Technical Paper

ISSN 1997-1400 Int. J. Pavement Res. Technol. 7(2):153-158
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Reliability Based Cost Effective Design of Asphalt Pavements Considering Fatigue and Rutting

Pabitra Rajbongshi[1+]

Abstract: Structural design of an asphalt pavement is associated with the multilayer design thicknesses and multiple failure modes. Therefore, multiple design solutions can be obtained in the form of different thickness combinations. The cost of each design solution may be different. This paper shows that a cost-effective design section of the pavement exists. This work focuses on the reliability based cost-effective design methodology considering fatigue and rutting failures. Two parameters, namely asphalt layer and granular layer thicknesses, are taken as the design variables. An automated scheme has been developed to identify the lowest cost design solution by using the simplex method of optimization technique.

DOI:10.6135/ijprt.org.tw/2014.7(2).153

***Key words**: Cost; Fatigue; Pavement design; Reliability, Rutting.*

Introduction

In the mechanistic-empirical (M-E) design process of asphalt pavements, fatigue, rutting, thermal fatigue cracking, low temperature cracking, etc., are considered different modes of pavement failure [1-8]. For a given set of input parameters (i.e. materials property, traffic, design reliability, etc.), the structural design of a multilayer pavement section may result in numerous alternative design solutions in terms of thickness combinations of the design layers [9]. The costs of these design sections of the initial pavement are expected to be different, which ultimately would affect the life cycle cost as well. Thus, the initial cost of pavement shall be considered an important factor for the design satisfaction.

The objective of the present paper is to develop an automated pavement design process where the least cost design solution, out of various possible thickness combinations can be identified. The reliability based M-E design method is used, and the fatigue and rutting are taken as two modes of pavement failure. For simplicity, the pavement structure is considered to be made up of three layers, namely, asphalt layer, granular layer, and the subgrade.

The next section of this paper presents a brief background on the related aspects of M-E pavement design. The concept of cost-effective pavement design is then discussed followed by the proposed approach on the cost-effective automated design process. The fifth section elaborates it further through the numerical example. Finally, the proposed design methodology is discussed.

Background

The structural pavement design process attempts to estimate the appropriate thickness values of design layers. The design thicknesses are determined iteratively based on the traffic repetitions (T) over the design period and the repetitions that a pavement can sustain before failure (N) for the given mode of failure. The parameter N is also termed as pavement life. Considering fatigue and rutting failures, a pavement can have two different life types, namely, fatigue life (N_f) and rutting life (N_r). In the M-E design process, N_f and N_r for a given section are obtained using filed calibrated regression equations. The generic forms of such fatigue and rutting equations can be expressed as given in Eqs. (1) and (2) respectively.

$$N_f = k_1 \times \left(\frac{1}{\varepsilon_t}\right)^{k_2} \times \left(\frac{1}{E_1}\right)^{k_3} \tag{1}$$

$$N_r = c_1 \times \left(\frac{1}{\varepsilon_z}\right)^{c_2} \tag{2}$$

where, ε_t is initial critical horizontal tensile strain at the bottom of asphalt layer; ε_z is initial critical vertical compressive strain at the top of subgrade; E_1 is elastic modulus of the asphalt layer; and k_1, k_2, k_3, c_1 and c_2 are the regression constants.

The traffic repetitions (T) over the design period may be estimated as given in Eq. (3).

$$T = 365 \times \frac{(1+r)^{yr} - 1}{r} \times A \tag{3}$$

where, A is daily traffic (standard axles) repetitions at the time of opening the pavement to traffic; r is annual traffic growth rate; and yr is the design period in years. The traffic with different axle loads under the mixed loading condition can be converted into the standard axle load by using different empirical load equivalency factors [1, 5, 10-11]. Such mixed loadings can also be accounted by using Miner's hypothesis of linear damage accumulation [6, 12-15].

