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Support Vector Machine Models for Prediction of Flow Number of Asphalt Mixtures

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Abstract: Permanent deformation is one of the most critical distress types affecting the serviceability of flexible pavement structures. Predicting the rutting potential of asphalt concrete is a complicated task. Flow number (F_n) of asphalt mixture is an explanatory index for the evaluation of rutting. This paper examines the potential of the Support Vector Machine (SVM) to predict the flow number of dense asphalt-aggregate mixtures. This SVM is firmly based on the statistical learning theory and uses the regression technique by introducing accuracy (ε) intensive loss function. The results are compared with those from gene expression programming (GEP) and multiple-least-squares-regression (MLSR). Overall, SVM shows good performance and proves to be better than the GEP model and MLSR model. A sensitivity analysis is also performed to investigate the importance of the input parameters. The study shows that the SVM has the potential to be a useful and practical tool for prediction of flow number of asphalt mixtures.

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***Key words**: Asphalt mixtures; Flow number; Sensitivity; Rutting; Support vector machine.*

Introduction

Pavement deformation or rutting is a common distress in asphalt pavement. The result of rutting is a deflection in the pavement surface at the wheel path. It is generated due to traffic loading through the service life of pavement. Rutting failure leads to poor serviceability of pavement, making a vehicle's ride rough and unsafe. Permanent deformation usually occurs in hot climates and under slow moving traffic conditions of heavy truck loads. Properties of asphalt and aggregate and volumetric portion in asphalt mixtures affect rutting resistance of asphalt mixtures. Evaluation of the rutting potential of asphalt mixtures has been the focus of much research in pavement engineering over the last decade. The majority of the available permanent deformation models are empirical or semi-mechanistic models with limited fundamental material characterization. Some of the empirical models are derived from limited sets of materials and environmental conditions. They lack robustness and are not transferable to other conditions [1]. Thus, prediction of rutting potential of asphalt concrete has been a complicated task.

It is important to identify practical laboratory test methods to predict the rutting of asphalt mixtures. With the evaluation of mixture designs from conventional Marshall mixture design to the Superpave design, researchers have sought for a simple yet reliable testing procedure to assess rutting potential of asphalt mixtures for more than a decade. The dynamic creep test is found to be one of the best methods for assessing the permanent deformation potential of asphalt mixtures [2]. The time to tertiary flow failure is thought to be a good indicator of the permanent deformation resistance of asphalt mixtures [1, 3]. It can be quantified by the flow number (F_n), as measured in a repeated load permanent deformation test. Witczak et al. defined the flow number as loading-cycle number where tertiary deformation starts [4]. Flow number attempts to identify the resistance of a mixture to permanent deformation by measuring the shear deformation that occurs because of dynamic loading [5]. Previous research has shown reasonable correspondence of the permanent strain and the flow number with rutting. Besides, regarding the emphasis on permanent strain, the experts generally agree on the flow number as the best indicator of the rutting potential of the asphalt mixtures [6]. The dynamic creep pretesting procedures are more complicated, time-consuming, and cost-consuming. Furthermore, experimental errors are inevitable.

Therefore, it is necessary to develop a relationship between the flow number obtained from the test and parameters from the Marshall mix design. The soft computing approaches such as artificial neural network (ANN), genetic expression programming (GEP), and Support Vector Machine (SVM) have recently emerged as promising approaches. These modeling techniques are becoming increasingly important tools in engineering areas as a result of rapid development of information and computer technology [7-10]. Pattern recognition, classification, design of structure, and modeling of material behavior are primary topics in which soft computing approaches are employed [11-15]. Recently, Mirzahosseini applied multi-expression programming (MEP) and multilayer perceptron (MLP) of artificial neural networks to evaluate the rutting potential of dense asphalt-aggregate mixtures [16]. Gandomi et al. developed models to predict the flow number of asphalt mixture using gene expression programming [15]. Also, Alavi et al. utilized GP/SA (combined Genetic programming and Simulated annealing) technique to evaluate flow number of asphalt mixture [1].

The SVM is another efficient machine learning technique derived from statistical learning theory by Vapnik [17]. Some recent SVM applications in the pavement engineering domain include pavement backanalysis [18] and pavement management [19]. This paper is the first one to explore the feasibility of the SVM application to evaluate the flow number of asphalt mixtures. The theory and

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procedure of SVM are briefly reviewed. The SVM model is developed relating the flow number to the particle size distribution of natural soil, air voids, and voids in mineral aggregate, Marshall stability and flow. The data set for training and testing are obtained from the work of Gandomi [15]. The feasibility of the SVM model for predicting the flow number of asphalt mixtures is investigated and the performance of predictive model are discussed. A sensitivity analysis has also been performed to investigate the importance of the input parameters.

Support Vector Machine Regression

SVM is based on the structural risk minimization (SRM) principle. The objective is to find a hyperplane which best separates the positive/negative data in the feature space. Assume that the training dataset $(x_1,y_1),(x_2,y_2),\cdots(x_l,y_l),\ x_i\in X\subseteq R^n, y_i\in Y\subseteq R$, l is the total number of samples. A primal space is transformed into a high-dimensional feature space by a nonlinear map $\phi(x)=(\phi_1(x),\phi_2(x)\cdots\phi_n(x))$. Approximating the dataset with a nonlinear function:

$$f(x)=\omega^T\phi(x)+b \tag{1}$$

The coefficients ω and b can be obtained by minimizing the regularized risk function as following:

$$\begin{cases}\min\left(\frac{1}{2}\|\omega\|^2+CR_e\right)\\ R_e=\frac{1}{l}\sum_{i=1}^{l}L(y_i,f(x_i))\end{cases} \tag{2}$$

where C is referred as the regularization constant. The first term $\|\omega\|^2$ is regularization term, i.e. confidence interval, which controls the function capacity. The second term R_e is the empirical error measured by the loss function. The initial choice for loss function $L(y_i,f(x_i))$ is the ε –insensitive loss function:

$$L(y_i,f(x_i))=|y_i-f(x_i)|_\varepsilon \tag{3}$$

$$|y_i-f(x_i)|_\varepsilon=\begin{cases}|y_i-f(x_i)|-\varepsilon & |y_i-f(x_i)|\geq\varepsilon\\ 0 & otherwise\end{cases} \tag{4}$$

where ε is called the tube size.

