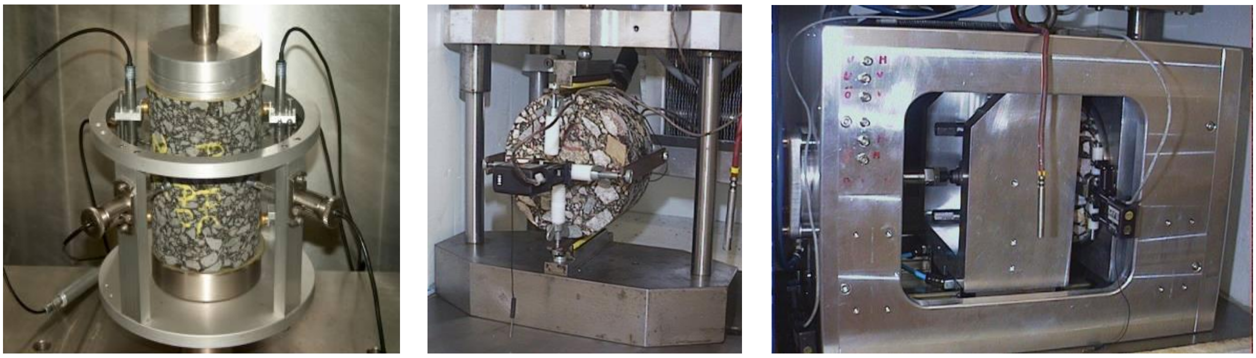

(a) (b) (c)

Fig. 2. (a) Actual ASU's E^* Test; (b) Actual VTI's Stiffness Modulus or Cyclic Indirect Tensile (IDT) Test (c) Actual VTI's Dynamic Shear Modulus Test Setup.

Table 1. Mixture Properties of Frequency Sweep Tests – Data Collection.

Mix	Mix ID	Agg. Grad. Type	Asphalt Content (%)	Target Air Voids (%)	Number of Replicates	Total Data Points
ASU – *E** Dynamic Modulus Test						
ASU-Conventional Dense Graded	CONV	Dense Graded	5.4	7.0	3	90
ASU-Asphalt Rubber Friction Course	ARFC	Open Graded	8.8	21.0	3	90
ASU-Fiber-Reinforced Asphalt	FRAC	Dense Graded	5.1	7.0	3	90
ASU-Asphalt Rubber Asphalt Concrete	ARAC	Gap Graded	6.7	2.0	3	90
VTI – *G** Shear Modulus Test						
Road Base mix	AG 100/150	Open Graded	4.5	5.5	2	80
Polymer-modified Road Base Mix	AG100/150 SBS	Open Graded	4.5	3.9	2	80
Binder Layer	ABb 50/70	Dense Graded	5.2	1.4	2	80
VTI – Stiffness Modulus or Cyclic IDT Test						
High Modulus Asphalt with 25% Reused Asphalt	EME 25%RA	Dense Graded	4.1	4.6	1	32
High Modulus Asphalt with 40% Reused Asphalt	EME 40%RA	Dense Graded	3.3	2.5	1	32
High Modulus Asphalt	EME	Dense Graded	5.5	2.1	1	32

G Dynamic Shear Modulus Test*

The *G** test is based on the method and the equipment developed at VTI-Swedish National Road and Transport Research Institute. In this method, the asphalt mix specimen is bound between two steel plates with epoxy glue. One of the plates can be exposed to sinusoidal or repetitive loading over a range of frequencies [15]. The test is performed on cylindrical samples of 150 mm in diameter and the thickness of the specimen is less than ¼ of the size of the diameter. Further information on the shear test is reported by Said [16]. The *G** testing procedure involves five temperatures: -7, 5, 20, 35 and 50^{o}C; and 8 loading frequencies: 16, 8, 4, 2, 1, 0.5, 0.1, and 0.05 Hz. Fig. 2(c) shows the shear box apparatus, which is used for dynamic shear modulus testing. The outputs of *G** test are *G** and ϕ.