The input parameters in Eqs. (1), (2) and (3) (except 'yr') show significant variability [9]. These variabilities can be accounted by estimating the probability/reliability [16-23]. Various researchers [24-27] studied the reliability analysis on the flexible pavement design approach. Dilip *et al*. [25] concluded that the surface layer

[1] Civil Engineering Department, National Institute of Technology Silchar, Silchar-788 010, Assam, India.

[+] Corresponding Author: E-mail prajbongshi@yahoo.com

Note: Submitted December 20, 2012; Revised September 22, 2013; Accepted September 23, 2013.

thickness is the most critical parameter affecting the design reliability for both fatigue and rutting failure. The reliability issues and the development of the concept of cost-effective pavement design are discussed in the next section.

Concept of Cost Effective Pavement Design

The reliability (for a given failure mode) of a pavement section can be defined as the probability that the number of repetitions a pavement can sustain (N) is greater than the predicted traffic repetitions (T) over the design period. Thus, the reliability (R) can be expressed as given in Eq. (4).

$$R = \int_0^1 f_D(x)dx \quad (4)$$

where, $f_D(x)$ is probability density function (pdf) of the damage factor D; and x is a variable of integration. The parameter D can be defined as given in Eq. (5).

$$D = \frac{T}{N} \quad (5)$$

where, T and N (N_f or N_r) are in terms of the same axle load (standard axle load).

The pdf $f_D(d)$ can be obtained for any known pdfs of T and N [15, 28-29]. The pdfs of T and N, and their parameters of distribution, can be obtained for known variability of the input parameters [12, 16, 19, 21, 30].

To initiate reliability based pavement design, a trial pavement section is assumed and the reliability is estimated. The trial thicknesses of the design layers are adjusted iteratively so that the estimated reliability is equal (or little higher) to the design reliability level (which is a designer specified input) for the given failure mode. In other words, the reliability for various combinations of asphalt layer thickness (h_1) and granular layer thickness (h_2) can be obtained, and subsequently a reliability contour can be developed. A similar process can be repeated for the other failure mode(s). Fig. 1 presents such a contour plot schematically for fatigue (R_f) and rutting (R_r) reliability. Also, the reliability value increases as the thickness increases.

From Fig. 1, a suitable contour can be chosen, which matches with the design reliability level (for fatigue or rutting case). This has been presented in Fig. 2, where the curve POQ corresponds to the fatigue design reliability (R_f^d), and curve XOY corresponds to the rutting design reliability (R_r^d). Thus, the curve XOQ may be considered as safe design curve where any point on this curve satisfies $R_f \geq R_f^d$ and $R_r \geq R_r^d$. Thus, any thickness combination (h_1, h_2) on XOQ indicates a safe design solution. At point ‘O’, both the fatigue and rutting design curves intersect, where $R_f = R_f^d$ and $R_r = R_r^d$. Given the nature of fatigue and rutting curves [1, 2, 7], it is expected that these curves would intersect at one point within the zone of realistic pavement thickness combinations.

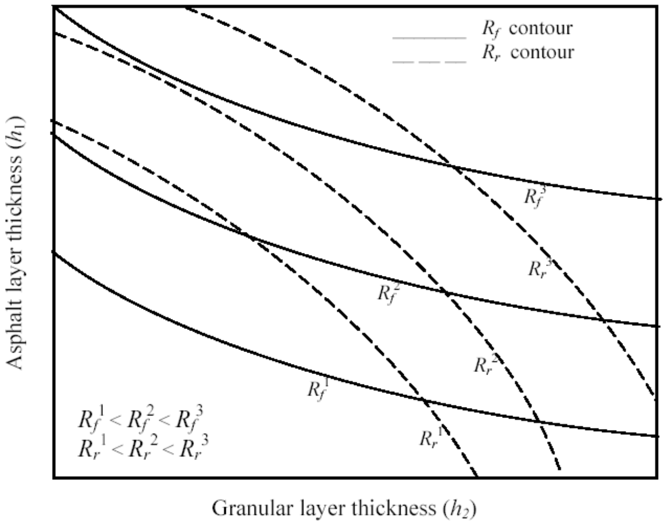

Fig. 1. Schematic Diagram of Fatigue and Rutting Reliability Contours.