Optimization problem Eq. (2) can be further transformed to the following primal objective function:

$$\begin{cases}\min\left(\frac{1}{2}\|\omega\|^2+C\sum_{i=1}^{l}\left(\xi_i+\xi_i^*\right)\right)\\ s.t.\begin{cases}y_i-\omega^T\phi(x_i)-b\leq\varepsilon+\xi_i\\ \omega^T\phi(x_i)+b-y_i\leq\varepsilon+\xi_i^*\\ \xi_i,\xi_i^*\geq 0\end{cases}\end{cases} \tag{5}$$

where ξ_i,ξ_i^* are positive slack variables. The constant $0<C<\infty$ determines the trade-off between the flatness and the amount up to which deviations larger than ε are tolerated.

The Lagrange function is constructed from both objective function and corresponding constraints in Eq. (4) as follows:

$$\begin{aligned}L=&\frac{1}{2}\|\omega\|^2-\sum_{i=1}^{l}\alpha_i\left[\varepsilon+\xi_i-y_i+\omega^T\phi(x_i)+b\right]\\ &-\sum_{i=1}^{l}\alpha^*_i\left[\varepsilon+\xi_i^*+y_i-\omega^T\phi(x_i)-b\right]\\ &+C\sum_{i=1}^{l}\left(\xi_i+\xi_i^*\right)-\sum_{i=1}^{l}\left(\eta_i\xi_i+\eta_i\xi_i^*\right)\end{aligned} \tag{6}$$

This Lagrange function has a saddle point with respect to the primal and the dual variables at the optimal solution. The dual variables in Eq. (6) satisfy positive, i.e. $\alpha_i,\alpha_i^*,\eta_i,\eta_i^*\geq 0$. The partial derivatives of L with respect to $\omega, b, \xi_i, \xi_i^*$ have to vanish to satisfy the saddle point condition:

$$\begin{cases}\frac{\partial L}{\partial\omega}=\omega-\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)\phi(x_i)=0\\ \frac{\partial L}{\partial b}=\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)=0\\ \frac{\partial L}{\partial\xi_i}=\sum_{i=1}^{l}(C-\alpha_i-\eta_i)=0\\ \frac{\partial L}{\partial\xi_i^*}=\sum_{i=1}^{l}(C-\alpha_i-\eta_i^*)=0\end{cases} \tag{7}$$

Substituting Eq. (7) into Eq. (6), and a kernel function introducing to reduce computational demand, the optimization problem can be written as:

$$\begin{cases}\max\begin{cases}-\frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)K(x_i,x_j)-\\ \varepsilon\sum_{i=1}^{l}(\alpha_i+\alpha_i^*)+\sum_{i=1}^{l}y_i(\alpha_i-\alpha_i^*)\end{cases}\\ s.t.\begin{cases}\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)=0\\ 0\leq\alpha_i\quad\alpha_i^*\leq C\end{cases}\end{cases} \tag{8}$$

where $K(x_i,x_j)=\phi^2(x_i)\phi(x_j)$ is called kernel function. The introduction of kernels according to Mercer's theorem avoids an explicit formation of the nonlinear mapping, makes the dimension of feature space high or even infinite, and reduces the computational load greatly by enabling the operation in low dimensional input space instead of high dimensional feature space [17]. Various kernels can be used as follow:

(1) Linear kernel (LIN) is shown as

$$K(x,x_i)=x^Tx_i \tag{9}$$

(2) Radial basis function (RBF) kernel is shown as

$$K(x,x_i)=\exp(-\gamma\|x-x_i\|^2) \tag{10}$$

Table 1. General Performance of SVM for Different Models.

	SVM-Ⅰ		SVM-Ⅱ		SVM-Ⅲ	
Kernel	RBF	Polynomial	RBF	Polynomial	RBF	Polynomial
C	200000	800000	900000	900000	500000	900000
ε	0.10	0.10	0.16	0.08	0.20	0.15
Parameter	$\gamma = 3.6$	$d = 5$	$\gamma = 0.01$	$d = 4$	$\gamma = 0.09$	$d = 4$
Training Performance	0.9679	0.9560	0.9348	0.9436	0.9112	0.9252
Testing Performance	0.9651	0.9578	0.9591	0.9520	0.9522	0.9395

(3) Polynomial kernel (POL) is shown as

$$K(x, x_i) = \left[(x^T x_i) + 1\right]^d \quad d = 1, 2, \cdots, N \tag{11}$$

where γ, d are the Kernel parameters.

By solving quadratic program Eq. (8), regression function Eq. (1) is rewritten as:

$$f(x) = \sum_{i=1}^{l} \left(\alpha_i - \alpha_i^*\right) K(x_i, x) + b \tag{12}$$

where α_i , α_i^* satisfy $\alpha_i \alpha_i^* = 0$, $\alpha_i \geq 0$ and $\alpha_i^* \geq 0$.