Mixture Properties

Two different datasets were collected for analyses purposes. One dataset consisted of *E** test results of four different variants of asphalt mixtures from Arizona State University (ASU) asphalt pavement materials characterization database. The second dataset comprised of *G** and stiffness modulus (cyclic indirect tensile) test results from the VTI-Swedish National Road and Transport Research Institute routine pavement performance characterization database. Table 1 provides a summary of the mix types, mixture unique identification, aggregate gradation type, asphalt content, air voids levels, number of replicates per mix, and total number of data points collected for analytical purposes.

ASU Database Asphalt Mixtures

Four different variants of asphalt mixtures which were used for *E** testing (five temperatures and six frequencies) were collected that had different mixture properties such as aggregate gradation, air voids, asphalt content and crumb-rubber modification [17-20]. Three replicates per mix were used resulting in a total of 360 data points. Thus, 360 pairs of E*-ϕ values were estimated as part of the data analyses.

VTI Database Asphalt Mixtures

*G** test (five temperatures and eight frequencies) and cyclic IDT test (four temperatures and eight frequencies) data from VTI database were also collected in this study. Three different types of asphalt mixtures were used for each type of test. Two replicates per mix were used for *G** test, resulting in a total of 240 data points. For the IDT test, 1 replicate per mix was tested resulting in a total of 96 data points. Both tests were conducted at VTI. The results of these mixes were obtained from the ongoing projects and so, the test results are not yet published.

Data Analyses

Fast Fourier Transform Technique – Theoretical Analyses

Based on the FFT technique as illustrated previously, ϕ and *E** (or *G**) values were estimated using the raw data for all the mix types and the various mechanical tests in the database. Since it is difficult to show all the analyses and calculations in this paper, only typical calculations performed for one temperature for each mechanical test and at selected test frequencies are shown.

ϕ Estimations from Frequency Sweep Tests

Fig. 3 presents typical results of the estimated ϕ values from the raw data of the *E** test (conducted at ASU) at a selected test temperature of 21°C, and for one high and one low frequency (25 and 0.1 Hz, respectively). The calculated ϕ values for the *G** test conducted at VTI for the raw data tested at 20°C, and at 15 and 0.05 Hz are shown in Fig. 4. Likewise, Fig. 5 depicts results of the estimated ϕ values from the IDT test (also conducted at VTI) results at 15°C,