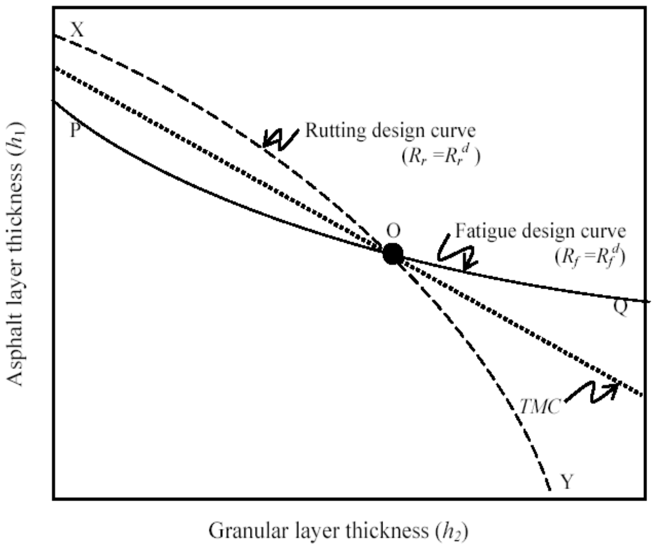

Fig. 2. Schematic Diagram of Pavement Design Curve and Cost Line.

To incorporate the cost factor, it may be considered that for each road material [31], the material cost per unit volume is constant. Therefore, for given road length and road width, the total material cost per unit area of pavement surface varies with the layers thickness only. Thus, the total material cost (TMC) per unit area of pavement may be expressed as given in Eq. (6).

$$TMC = ALC \times h_1 + GLC \times h_2 \quad (6)$$

where, ALC and GLC are material cost per unit volume of asphalt and granular layer respectively, and h_1 and h_2 are the design thicknesses of asphalt and granular layers respectively. Eq. (6) is a linear equation and thus, the cost contour in ($h_1 - h_2$) space can be represented by family of straight lines. The line that would touch the XOQ curve first would represent the design solution with the minimum cost possible. That is, this case point ‘O’ is the lowest cost design solution, as depicted in Fig. 2. It may be mentioned that at point ‘O’, both $R_f = R_f^d$ and $R_r = R_r^d$, which is unique for traditional nature of the fatigue and rutting curves. The next task is to develop an automated scheme to determine the cost effective design point in the ($h_1 - h_2$) space, without having to draw the entire design chart. This would enable a pavement designer to quickly locate the best design solution without computing various possible design alternatives and subsequently comparing them. This approach will, therefore, save a nation a considerable amount of computational effort as well as revenue.

Cost Effective Design Solution

While performing a reliability based pavement design (for assumed h_1 and h_2 values), the fatigue (R_f) and rutting (R_r) reliability values can be obtained using Eq. (4). Subsequently, R_f and R_r are compared with required reliability levels (i.e., R_f^d and R_r^d). Finally, a combination of h_1 and h_2 is chosen so that both R_f and R_r values are as close as possible to R_f^d and R_r^d values respectively. That is the point ‘O’ as shown in Fig. 2 where, $R_f = R_f^d$ and $R_r = R_r^d$. Thus, an objective function O_f may be formulated as given in Eq. (7).

$$Minz.O_f = \left|R_f(h_1,h_2) - R_f^d\right| + \left|R_r(h_1,h_2) - R_r^d\right| \tag{7}$$

In Eq. (7), the absolute minimum value of O_f is 0, i.e., at point ‘O’ which also represents the minimum *TMC* for a safe pavement section. For different combinations of h_1 and h_2, the O_f values can be obtained and a contour plot of O_f can be developed. Such a contour plot has been presented schematically in Fig. 3.