Only a number of coefficients $\alpha_i - \alpha_i^*$ are nonzero values, and the corresponding training data points have approximation errors equal to or larger than ε . They are called support vectors.

Analysis of Support Vector Machine

Flow number is influenced by several factors. In order to provide accurate assessment of the flow number, the effects of several influencing factors should be incorporated into the model developed. Based on the results of previous research, the dynamic creep test was chosen as an appropriate laboratory method to investigate the rutting potential of dense bituminous mixtures [1, 2]. Several uniaxial dynamic-creep tests were carried out utilizing UTM-5 to develop the database by Gandomi et al. [15]. The detailed description about the test process and the statistics of variables can be found in Gandomi et al. [15].

The main objective of this study is to implement the SVM methodology to estimate the flow number of asphalt mixtures. The database includes the measurements of coarse aggregate (*C*), fine aggregate (*S*), filler (*FP*), air voids (V_a), voids in mineral aggregate (*VMA*), bitumen (*BP*), Marshall stability (*M*), Marshall flow (*F*), and flow number (F_n). They were selected based on some previously suggested values [1, 15]. C/S, VA (%), VMA (%), and M/F are considered as the input parameters based on the analysis of factors affecting rutting and after an extensive literature review. In this study three SVM models (SVM-Ⅰ, SVM-Ⅱ, and SVM-Ⅲ) are developed. In the SVM-Ⅰ model, four input parameters are preferred: C/S, Va, VMA, and M/F. In the SVM-Ⅱ model, three input parameters are preferred such as C/S, VMA, and M/F. In SVM-Ⅲ model, two input parameters are used: C/S and VMA.

In performing the formulation, the data were randomly divided into training and testing subsets. Out of 118 data, 89 data (about 75%) were used as the training data and the remaining 29 data (about 25%) were considered as testing data set. The data were scaled between 0 and 1 before being used in the model. In the case of SVM training, two types of kernel functions were used, namely radial basis function and polynomial function. The parameters of *c*, *ε*, and other kernel-specific parameters were chosen by a trial-and-error approach. In this study, training, testing, and sensitivity analysis of SVM were carried out using the SVM tool-box in MATLAB. The best simulation performances of SVM are summarized in Table 1.

Sensitivity analysis is of utmost concern for selecting the important input variables. The contribution of each predictor variable to the prediction of the flow number is evaluated by sensitivity analysis. For this purpose, the cause-and-effect relationship between the inputs and the outputs of the SVM model were obtained. The main idea is that each input parameter of the model is offset slightly, and the corresponding change in the output parameter is reported. The sensitivity (*S*) of each input parameter can be calculated by the following formula [20]:

$$S\% = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\%change\ in\ ouput}{\%change\ in\ input} \right)_i \times 100\% \tag{13}$$

where *N* is the number of data points. The analysis is carried out on the trained model by varying each input parameter, one at a time, at a constant rate of 20% [19].

Results and Discussion

This paper investigates the potential of the SVM model in forecasting the flow number of asphalt mixtures, which has great significance for pavement engineering. For quantitative assessments of the model's predictive abilities, the results obtained from these approaches are comprehensively evaluated in statistics. In order to learn the performance of the developed models, several statistical verification criteria are used, such as correlation coefficient (*R*), root mean squared error (RMSE), and mean absolute error (MAE).

In statistics, the overall error performances of the relationship between predicted and experimental values can be interpreted from the R value. If R value of a relationship between predicted and experimental values is greater 0.8, this correlation is considered satisfactory according to statistics [22].

Figs. 1-6 show the results of the SVM model for the training dataset using polynomial function and radial basis function, respectively, and Figs. 7-12 show the performances of the SVM model for the testing dataset using polynomial function and radial basis function. The best results in terms of the R value are obtained as 0.956 and 0.968 for the SVM-Ⅰ model. However, the SVM-Ⅲ model gives a fairly high R value of 0.91 and 0.95, respectively. It

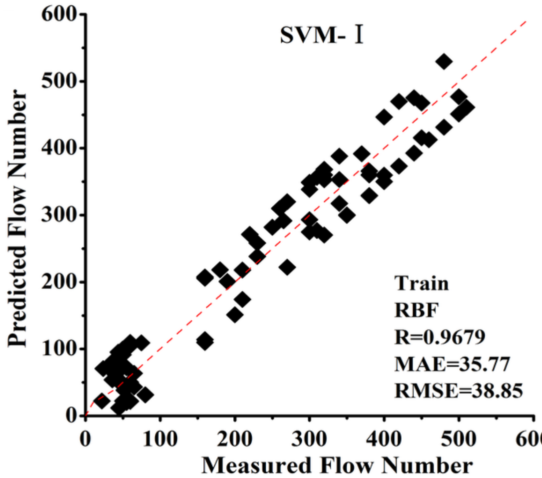

Fig. 1. Performance of SVM-I_{RBF} for Training Dataset.

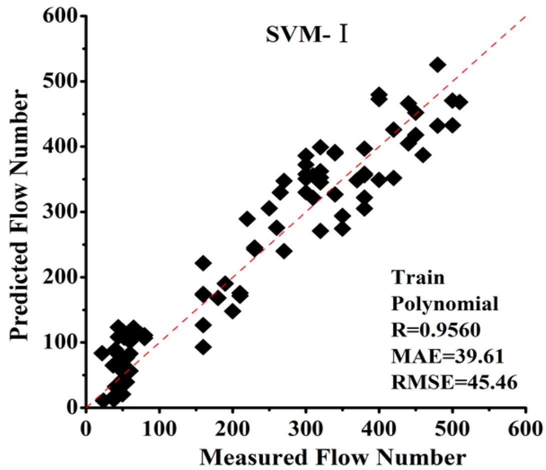

Fig.2. Performance of SVM-I_{POL} for Training Dataset.