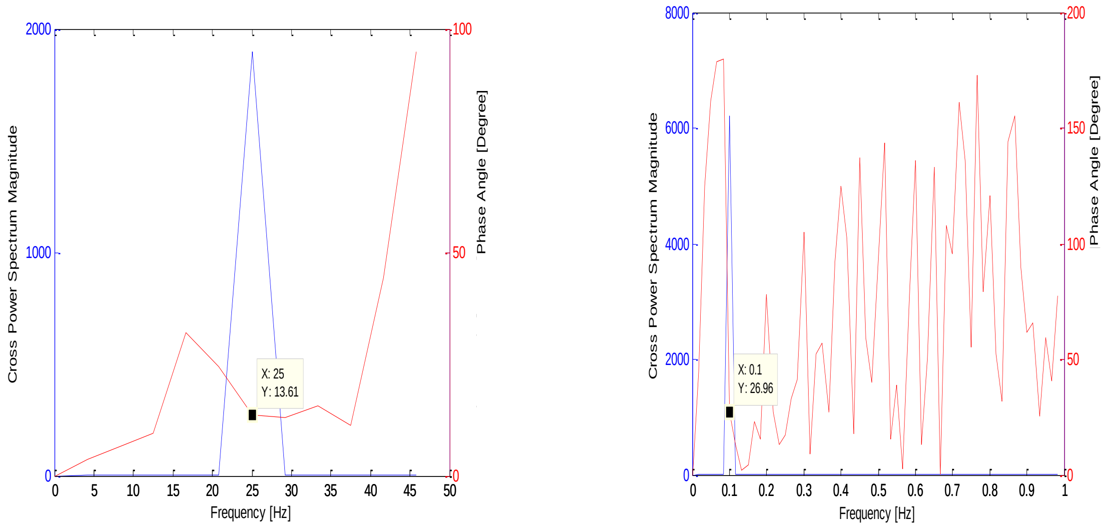


Fig. 3. FFT Technique – Phase Angle Estimation of ASU Conventional Asphalt Mixture Using *E** Test at 21°C, and at Frequencies of 25 Hz (left) and 0.1 Hz (right).

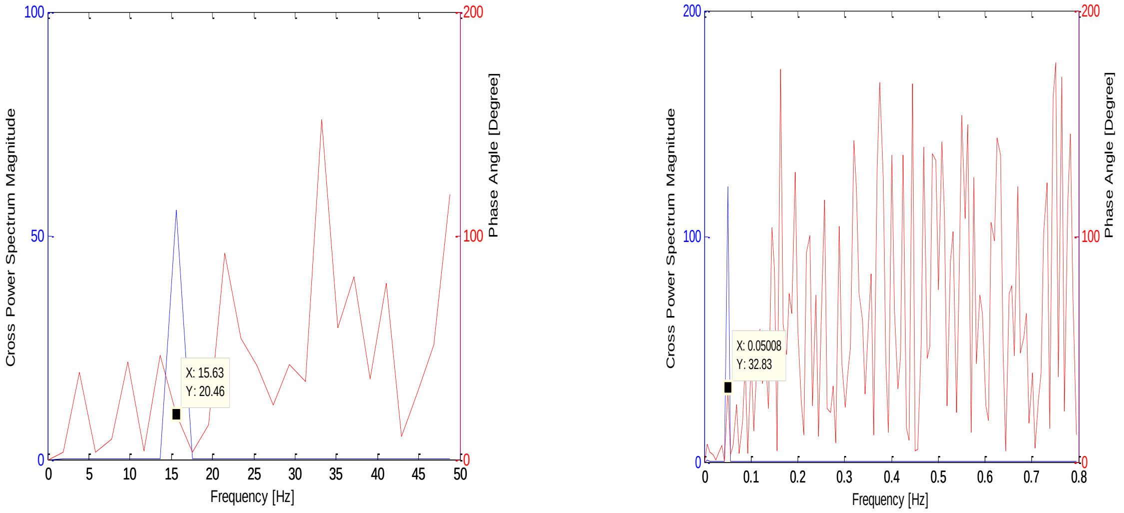


Fig. 4. FFT Technique – Phase Angle Estimation of VTI Conventional Asphalt Mixture Using *G** Test at 20°C, and at Frequencies of 15 Hz (Left) and 0.05 Hz (Right).

