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# Numerical Modelling of Thermal Cracking of Pavements

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**Abstract:** Thermal cracking of pavements may occur due to the top of the pavement being exposed to cold atmospheric conditions. The formation of cracks may cause a deterioration of ride quality and ingress of water to the base. Developing of capabilities to model thermal cracking of pavements is beneficial because it will allow the assessment of susceptibility of pavement materials and geometries to thermal cracking. Fracture resistant asphalt mixtures can then be identified. A methodology is proposed by which numerical modelling of thermal cracking of pavements is carried out using the distinct element method and cohesive cracks. The distinct element method efficiently handles the subdivision of the originally intact material. The cohesive crack method allows the inclusion of experimentally determinable fracture properties into the formulation. Results obtained were consistent with an analytical model available in literature. Thermal cracking observed in a field experiment could then be numerically replicated.

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***Key words**: Cohesive cracks; Distinct element method; Numerical modeling; Pavements; Thermal cracking.*

## Introduction

### Thermal Cracking of Pavements

Thermal cracking of pavements occurs when cooling of the road surface causes unsupportable tensile stresses to be generated in it. The restraint provided by the base to the road surface aids in the development of such cracks. Thermal cracking of pavements causes significant damage to pavements in the colder regions of the world. For example, Marasteanu et al. [1] report that thermal cracking is the predominant cause of damage to road pavements in northern regions of the United States of America and Canada. As such, it is important that the modelling capabilities of thermal cracking in pavements be advanced. Then it will be possible to check the cold weather performance of an asphalt concrete mix after obtaining the required material properties by laboratory experimentation. This paper proposes the distinct element method as a valuable tool in modelling thermal cracking of pavements.

The composition of this paper is as follows. Some previous models of thermal cracking of asphalt pavements are discussed next. Then a review of fracture properties of asphalt concrete is made after which a brief introduction to the cohesive crack method, which is used to model fracture in this paper, is presented. This method was incorporated into a distinct element program. Some special considerations to be made when multiple interacting cracks are present are discussed. A parametric study was carried out numerically to assess the effects of varying 3 key material properties in the modelling of a pavement section. The results are compared with those predicted by analytical modelling. Subsequently, thermal cracking of an asphalt pavement observed in a field experiment is replicated numerically.

### Previous Models of Thermal Cracking of Pavements

Some of the models used for thermal cracking of pavements have an empirical basis. Empirical formulae, while easy to use, have the drawback that it is difficult to extrapolate from the physical conditions they were derived. Fromm and Phang [2] produced regression equations for this purpose, basing their recommendations on a testing program on 33 pavement sections in Ontario. The parameters used in the empirical derivations included the viscosity ratio, freezing index, critical temperature, air voids, stripping rating and aggregate and base characteristics. The $R^2$ obtained from the fitting varied between 0.6 and 0.7.

Haas et al. [3] conducted a similar study based on data obtained from 26 airport pavements in Canada. This model predicts the transverse crack spacing. The input to the data includes thickness of asphalt concrete, minimum temperature recorded on site, penetration, viscosity and coefficient of thermal contraction of asphalt. The equation, with an $R^2$ of 0.7, predicts less cracking with increasing asphalt thickness.

Predictions on cracking can also be made based on principles of mechanics. Hills and Brien [4] devised a method by which the temperature of cracking could be estimated but not the frequency or depth of cracks. Finn et al. [5] developed the computer program COLD based on the Hills and Brien approach. The motivation for the program was as an aid to the selection of binders for locations susceptible to thermal cracking. The program is able to determine the evolving temperature distribution by solving the governing equation by the finite difference method. COLD has the capability to accommodate varying strength with temperature.

The thermal cracking TC Model was developed by Hiltunen and Roque [6] and subsequently modified and refined by National Cooperative Highway Research Program (NCHRP). The thermal stresses are calculated by using the coefficient of thermal contraction and relaxation modulus of the asphalt mixture. The temperature profile within the asphalt is determined by a climatic model. Crack propagation is determined by a two dimensional finite

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element program using fracture toughness. Because this is not practical for pavement design, this program has been run for a broad range of pavement conditions and a simple regression equation has been generated. However, the spacing of cracks is determined by an empirical relation.