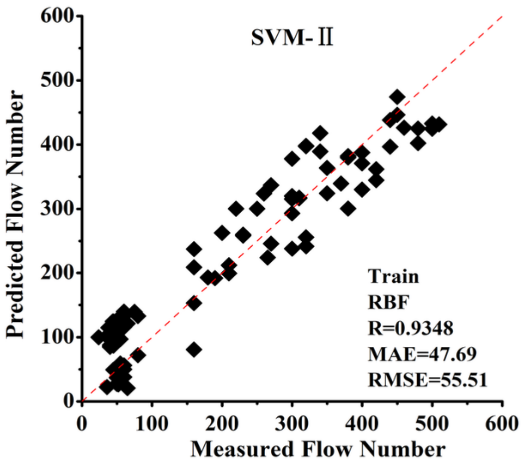

Fig. 3. Performance of SVM-II_{RBF} Fortraining Dataset.

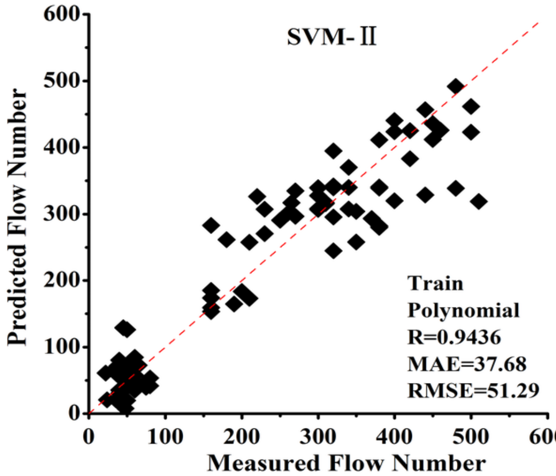

Fig. 4. Performance of SVM-II_{POL} for Training Dataset.

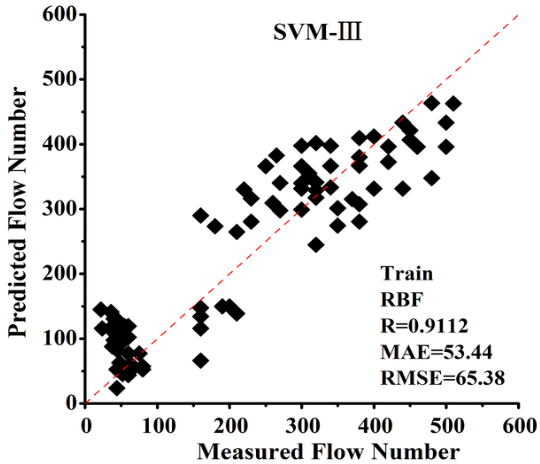

Fig. 5. Performance of SVM-III_{RBF} for Training Dataset.

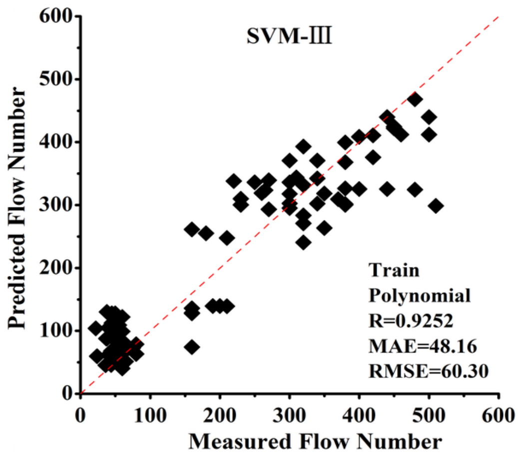

Fig. 6. Performance of SVM-III_{POL} for Training Dataset.

clearly appears that the results from the SVM models are in good agreement with the experimental values. For convenient comparison, the experimental and predicted results are plotted in Fig. 13, which shows that SVM has good ability to predict the flow number of asphalt mixtures. This also shows that all models are capable of learning the complex relationship.

A comparative study has been made between the developed SVM model and other models (GEP model and MLSR-based model) proposed by Gandomi for the testing dataset [15]. Figs. 14-15 are bar graphs comparing the MAE values and the RMSE values of GEP model, MLSR-based model and the SVM model, respectively. From these figures, it is clear that there is no major difference in performance between the polynomial and radial basis function kernels. However, in general, the radial basis function kernel exhibits slightly better performance than the polynomial kernel. When comparing the performance of the proposed models, the

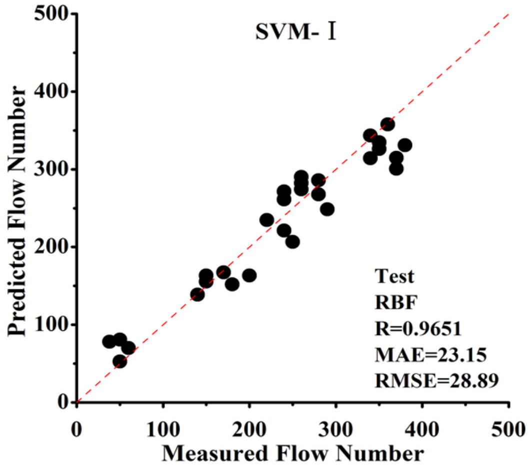

Fig. 7. Performance of SVM-I_{RBF} for Testing Dataset.