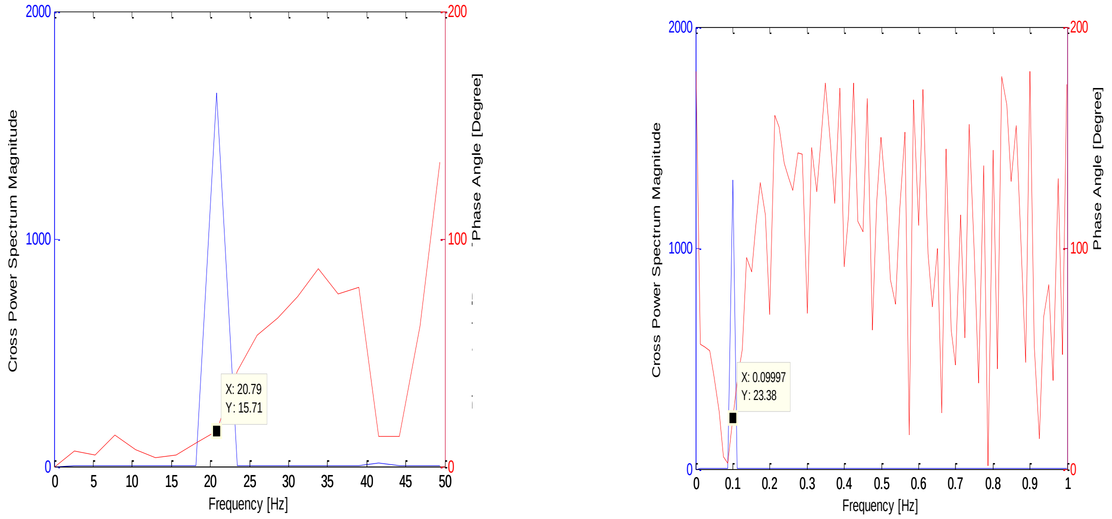


Fig. 5. FFT Technique – Phase Angle Estimation of VTI Conventional Asphalt Mixture Using Cyclic IDT Test at 15°C, and at Frequencies of 15.7 Hz (Left) and 0.1 Hz (Right).

and at 15.7 and 0.1 Hz. Note that the indicated figures show pictorial estimations of ϕ for only CONV mixtures, from ASU and VTI materials databases. The figures demonstrated that the FFT method used on the different types of frequency sweep test results enabled to identify the test frequencies as well as the corresponding estimations of the ϕ values. Each figure shows plots of the CPS value (primary y-axis; ordinate) shown in blue-colored lines as a function of frequency (abscissa); and ϕ (secondary y-axis; ordinate) shown in red-colored lines versus frequency. The frequencies corresponding to the maximum value of the cross correlation are the frequencies contained in the signal and the ϕ coordinate corresponding to these frequencies are reported as the test ϕ.

Rationality and Comparison of Different Analyses Methods

Generally, E^* and ϕ obtained from frequency sweep tests are computed by locating the peak values of stresses and strains using the raw data. However, the time delays between the peak stress and strain are used to estimate ϕ. The choice of the peak stresses and strains can be done manually. However, as mentioned previously, to avoid any sort of computation of erroneous results through the manual computations, it is better to use computer-based mathematical software tools to estimate E^* (or G^*) and ϕ.

The rationality and accuracy of the FFT method developed in this study was checked by means of a comparison made on the obtained ϕ estimations using the FFT method and the currently adopted analysis techniques, both at ASU and VTI. Currently, ASU is using polynomial regression fitting of the data to obtain ϕ (and E^*), which will be henceforth referred to as "POLY" for brevity in the paper; the details of this analysis technique is as follows. In the POLY method, the maximum and minimum points of the test data are used to fit the stresses and strains signals using polynomial functions over 25% of the raw data, both on the left and right side of the curves. The ϕ and E^* of the mixtures are then obtained from the fitted data.

On the other hand, currently VTI is in the process of selecting a rational method out of the two methods to analyze dynamic frequency sweep test data to estimate ϕ, which includes regression fitting technique and a method very similar to the FFT method [21]. Thus, the authors anticipate that this study would help assist the researchers at VTI (and other institutes) to choose the appropriate rational method to estimate ϕ and E^* (or G^*) in future analyses.

Apart from using FFT method, the E^* test outputs (from the ASU database) were also analyzed using the POLY analysis method. Concurrently, the data from VTI database (G^* test and IDT test) were analyzed using the currently used VTI method. The results from the FFT, POLY fitting and VTI analyses methods were evaluated and compared to understand the precision and bias of those methods.

Fig. 6(a), (b), (c) and (d) present a comparison of the E^* test ϕ values for CONV, ARFC, FRAC, ARAC, and different types of asphalt mixtures tested at ASU using FFT and POLY fitting methods. The number of data points used for each mix and the two analyses methods are also marked on each figure. From the figure, it is observed that the computed ϕ for all the mix types were influenced by the analysis methods. For all the mix types, the predicted ϕ from the POLY method were higher in comparison with the FFT method. At moderate and higher temperatures, the FFT analysis method resulted in lower values of ϕ than the POLY fitting method with range of values in the viscoelastic regime of 5-50^{o}.