Marasteanu et al. [1] carried out finite element modelling of themal cracking of asphalt pavements by incorporating interface elements. The cohesive zone method, similar to the cohesive crack method used in this paper, was used. This model encompasses many features not seen in previous models. No rational basis for multiple crack interaction has been provided, which may be a critical factor as for example Amarasiri and Kodikara [7] show that modelling only a single crack significantly overestimated crack depth and opening in shrinking soil when compared to modelling with multiple cracks allowed.

Timm et al. [8] developed an analytical model for predicting thermal crack spacing, which is discussed in depth in Section 1.4.

## Fracture Properties of Asphalt Concrete

Majidzadeh et al. [9] tested single edge notched beams made of Ottawa sand and limestone mixed with bitumen. The data generated were used to predict fatigue life of asphalt mixes considering material constraints, geometry, boundary conditions, and the state of stress. Fatigue failure was based on the processes of damage initiation, crack growth and final failure. Ramsamooj et al. [10] also found that simple fracture tests could be used to predict the fatigue life of asphalt concrete.

Labuz and Dai [11] used closed loop, computer controlled testing on three point bend specimens to determine fracture toughness of asphalt concrete. Unload-reload cycles were used to obtain multiple data points from a single specimen. Crushing and non-linear behaviour at the roller points caused the deflection measurements needing to be taken with respect to points directly above roller supports. The Young’s modulus could also be estimated by using the compliance of the beam. Assuming the validity of LEFM, fracture toughness values of between 0.25 MPam$^{0.5}$ and 0.53 MPam$^{0.5}$ were found between temperatures of 0$^{o}$C and -34$^{o}$C.

Kim et al. [12] conducted fracture tests on asphalt concrete specimens between 0$^{o}$C and -30$^{o}$C. It was found that the fracture toughness had a maximum at about -15$^{o}$C, with it decreasing on either side of this temperature. Asphalt concrete made with granite showed slightly higher fracture resistance than with limestone. Wagoner et al. [13, 14] fractured single edge notched beams constructed with aggregates with 3 nominal maximum sizes and 3 binders. The smallest aggregate size in combination with the polymer modified binder delivered the highest fracture energy.

However, most fracture tests on asphalt concrete have been made on cylindrical specimens due to ease of laboratory preparation and extracting field cores. The semi-circular bend (SCB) test originated by Chong and Kuruppu [15, 16] was used by Molenaar et al. [17] to determine the fracture properties of asphalt mixes. Test temperatures ranging from -10$^{o}$C to 25$^{o}$C were deployed, and it was thought that excessive plastic deformation at higher temperatures may negate conditions assumed in computing fracture toughness. Amongst other findings, they concluded that the SCB test had higher discriminative ability than the indirect tensile strength test.

Li et al. [18] and Li [19] tested the fracture properties of asphalt concrete with SCB specimens using three grades of asphalt binders at the temperatures of -20$^{o}$C, -30$^{o}$C and -40$^{o}$C. Both the fracture toughness and fracture energy of asphalt mixtures were determined. The authors concluded that the fracture energy is a better way to compare mixtures than fracture toughness due to non-dependence on linear elasticity of the bulk medium. In general, the lower the temperature, the lower the fracture energy. However, a plateau value of fracture energy seemed to appear when the temperature dropped below the performance grading (PG) critical temperature.