Eq. (7) represents a multi-variable optimization problem without any constraint. In this case, a standard optimization technique namely the “Simplex method” [32-33] can be used to minimize O_f. Moreover, Simplex is not a convenient method with more than three variables. This is a direct search method which uses the functional values. For a function with two variables (as is the present case), three initial points are necessary to form a triangular ‘simplex’ within the search space. In the next step, a new point is searched through reflection, expansion, and contraction operations which provides the improvement on the functional value of O_f. This new point replaces the worst point, showing maximum functional value of the existing simplex. Thus, a new simplex is formed. This process of searching continues until a certain specified terminating condition is satisfied. In the present analysis, a terminating criterion $O_f \leqq 0.01$ has been adopted. The optimization process for such automated identification of a point with $O_f \leqq 0.01$ is further explained through a numerical example in the next section.

Numerical Example

This section elaborates the proposed cost effective pavement design approach through a numerical example.

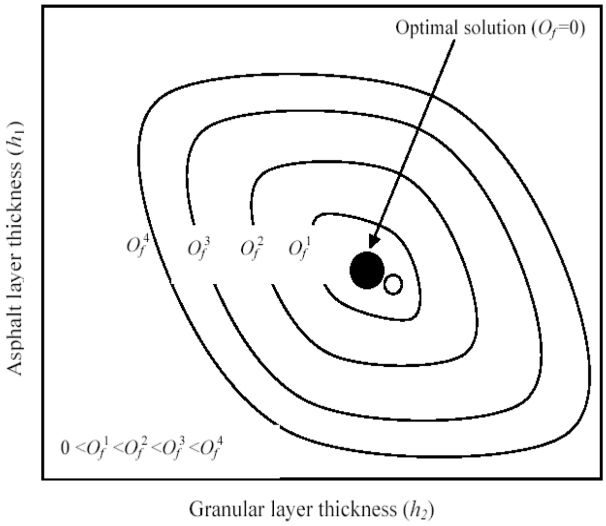

Fig. 3. Schematic Diagram of O_f Contours.

Problem Statement

The elastic moduli of a three-layered pavement with asphalt, granular, and subgrade layers are given as 1,500, 250, and 80 MPa respectively. The values of Poisson's ratio are 0.4, 0.35, and 0.45 for asphalt, granular, and subgrade layers respectively. The coefficients of fatigue (Eq. (1)) and rutting (Eq. (2)) equations are given as, $k_1 = 2.21\times10^{-4}$, $k_2 = 3.89$, $k_3 = 0.854$, $c_1 = 4.1656\times10^{-8}$, and $c_2 = 4.5337$ [5]. The present traffic (A) is 1210 standard axles (82 kN, dual wheel) per day. The annual rate of traffic growth (r) is 7.5% and the design period (yr) is 20 years. The cost optimal pavement section for $R_f^d = 0.70$ and $R_r^d = 0.65$. Assuming that the parameters T, N_f, and N_r are normally distributed, the coefficient of variation (COV) of T, N_f, and N_r are 25%, 35%, and 40% respectively.

Solution

For the given data and using Eq. (3), the total estimated traffic repetitions (T) are obtained as 25 million standard axles (msa). For any assumed pavement section, the strain (ε_t and ε_z) values may be evaluated using multilayer elastic analysis [34]. For example, for h_1 = 100 mm and h_2 = 200 mm, these values are ε_t = 0.000402 and ε_z = 0.000728. Accordingly, using Eqs. (1) and (2), the N_f and N_r values are obtained as 6.9 msa and 7.0 msa respectively.

The next task is to determine the reliability (R) of the pavement structure. It is given that T and N are normally distributed. Thus, the marginal pdf ($f_D(d)$) of the damage factor D can be derived as given in Eq. (8) [28-29].

$$f_D(d) = \frac{1}{2\pi \times sd_T \times sd_N} \int_0^\infty n e^{-\frac{1}{2}\times\left(\frac{d\times n - E[T]}{sd_T}\right)^2} \times e^{-\frac{1}{2}\left(\frac{n-E[N]}{sd_N}\right)^2} dn \tag{8}$$

where, *E[X]* and sd_X represent the expected value and standard deviation of the random variable X; and n is an integrating variable.