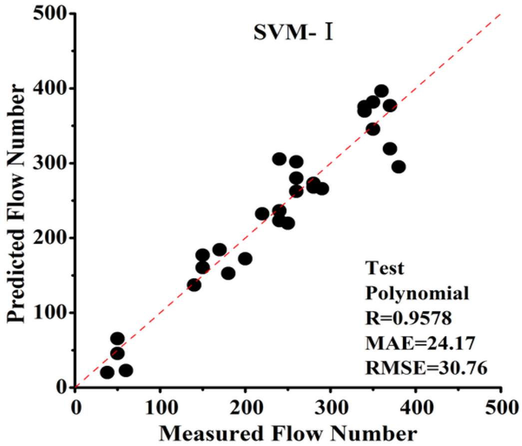

Fig. 8. Performance of SVM-I_{POL} for Testing Dataset.

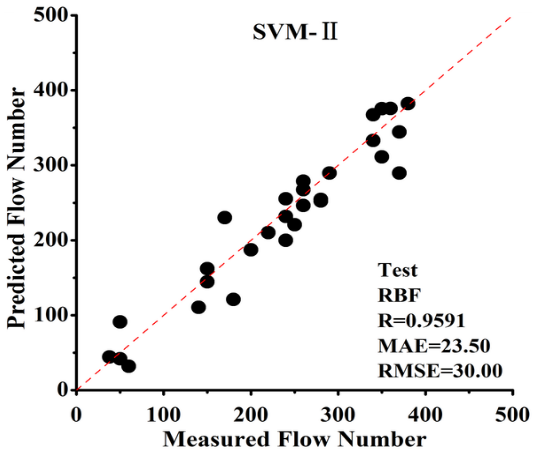

Fig. 9. Performance of SVM-II_{RBF} for Testing Dataset.

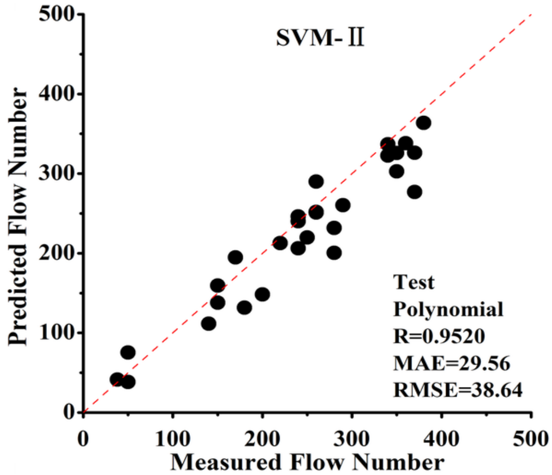

Fig. 10. Performance of SVM-II_{POL} for Testing Dataset.

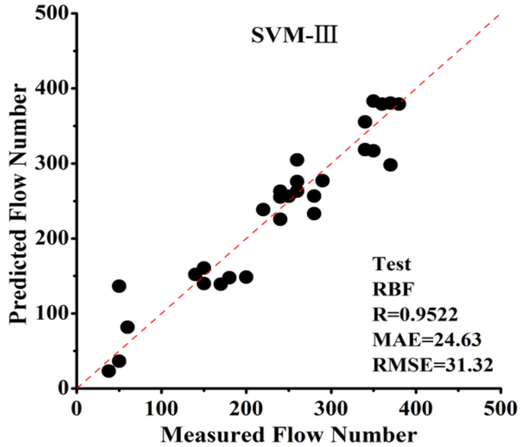

Fig. 11. Performance of SVM-III_{RBF} for Testing Dataset

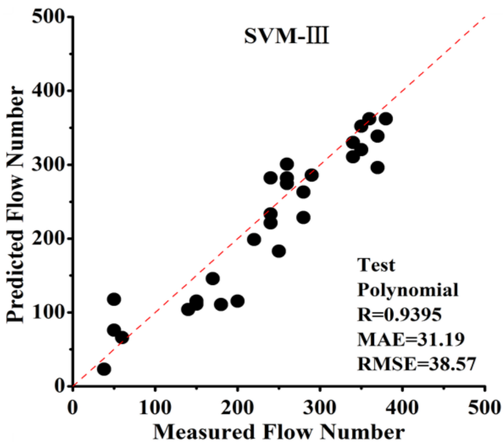

Fig. 12. Performance of SVM-III_{POL} for Testing Dataset.

SVM- Ⅰ model has produced the best results in predicting the flow number of asphalt mixtures. Also, the SVM- Ⅱ model, which has accounted for the effects of C/S, VMA, and M/F, has better performance than the SVM-Ⅲ model developed using C/S and VMA. Overall, the proposed models using more variables as inputs outperform those developed with fewer input variables.

The results also clearly show that the SVM model's performance is superior to the GEP model and MLSR model developed with the same parameters as input. Furthermore, the SVM models provide other significant advantages in addition to their good performance. SVM has the ability to avoid overtraining; hence, it has good generalization capability. Notwithstanding this, SVM formulation does not try to fit data. Instead, it tries to capture underlying functions from which the data are generated irrespective of the presence of noise. For SVM, this insensitivity to noise in the data is attributed to the ε-insensitive loss function in the model formulation.