Marasteanu et al. [1] and Marasteanu et al. [20] carried out a comprehensive experimental program to determine the fracture properties of asphalt concrete at low temperatures. Temperatures ranging from -12$^{o}$C to -42$^{o}$C were examined. Modified and unmodified bitumens of several grades were used with limestone and granite aggregate. Three replicates were tested for each data point. SCB test results yielded fracture energies ranging from 205 N/m to 1479 N/m and fracture toughness ranging from 0.3 MPam$^{0.5}$ to 1.3 MPam$^{0.5}$. Disc-shaped compact tension tests yielded fracture energies ranging from 230 N/m to 3505 N/m. Indirect tensile strength (IDT) specimens were used to determine the tensile strength of the asphalt concrete mixtures. At the slowest loading rate of 1 mm/min, which is probably applicable to fracture due to thermal cooling, the tensile strength obtained varied between 1.66 MPa and 9.22 MPa. There is a clear trend of fracture toughness increasing and fracture energy decreasing with cooling. ASTM E399 [21] requires that the uncracked ligament length ahead of a crack at the start of the test of specimen be longer than 2.5 $(K_{app}/\sigma_{ys})^2$ where $K_{app}$ is the apparent fracture toughness calculated using the failure load and $\sigma_{ys}$ is the yield strength of the material. This is to make sure that the size of the fracture process zone is sufficiently small compared to the specimen, and thereby guaranteeing the applicability of LEFM. In this case, as $\sigma_{ys}$ is unavailable, the reported tensile strength could be used. It can be seen from data presented by Marasteanu et al. [1] that this condition is indeed violated for over two thirds of the fracture tests where the tensile strength has also been reported. However, no clear trend in magnitude of deviation of behaviour from LEFM with temperature could be discerned.

Li et al. [22] carried out tests to determine fracture properties of asphalt concrete at low temperature using SCB specimens. Three temperatures were examined, -6$^{o}$C, -18$^{o}$C, and -30$^{o}$C. Several grades of bitumen and both limestone and granite aggregates were used. The fracture toughness obtained varied between 0.29 MPam$^{0.5}$ and 1.17 MPam$^{0.5}$. Fracture energies ranging from 195 N/m to 1479 N/m were obtained. As observed by previous researchers, fracture energies decreased with cooling and fracture toughness increased with cooling. They also investigated the effect of aggregate, air voids, bitumen content, modifier and binder on fracture properties. They state that LEFM conditions most likely do not exist for the material under investigation.

## Analytical Model Extended upon Numerically

Timm et al. [8] produced an analytical model of cracking in thin films. The current methodology extends upon this work by including fracture energy amongst other aspects. The central contribution by Timm et al. [8] is the development of a one-dimensional analytical model capable of obtaining crack spacing in a pavement section due to cooling. The cooling could be uniform over the depth of the pavement or there could be a temperature differential between the top and bottom of the pavement. The pavement rests on a base that is taken to be softer than it, and shear stresses are generated between the base and the bottom of the pavement. These foundation stresses are assumed to be of the Winkler-type with a cohesive-frictional interface added to it. The analytical model has been validated by comparison with two dimensional numerical modelling. Furthermore, two practical applications have been tested, i.e. thermal cracking of a test pavement section and the average crack density of a thin ceramic film subjected to axial strain.

The geometry (Fig. 1) is simplified to that of an elastic slab of length $\lambda$, thickness $h$, Young's modulus $E$, tensile strength $S$, Poisson's ratio $v$, coefficient of thermal expansion $\alpha$, and unit weight $\gamma$ in Timm et al.'s formulation. It is subjected to cooling by $\Delta T$ while resting on a base made of elastoplastic material. The base has Young's modulus $E_b$, Poisson's ratio $v_b$ and thickness $h_b$. It is assumed that the slab is cool enough that viscous effects can be neglected. Shear stresses in the base can be taken to be limited by the Mohr-Coulomb failure criterion.

If the assumption of negligibly small transverse tensile stress in the cooling layer is made, an analytical approximation of the above geometry can be made by an elastic slab resting on a Winkler-type foundation [8]. The Winkler foundation can be augmented by an interface having cohesive-frictional properties. In the analytical solution, bending effects due to the eccentricity of the shear stresses at the interface are neglected. If the horizontal displacement at the slab-base interface at a point is $u_x$, then the reaction $R_x(x)(N/m)$ at the augmented Winkler foundation is given by:

$$R_x(x) = ku_x \quad \text{if} \quad |u_x| < R_x{}^f / k \tag{1}$$

$$R_x(x) = R_x{}^f \, sign \quad u_x \quad \text{if} \quad |u_x| \geq R_x{}^f / k \tag{2}$$

within the region $0 \leq x \leq \lambda$ when monotonic thermal loading occurs, where $k$ stands for the elastic spring coefficient (N/m$^2$) of the slab-base interface, and $R_x{}^f$ (N/m) is the maximum possible shear stress at the interface. $R_x{}^f$ can be calculated to be:

$$R_x{}^f = w\tau_f = w(c + p_z \tan\phi) = w(c + \gamma h \tan\phi) \tag{3}$$

within the region $0 \leq x \leq \lambda$ where $w$ is the width of the slab, $\tau_f$ is the maximum possible shear stress, $p_z = \gamma h$ is the vertical stress due to the self-weight of the slab and $c$ and $\varphi$ are the cohesion and friction of the base material. A uniform temperature change $\Delta T < 0$ is applied to the slab, resulting in its net shrinkage and development of shear stresses along the interface. Depending on $\Delta T$, these stresses could be elastic or elastoplastic. Timm et al. [8] also consider differential cooling within the slab thickness, with a cooling of $T_1$ and $T_2$ respectively at the top and bottom of the slab, and an average cooling of $\Delta T$.

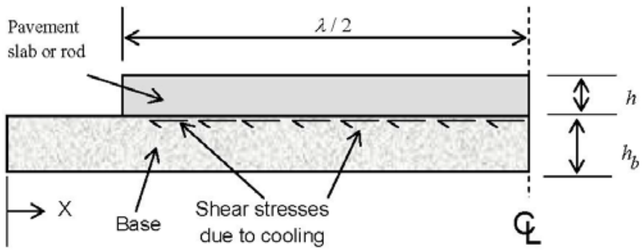


**Fig. 1.** Schematic View of Slab Resting on Base.

Timm et al. [8] suggest that the final crack spacing should be bounded by:

$$\frac{\lambda_s}{2} < \lambda_0 = \frac{L}{n+1} < \lambda_s \tag{4}$$

where $\lambda_s$ can be calculated according to the geometry of the slab, $\lambda_0$ is the average crack spacing obtained in the field or by experimental or numerical modelling, $L$ is the length of the intact slab, and $n$ is the number of cracks developed in it.

The partial slip solution for $\lambda_s$ is:

$$\lambda_s = 2x_t + \frac{2}{\beta}\left( \frac{1+\sqrt{1-\left(\dfrac{1-\psi}{1-\beta\eta x_t}\right)^2}}{\dfrac{1-\psi}{1-\beta\eta x_t}} \right) \tag{5}$$

where

$$x_t = \frac{\lambda}{2} - \frac{1}{2\beta}\log\left(\frac{1-\beta\eta x_t + \eta}{1-\beta\eta x_t - \eta}\right) \tag{6}$$

$$\psi = \frac{S - \dfrac{\alpha E(T_2 - T_1)}{2}}{\alpha(-\Delta T)E} \tag{7}$$

$$\beta = \sqrt{\frac{k}{EA}} \tag{8}$$

$$\eta = \frac{\beta\tau_f w}{k\alpha(-\Delta T)} \tag{9}$$

$x_t$ is the distance from the free end to the interior point of each subdivision where shear displacement has been sufficient for the full shear resistance $R_x{}^f$ to be developed, $A = hw$ is the cross section area of the slab, and $w$ is the width of the slab. It should be noted that in Eq. (6) a recursive solution is required for $x_t$.

The above analytical model will be used as a basis of comparison for results obtained from the current numerical model.

## Advancements in Current Methodology

Several aspects are improved by this work over current practices. Most analytical solutions such as proposed by Timm et al. [8] are feasible only up to the formation of the first crack. A different approach is required after that. Furthermore, Timm et al. [8] gives only bounds for the average crack spacing. The distinct element method used herein has the advantage over other analytical and numerical methods of modelling that progressive subdivision into smaller blocks of the originally intact material is easily accommodated. Furthermore, shear stresses at the base may reverse in some segments after fracture due to release of accumulated tensile stresses, and are difficult to take into account in analytical models. The distinct element method easily accommodates these stresses. Crack widths are of significance in determining the damage done to a pavement, and the current methodology determines them, unlike in many other formulations. If present, the differential in cooling between the top and bottom of the slab will cause some curling. Therefore, the tensile strength and normal stiffness of the slab-base interface may have an effect on fracture behaviour of the slab. Amarasiri and Kodikara [17] reported that the prescribed tensile strength between a desiccating soil and its base had a significant effect on the crack patterns obtained by numerical modelling. The tensile strength of the slab-base interface was prescribed sufficiently high in the current work so that curling was precluded in line with the analytical solutions of Timm et al. However, with the current methodology, it is easily possible to prescribe the tensile strength of the slab-base interface according to values determined by laboratory experimentation. Though analytical solutions are limited to simple geometries, multiple layers of differing materials under the pavement can also be easily accommodated using the current methodology. Though the current work assumes linear elastic behaviour in the bulk medium, UDEC (Universal Distinct Element Code) accommodates plastic behaviour too, and furthermore, the cohesive crack method also does not require linear elastic behaviour in the bulk medium. Concurrent changes in moisture content and possible freezing of the base materials could be incorporated in the numerical analysis if required. It is possible to include evolving of material properties with cooling in the current methodology as well. This can be accomplished by using the programming language FISH available with UDEC that allows a user make modifications in material and interface properties assigned to individual elements in the model.