Thus, using Eq. (4) through Eq. (8), the reliability R (i.e. R_f or R_r) can be obtained numerically for both fatigue and rutting cases. For example, for h_1 = 100 mm and h_2 = 200 mm, the reliability values are obtained as R_f = 0.0035 and R_r = 0.0043. Thus, at this point, the value of the objective function (refer to Eq. (7) O_f is 1.3422.

In a similar way, O_f value can be evaluated for different combinations of (h_1, h_2) of the simplex. The (h_1, h_2) combinations of the initial triangular simplex are taken as A(50,100), B(80,120), and C(60,150), and their corresponding O_f values are 1.4103, 1.3532, and 1.3712 respectively. Accordingly, to minimize the O_f value, the new point is chosen automatically through the simplex search technique. The search process continues until it satisfies the condition $O_f \leqq 0.01$. Fig. 4 shows the sequence of how various points are chosen during the optimization process for the present example problem. Thus, the cost effective design solution is achieved as h_1 = 157 mm and h_2 = 194 mm. A flow chart of the design process is presented in Fig. 5.

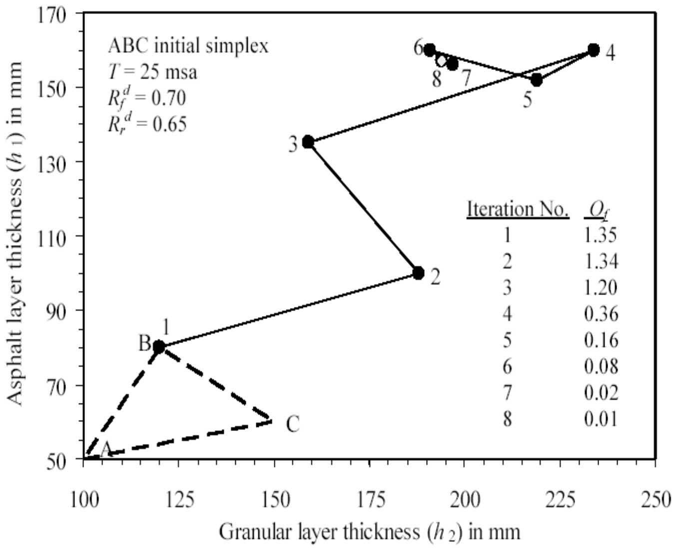

Fig. 4. Design Point at each Iteration of the Numerical Example.

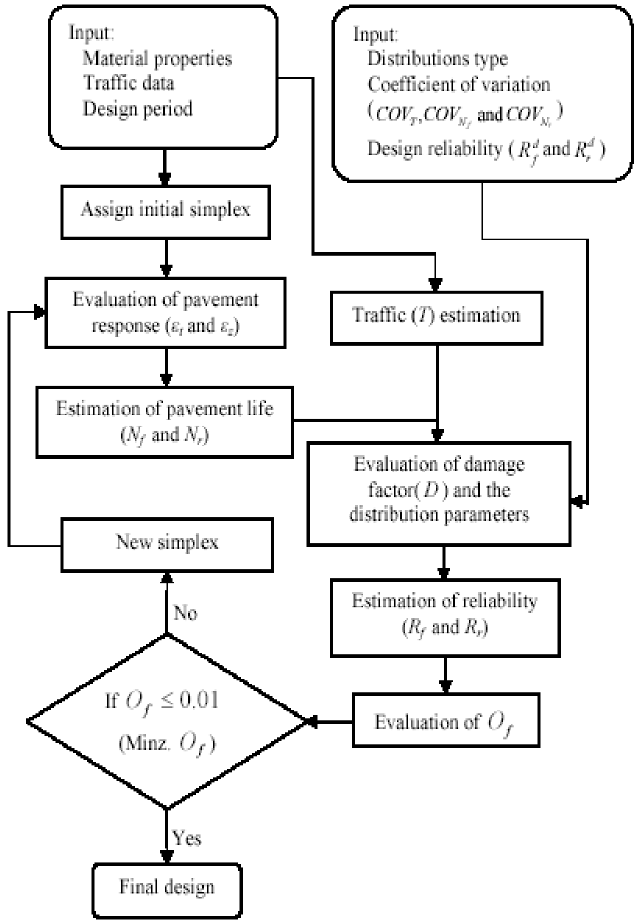

Fig. 5. Flow Chart of Cost-effective Pavement Design.