The influence of the analysis method on the calculated ϕ was more significant for the ARFC mixes (Fig. 6(b)). A case of high bias and high precision was observed for the ARFC mix. It is clearly observed from Fig. 6(b) that there are certain ϕ values greater than 60^{o} as predicted by the POLY method, which are plausibly unreasonable estimates and a deviation of viscoelastic nature of asphalt mixtures. Since the POLY method employs curve fitting method where the data points are regressed using a polynomial curve, the method only picks the ϕ estimates that could be outliers of the chosen dataset. However, at very low temperatures there existed a very good agreement between the ϕ computed using the two methods. Magnitude-wise, differences of 10–40% between the FFT and POLY methods were observed for ϕ of CONV mixes at low to high temperatures. Furthermore, for intermediate to higher temperatures, differences of 10–50% for ARFC, 10–20% for FRAC, and 10% for ARAC mixes were observed for the ϕ between FFT and POLY methods.

The ϕ calculated and compared using FFT and the VTI methods from G^* and cyclic IDT tests are shown in Fig. 7(a) and (b), respectively. As observed in Fig. 7(a), for the G^* test results conducted at VTI, the estimations of ϕ obtained from both FFT and VTI methods have no significant difference for all test temperatures and frequencies. This is mainly because the current VTI method uses a similar analogy to that of the FFT analysis technique, but with a difference in manual application of sinusoidal fitting of the stable part of the raw data and the technique for trend removal.

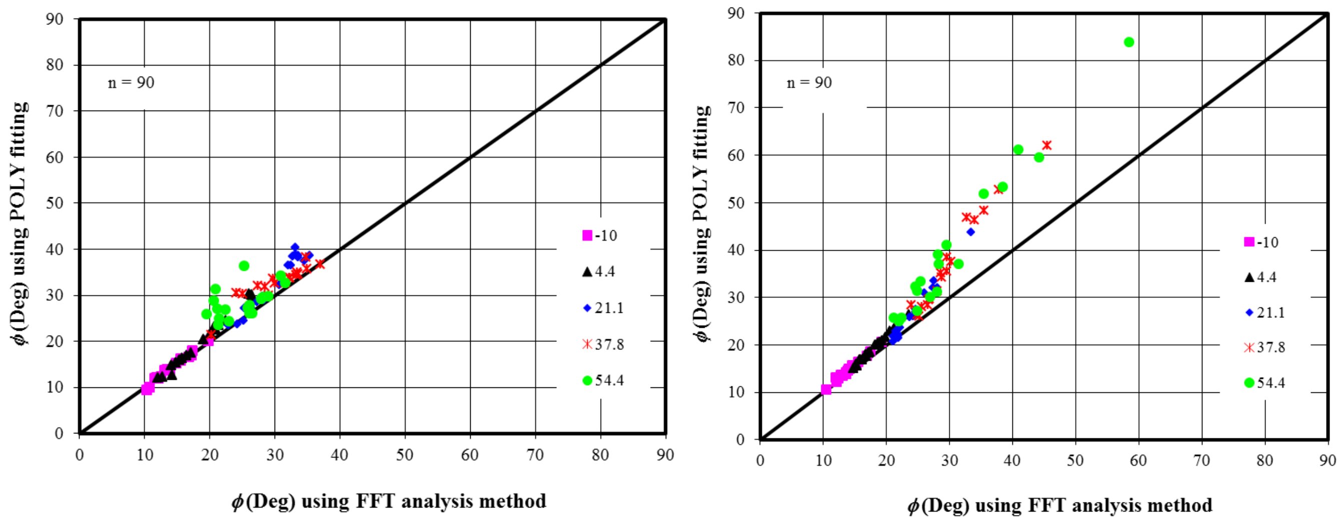
Furthermore, for the VTI's cyclic IDT test estimations of ϕ comparisons between FFT and VTI method (Fig. 7(b)), a similar observation was made as shown in Fig. 6 in that the estimated values by the VTI method were higher than the values from FFT with a significant difference of around 13-15%, specifically at intermediate to higher test temperatures.