## The Use of the Cohesive Crack Method to Model Fracture

The cohesive crack method was first proposed by Dugdale [23] and Barenblatt [24]. Crack initiation in an intact material is assumed to occur when normal stresses reach the tensile strength. In this method, it is assumed that bridging normal stresses $\sigma_n$ across a propagating crack progressively drop from the tensile strength to zero as the crack normal displacement $\delta$ increases (Fig. 2) [26]. The variation of crack normal stress with opening is termed the softening law. Bi-linear, linear, rectangular and other softening laws are used in practice. Concrete is commonly considered to have a distinctly bi-linear softening curve [25], while Amarasiri and Kodikara [26] and Amarasiri et al. [27] showed that linear softening may be sufficient in practice for an artificial soft rock and a clay soil respectively. This paper is based on linear softening, but nevertheless, any type of softening behaviour can be accommodated

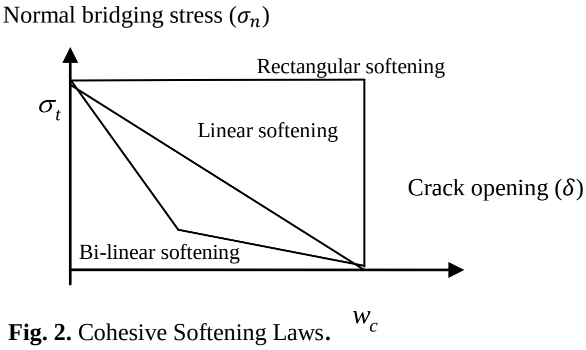


**Fig. 2.** Cohesive Softening Laws.

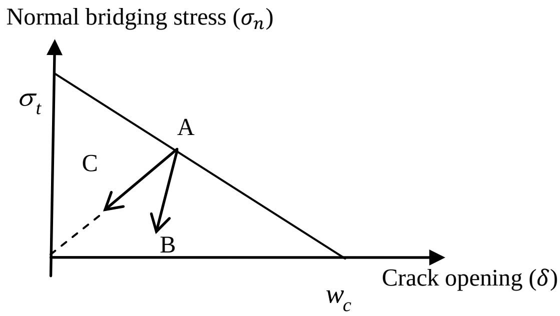


**Fig. 3.** Unloading of a Cohesive Crack.

by this methodology. This choice can be made as more experimental evidence on softening curves of asphalt concrete becomes available. When multiple cracks are present, the opening of some cracks may affect the behaviour of adjacent cracks. Therefore, provisions for crack closure were made as described by Amarasiri and Kodikara [7]. One option for unloading of a cohesive crack is path AB in Fig. 3 where most of the damage is taken to be plastic or permanent. In this work, stiffness degradation is thought to take place with crack opening, and unloading is considered to take place towards the origin (Path AC). However, materials displaying the former behaviour can also be readily accommodated. If a system of parallel cohesive cracks is provided, it has been observed that very often only some of them will open preferentially. This is because a semi open crack requires less energy to open further than an intact crack due to the form of the cohesive softening law. Bazant and Planas [25], Bazant [28] and Amarasiri and Kodikara [29] provide more detailed discussions on the mechanics of cohesive cracks.