Discussions

A methodology has been proposed to determine the cost effective design solution based on reliability requirements. This methodology can be adopted for any distribution of T, N_f, and N_r. Any value of the design reliability levels R_f^d and R_r^d can be used as desired by the designer. Table 1 presents the cost-effective design thicknesses for different R_f^d and R_r^d values. Data from the same example problem are used to develop the table. Table 1 shows that both asphalt layer thickness (h_1) and granular layer thickness (h_2) are sensitive to fatigue reliability (for constant R_r^d), whereas only the granular layer thickness (h_2) is sensitive to rutting reliability (for constant R_f^d).

Basically, the proposed design approach identifies a single design point where the estimated individual reliability is the same as that of required reliability value. Knowing the fact that both reliability and cost are increased with increasing the layer(s) thickness, the thickness combination which just fulfills the reliability requirements would bear the lowest cost, i.e., the point 'O' of Fig. 2. However, in some special cases (for example, TMC is non-linear with layer thickness, non-standard trend of fatigue-rutting design curve, etc) an exhaustive search technique may be adopted for obtaining such a cost-effective design solution. In that case, an objective function can be formulated as given in Eq. (9).

$$Minz.TMC(h_1,h_2) \tag{9}$$

subject to:

$$R_f(h_1,h_2) \geq R_f^d$$

$$R_r(h_1,h_2) \geq R_r^d$$

The numerical example presented in this paper has directly used the traffic repetitions in a standard axle load. Under mixed loading conditions, the empirical load equivalency factors can be used to convert the different axle loads repetitions into standard axle load repetitions. Alternatively, Miner's hypothesis of linear damage accumulation can also be used. Using Miner's hypothesis [12, 35], the expression for damage factor D can be written as given in Eq. (10).

Table 1. Cost-effective Design Thicknesses (mm) at Different Design Reliability Levels.

	R_r^d = 0.50		R_r^d = 0.65		R_r^d = 0.80	
	h_1	h_2	h_1	h_2	h_1	h_2
R_f^d = 0.60	153	188	151	207	149	231
R_f^d = 0.70	159	175	157	194	154	220
R_f^d = 0.80	169	155	165	178	162	206

$$D = \sum_{\forall i} \frac{T_i}{N_i} \tag{10}$$

where, T_i and N_i are the predicted traffic repetitions and pavement life (fatigue or rutting), respectively, for the i th axle load group. T_i and N_i can be obtained using Eq. (3) and Eq. (1) or (2), respectively, for each axle group. Now, equating Eq. (5) with Eq. (10), an equivalent traffic repetitions in terms of standard axles (T) can be obtained using Miner's damage principle as expressed in Eq. (11).

$$T = N \times \sum_{\forall i} \frac{T_i}{N_i} \tag{11}$$

where, N (N_f or N_r) in Eq. (11) is in terms of standard axles. T value as obtained from Eq. (11) may be used in Eq. (5) and the same procedure can be followed for reliability calculation in the design process.

Conclusions

This paper presents a simple automated pavement design approach to find the most cost-effective design solution, satisfying the pre-specified fatigue and rutting reliability levels. The proposed methodology is general enough to accommodate any fatigue and rutting equations and any probability distribution of T and N parameters as well. A deterministic cost-effective design can also be obtained by adopting the design reliability (R_f^d and R_r^d) of 0.5 in the proposed design method. Furthermore, the initial pavement section with lowest cost would also reduce the subsequent life cycle cost of the structure.

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