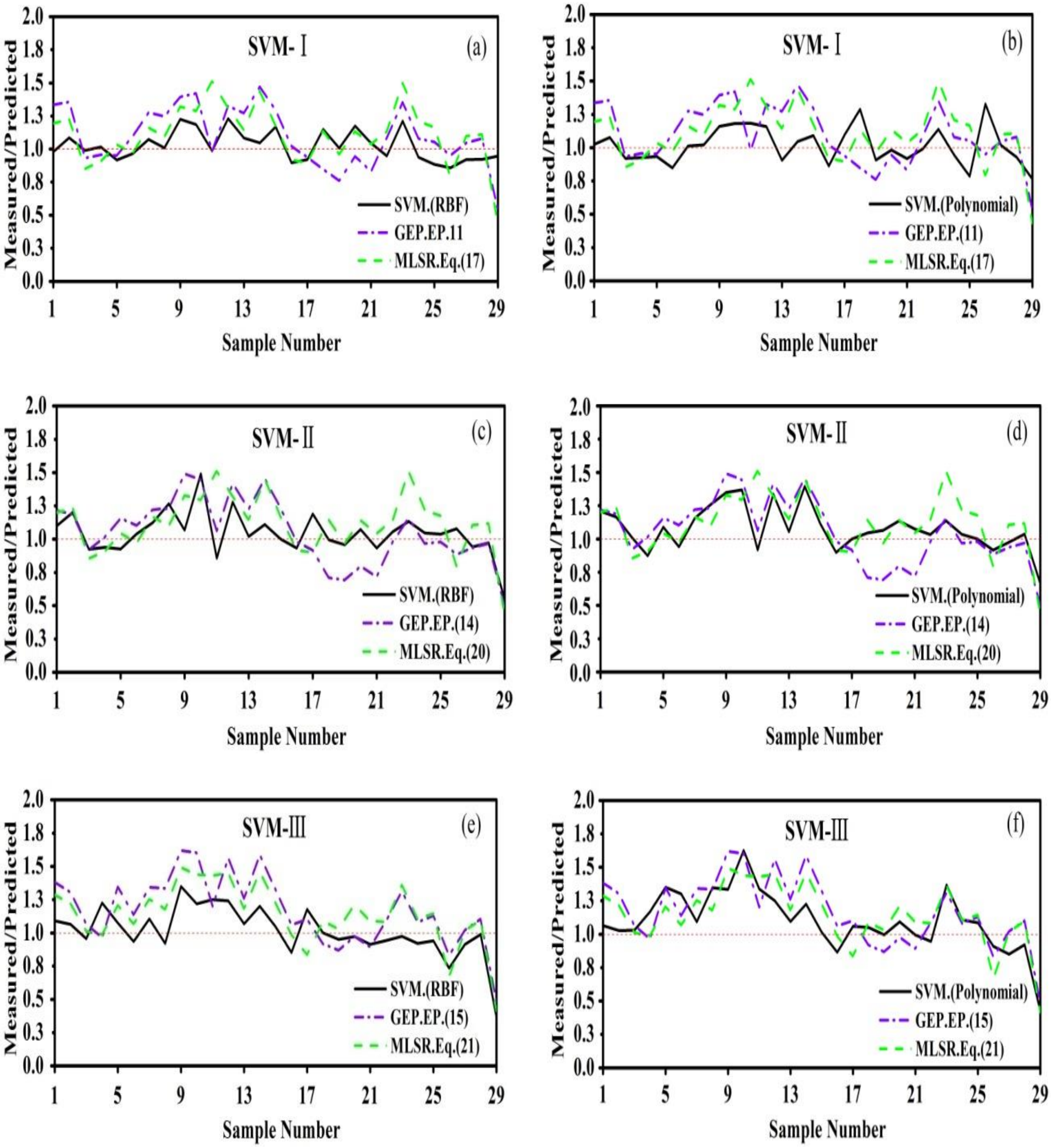

Fig. 13. A comparison of the Ratio between the Experimental and Predicted Flow Number Values Using Different Models for Testing Dataset.

This feature also provides control over model complexity in ways that alleviate the problems of over and underfitting. The evaluations shown above reveal that the SVM model has good prediction ability. The prediction accuracy of the model appears to be sufficient from the statistical point of view in the prediction of the flow number. Considering that the laboratory tests for determination of the flow number of asphalt mixtures can be laborious, time consuming, and costly, it can be concluded that using the developed SVM models is a reasonable way to predict the flow number of asphalt mixtures. Although the artificial network neural (ANN) has been also used widely in regression and prediction areas [1], there are some shortcomings for ANN, such as slow convergence speed, poor generalizing performance, arriving at local minimum, and over-fitting problems. Furthermore, there is no proper method to determine the number of hidden layers. SVM has ability to avoid overtraining, and has better generalization capability than the ANN model. Moreover, the SVMs can always be updated to get better results by presenting new training examples as new data become available [21]. The drawback of the SVM against other soft computing tools such as GP (GEP), etc. is determination of the parameters values of the constant C and the accuracy ε, as this is still a heuristic process.