The distinct element program UDEC by Itasca was used for the numerical modelling. These programs work on the principle of explicit time-marching to directly solve the equations of motion. The model comprises of continuum elements with UDEC interfaces for joints. These joints can be used to model the breaking up of once intact material. UDEC contains the inbuilt programming language FISH, which can be used to modify material properties with progression of cooling if required. A global stiffness matrix need not be assembled, as UDEC is based on explicit analysis i.e. dynamic time stepping. Conversely, when using the finite element method, formation of a global stiffness matrix is required. When the normal stress at an UDEC interface reaches the tensile strength assigned to it, it fractures and the stress reverts to the assigned residual tensile strength. Closely spaced vertical UDEC interfaces are distributed throughout the model as potential crack paths. It is possible using

**Table 1.** Some Material Properties Used in Numerical Model.

| Material Property | Value |
|---|---|
| $E$ | 14.0 GPa |
| $\nu$ | 0.20 |
| $\gamma$ | 21.6 kN/m$^3$ |
| $\alpha$ | 1.8 x 10$^{-5}$ 1/K |
| $S$ | 1.9 MPa |
| $h$ | 0.15 m |
| $E_b$ | 5.5 GPa |
| $\nu_b$ | 0.4 |
| $C$ | 15 kPa |
| $\Phi$ | 30$^o$ |
| $h_b$ | 2.0 m |
| $\Delta T$ | -15.0 K |
| $T_2 - T_1$ | 8.0 K |

FISH to vary the residual stress with crack opening, and thus achieve any of the softening paths shown in Fig. 2. The initial compliance of these joints is set to be negligible before the reaching of the peak stress. This methodology of programming cohesive cracks in UDEC used in this paper is described and validated by Amarasiri and Kodikara [29].

## Parametric Study

### Geometry and Properties used in Parametric Study

The bulk of the material properties used in this study were those reported by Timm et al. (2003) for a field experiment and are shown in Table 1. Two types of cooling can be analysed. The pavement can be taken to cool uniformly over its thickness. Alternatively, there can be a cooling differential over the thickness of the pavement of $T_2 - T_1$.

The tensile strength $S$, the shear stiffness at the base $k$ / $w$, and the residual limit in the cohesive crack $w_c$ were the subject of parametric studies. The pavement was taken to cool uniformly over its depth by 15$^o$C. This cooling was applied in small increments during the modelling. The parametric study was carried out with the material and properties set as in Table 1, except those being varied. The Young's modulus of the base was also taken to be very high at 110 GPa, effectively lumping all shear pliability into the base-pavement interface, in line with the analytical model. A schematic diagram of the numerical model is shown in Fig. 4. A segment of the numerical model after completion of cooling is shown in Fig. 5, while a fractured joint is shown in Fig. 6.

### Results of Parametric Study

#### *Sensitivity to S, Tensile Strength*

It can be seen from the results shown in Table 2 that the average crack spacing increases with increasing $S$. This result is consistent with the fact that relatively larger cooling and/or longer lengths of pavement may be necessary for adequate normal stresses for fracture to be generated with increasing $S$. When the tensile strength is relatively small, cracking starts earlier in cooling, and further