As mentioned before, the FFT technique could also be used simultaneously to estimate E^* or G^*. Raw data for the different asphalt mixtures were used to calculate the moduli values simultaneously with ϕ to investigate if there was a difference in magnitude of moduli between the different analyses methods. Fig. 8 shows comparison of the E^* calculated using FFT and POLY fitting method from the outputs of ASU's E^* test. As observed, the influence of the analysis method on the calculated E^* from the outputs of the predictions was insignificant. The same comparison patterns were obtained for E^* calculated from the outputs of IDT test, and for the G^* calculated from the G^* test. A difference in moduli of 1–8% for CONV, 1–7% for ARFC, 1–6% for FRAC and ARAC mixes were observed between FFT and POLY methods of analyses.

It is noteworthy that for all the comparisons shown in Figs. 6 through 8, a case of high bias and high precision was observed.

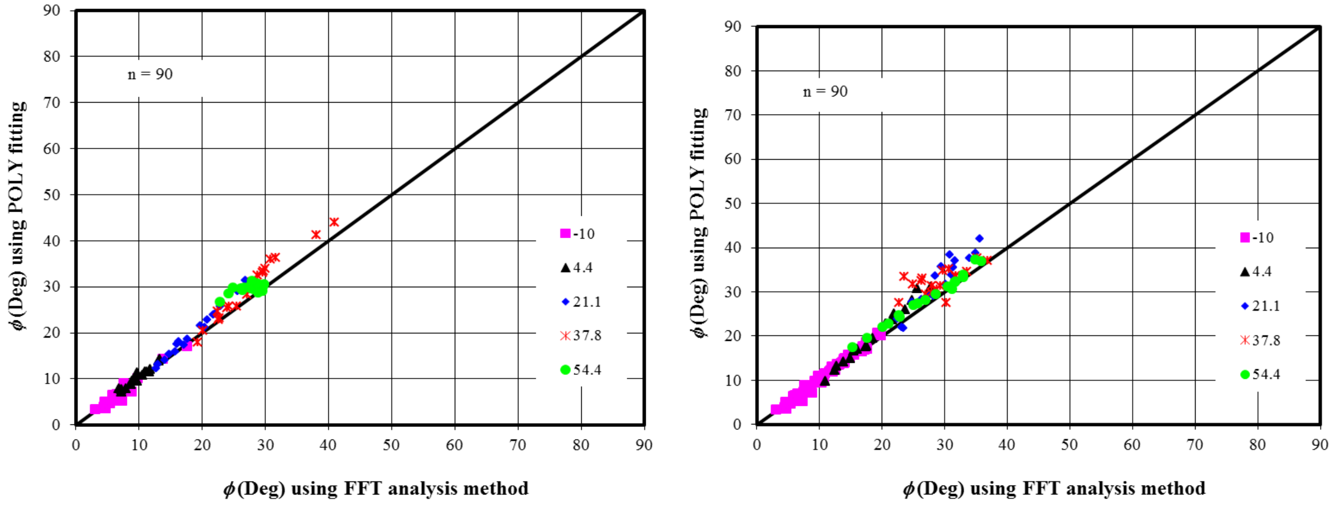
Statistical Analyses

To further understand the difference between the various analysis methods, statistical analyses using Analysis of Variance (ANOVA)