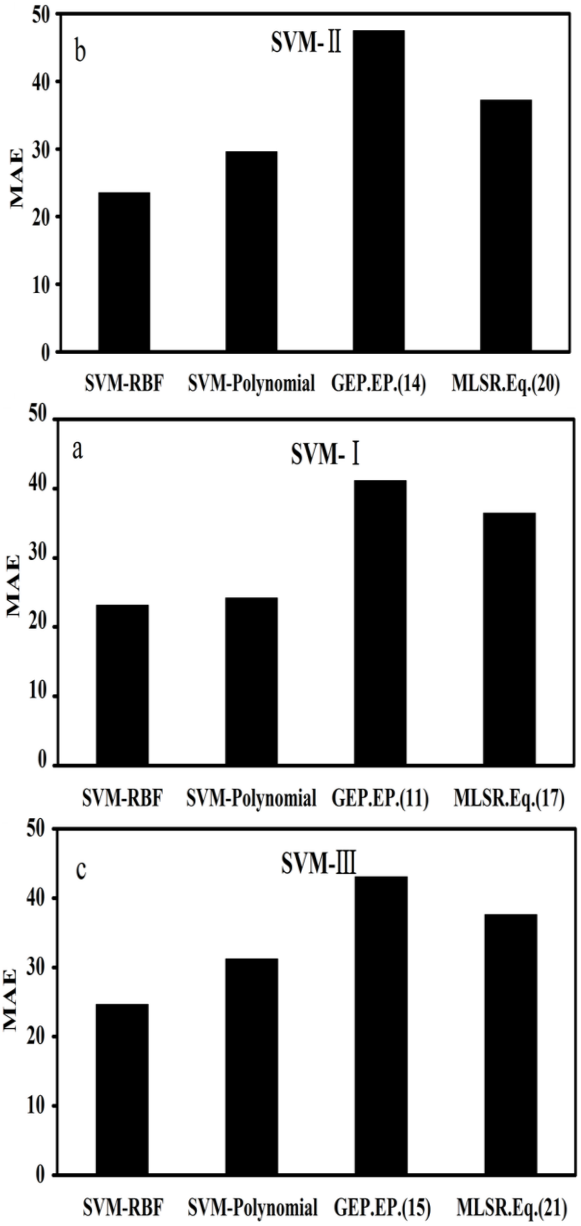

Fig. 14. Comparison between SVM and other models in terms of MAE.

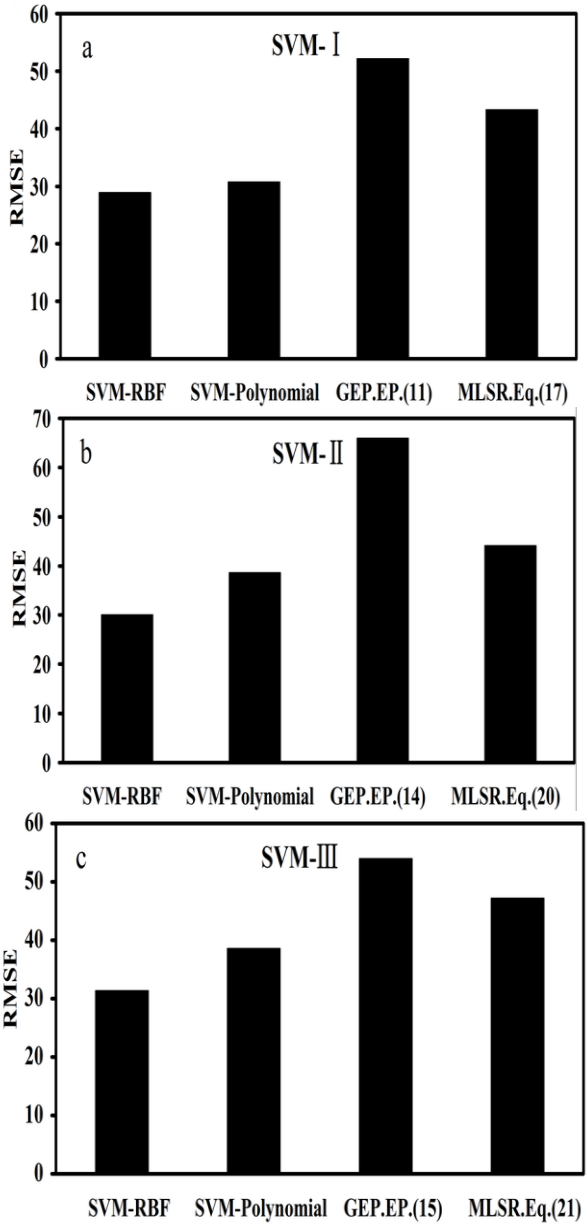

Fig. 15. Comparison between SVM and Other Models in terms of RMSE.

Sensitivity Analysis and Parametric Analysis

Sensitivity analysis is of utmost concern for selecting the important input variables. The sensitivity of the radial basis function has been examined. The reason for choosing the radial basis function kernel is that it gives better results than the polynomial kernel. The results of the sensitivity analysis are shown in Fig. 16. From this figure, it is clear that C/S and VMA are the most influential parameters on the flow number of asphalt mixtures, followed by *Va* and M/F. Earlier findings in the literature are in close agreement with this observation [23, 24]. The relative importance values of the input parameters are investigated by Mirzahosseini [16], who also found the flow number was more sensitive to C/S and VMA in comparison with the other effective parameters.

For further verification of the models, a parametric analysis has been performed to find the effect of each parameter on the flow number (F_n). Fig. 17 presents the tendency of the Fn predictions of C/S, *Va* (%), VMA (%), and M/F. As can be seen in Fig. 17(a), F_n continuously decreases due to increasing C/S. The fine aggregate content in asphalt mixtures is inversely proportional to C/S. It is well known that an increase in the fine aggregate will stiffen the asphalt mixtures, leading to higher Marshall stability values and better resistance to permanent deformation. Because the air void of asphalt mixtures is filled by the fine aggregate and a more integrate grading will be obtained. Besides the fine aggregate affect the load spreading characteristics of the mixture.

It is well known that the rutting resistance of the mixtures increase as the air void (V_a (%)) and voids in mineral aggregate (VMA (%)) decrease. This is verified completely by Fig. 17 (b) and (c). Because the specimens with higher V_a become less dense, and

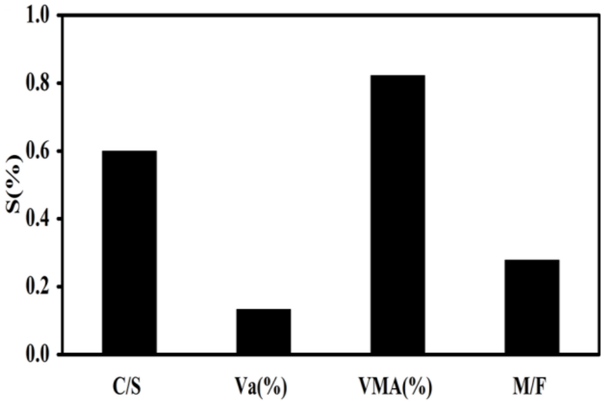

Fig. 16. Sensitivity Analysis of Input Parameters.

lead to less shear resistance for the asphalt mixtures, which increases deformation of the mix caused by loading. As the VMA value increasing, the asphalt content will increase too, which attributes to increasing the rutting potential and softening the sample. The results of several studies also indicate resistance against the permanent deformation increase as *Va* value and VMA value decrease [23, 25, 26].

Fig. 17 (d) indicated that F_n initially increases when M/F increases up to about 3.5; thereafter, it starts decreasing. According to the previous studies [1, 15], the effect of M/F on the rutting potential of asphalt mixtures is complex. There are no clear conclusions in the literature about the effect of M/F to the rutting resistance of asphalt mixtures. Several studies indicate that resistance against rutting potential increases as M/F increase [26-28]. Recently, Tayfur et al. [29] had investigated the rutting performance of asphalt mixtures and found that M/F may not be a good indicator for measuring permanent deformation.

Conclusions and Further Recommendations

The application of the SVM for predicting the flow number of asphalt mixtures has been investigated in this study. The results indicate that the SVM has the ability to predict the rutting potential of asphalt mixtures with an acceptable degree of accuracy. The results obtained also show that the SVM model outperforms the GEP model and MLSR-based model. The use of SVM is very advantageous for the prediction of flow number of asphalt mixtures because it can perform nonlinear regression efficiently for high dimensional data sets. Furthermore, its solution is global. The sensitivity analysis of each input parameters in the SVM model is evaluated and indicates that C/S and VMA are the most influential parameters on the flow number of asphalt mixtures. The results are supported by the experimental evidence and presented by other researchers. In summary, this paper has investigated the SVM and finds that SVM can be viewed as a powerful and practical tool for the determination of flow number of asphalt mixtures. In the future, it is necessary to incorporate more parameters into the model to obtain improved results and to find a good approach to determine the correct values, the constant *C*, and the accuracy ε of SVM models.

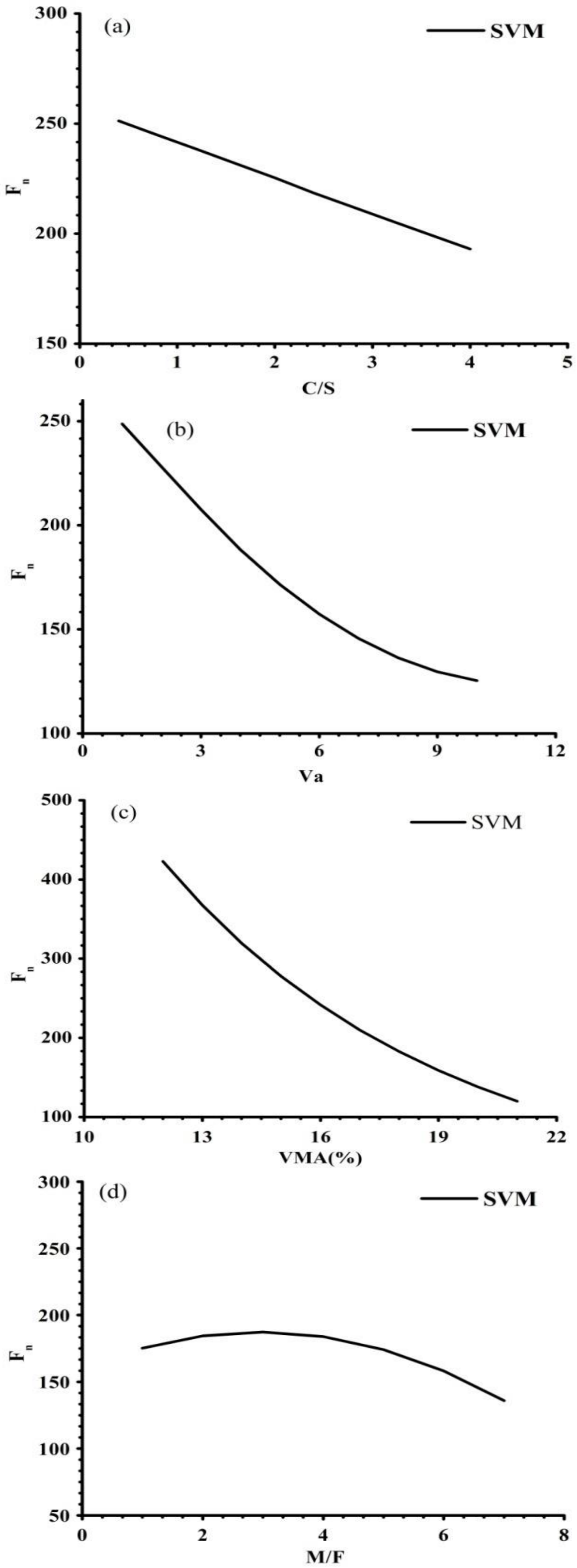

Fig. 17. Flow Number Parametric Analysis in the SVM Model.

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