(a) CONV

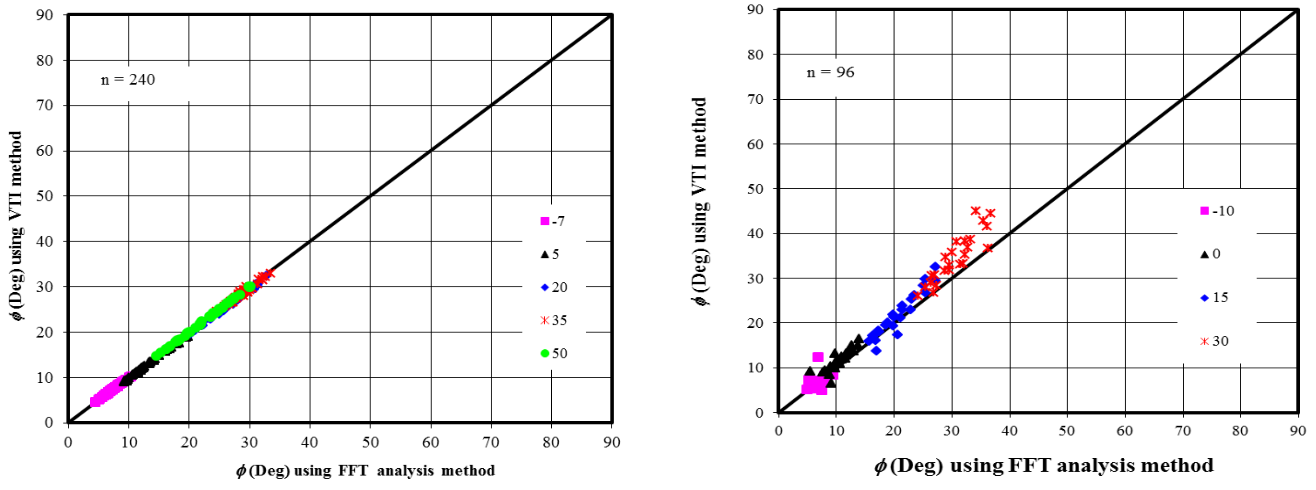
(b) ARFC



(c) FRAC

(d) ARAC

Fig. 6. Comparison of the Estimated ASU *E** Test Phase Angle (Degrees) from FFT and POLY Fitting Analysis Methods for (a) Conventional Dense Graded (CONV); (b) Asphalt Rubber Friction Course (ARFC); (c) Fiber Reinforced Asphalt Concrete (FRAC); and (d) Asphalt Rubber Asphalt Concrete (ARAC).



(a)

(b)

Fig. 7. Comparison of the Estimated VTI Phase Angle (Degrees) from FFT and POLY Fitting Analysis Methods for the Three Different Asphalt Mixtures Combined, Using (a) *G** Test; (b) IDT Test.

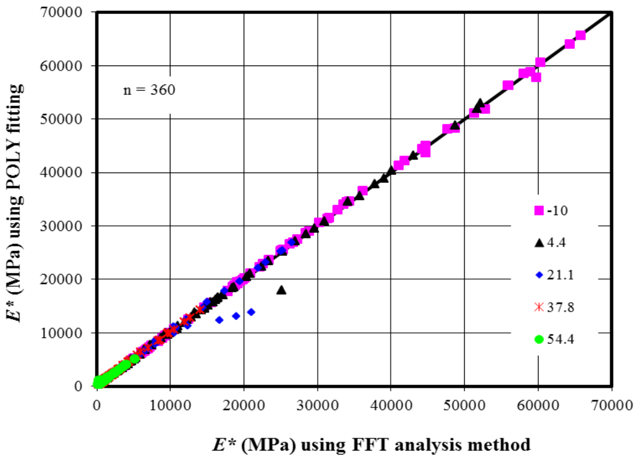


Fig. 8. Comparison of the Estimated ASU Dynamic Modulus Test *E** (MPa) from FFT and POLY Fitting Analysis Methods for Four Different Asphalt Mixtures Combined.

was performed at a confidence interval of 95% (or risk level of α = 5%) assuming a null hypothesis that the analyses are not significantly different between each other for estimating ϕ and moduli. Table 2 presents the ANOVA results for ϕ computed using the different methods for the three mechanical tests for all the asphalt mixtures used at both ASU and VTI. Results of the ANOVA for calculated dynamic or shear moduli are also indicated in Table 2.

The ANOVA results indicated that the two analysis methods were significantly different for estimating ϕ from both the *E** test and the IDT test. As noted in Table 2, for *G** test, the calculated ϕ were not significantly affected by the analysis technique. This is due to the fact that the VTI method employs a similar analogy to that of the FFT method. Admittedly, the estimated moduli obtained from the three test methods were not significantly affected by the different data analysis procedures as shown in Table 2.

Statistical analyses confirmed the theoretical estimations (observations) in that the comparison of the FFT method with the POLY technique of the different mixtures indicated a significant difference of the ϕ values obtained from the two analysis techniques at a confidence interval of 95%. Previous research studies [8] also had a similar finding regarding the difference in magnitudes of the ϕ results taken from different mechanical tests. This might be attributed to the fact that the ϕ values are acutely sensitive numbers unlike the moduli thus making the ϕ values' estimations influenced specifically by the analysis technique. However, the estimations of the dynamic or shear modulus was not significantly affected by the analysis technique type, both stochastically and statistically.

More so, the discrepancies in the ϕ of the two methods investigated in this study may be attributed to the inherent differences in the mathematical approaches adopted. Essentially, the POLY curve-fitting approach applies fitting a curve locally, *i.e.*, for each pulse of stress and strain signals or considers the last few cycles of the periodic loading; while, the FFT method fits the whole signal in a comprehensive sense, similar to the so-called "spectral methods" [9].

Conclusions

The main objective of this study was to develop a robust computational algorithm that can estimate rational ϕ of the ten different asphalt mixtures totaling 690 data points obtained from mechanical frequency sweep tests such as *E**, *G**, and cyclic IDT. In this direction, a simple yet powerful user-friendly method and algorithm was successfully developed using FFT techniques. Based on the FFT algorithm and the associated estimates, the following conclusions are made:

- ϕ *Estimations*: Theoretical observations revealed that there were significant differences for ϕ between FFT and other methods to the order of 10-50% particularly at intermediate to high temperatures with FFT methods predicting very reasonable estimates encompassing viscoelastic range of values (5-50^o). Statistical analyses using ANOVA corroborated the predicted estimates indicating significant differences between the two analytical methods in estimating ϕ.
- *Moduli Estimations*: Both the theoretical and statistical analyses were indicative of insignificant differences (1-8%) of the estimated moduli (*E** or *G**) between the FFT methodology and the other analytical techniques.
- Overall, it was successfully demonstrated that FFT analysis is scientifically a robust tool in estimating rational ϕ since the algorithm uses sound mathematical functions. Also, the analytical method is a user-friendly kit that can substantially obviate errors in the estimation of the acutely sensitive ϕ viscoelastic parameter.

Table 2. Summary of ANOVA Results for Phase Angle and Moduli between FFT and POLY Methods of Analyses.

Statistics	*E**-Test		*G**-Test		IDT-Test	
	ϕ (°)	*E** (MPa)	ϕ (°)	*G** (MPa)	ϕ (°)	*E** (MPa)
Sum of squares	9.508E+02	1.779E+06	6.750E+00	5.574E+01	1.208E+02	1.365E+03
Degree of Freedom	1	1	1	1	1	1
Mean Square Error	28.950	6.251E+07	8.570	79550.7	4.470	711347.2
Level of Confidence, %	95	95	95	95	95	95
F	32.85	0.03	0.79	0.0007	27.02	0.001919
$F_{crit(\alpha/2,\ \nu)}$	3.856	3.856	3.862	3.862	3.894	3.894
P-value	1.531E-08	0.8661	0.3752	0.9789	5.481E-07	0.9651
Statistical Difference	SIGNIFICANT	IN-SIGNIFICANT	IN-SIGNIFICANT	IN-SIGNIFICANT	SIGNIFICANT	IN-SIGNIFICANT

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