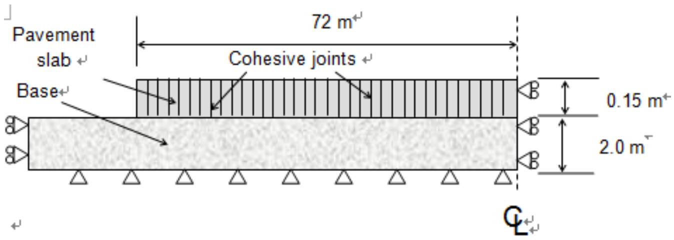


**Fig. 4.** Schematic Diagram of Numerical Model (Not to Scale).

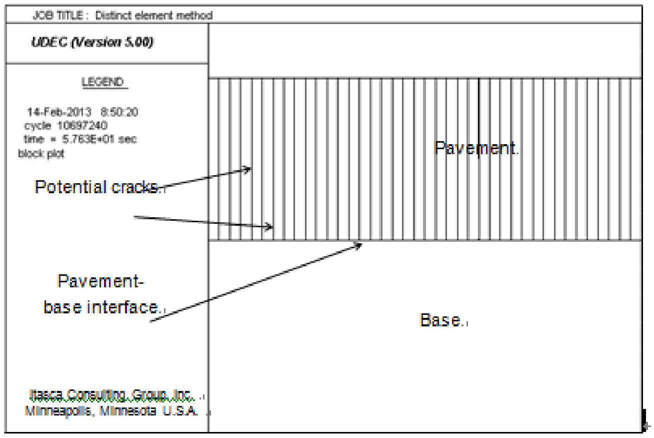


**Fig. 5.** A Segment of the Model Showing Cohesive Joints.

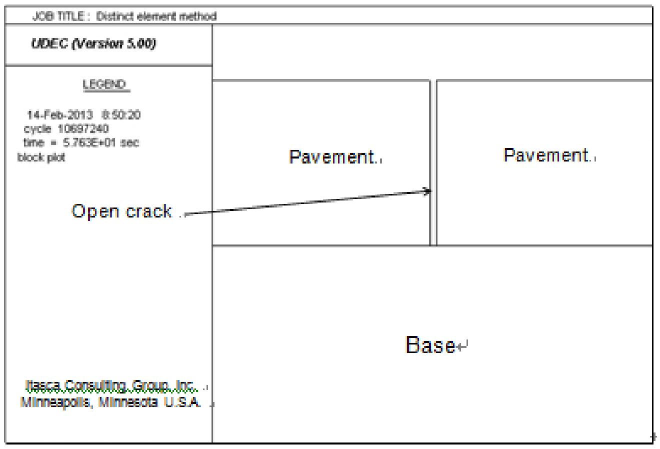


**Fig. 6.** A Fractured Joint.

subdivision can happen easily as cooling progresses. When $S$ becomes very small, the crack spacing becomes smaller than can be verified by the density of potential crack paths provided in these modelling runs. Amarasiri and Kodikara [7] showed that as long as only about every third potential crack opens, the results generated are reasonably independent of density of potential crack paths provided in UDEC when modelling desiccation cracking of soil. As this requirement is violated when $S$ = 100 kPa, a statement is made that the average crack spacing is much smaller than for other assigned values of $S$. The average crack spacings obtained are all within the bounds predicted by Timm et al. by Eq. (4), and thus the numerical model is consistent with Timm et al.'s analytical model.

#### *Sensitivity to k /w, Shear Stiffness at the Base*

On examining the data shown in Table 2, it is apparent that crack

**Table 2.** Results from the Parametric Study

| Sensitivity to Parameter | $S$ (kPa) | $W_c$ (μm) | $k/w$ (MPa) | $L/(n+1)$ (m) | $\lambda_S/2$ (m) | $\lambda_S$ (m) |
|---|---|---|---|---|---|---|
| $S$ | 100 | 100 | 100 | Very Small | NC | NC |
| | 250 | 100 | 100 | 4.11 | 2.36 | 4.72 |
| | 500 | 100 | 100 | 5.76 | 3.89 | 7.77 |
| | 1000 | 100 | 100 | 11.08 | 7.84 | 14.46 |
| | 1900 | 100 | 100 | 20.57 | 13.33 | 26.66 |
| $W_c$ | 1000 | 1000 | 100 | 13.09 | 7.84 | 15.67 |
| | 1000 | 200 | 100 | 11.08 | 7.23 | 14.46 |
| | 1000 | 100 | 100 | 11.08 | 7.23 | 14.46 |
| | 1000 | 50 | 100 | 6.86 | 5.82 | 11.63 |
| | 1000 | 25 | 100 | 4.65 | 5.02 | 10.03 |
| | 1000 | 10 | 100 | Very Small | NC[b] | NC[b] |
| $k/w$ | 1000 | 100 | 1 | 48.00 | CBC[a] | CBC[a] |
| | 1000 | 100 | 10 | 20.57 | CBC[a] | CBC[a] |
| | 1000 | 100 | 100 | 11.08 | 7.23 | 14.46 |
| | 1000 | 100 | 1000 | 8.47 | 6.37 | 12.74 |
| | 1000 | 100 | 10000 | 8.47 | 6.37 | 12.74 |

Notes: [a] CBC- Cannot be calculated due to the term within the square root in Eq. (5) being negative; [b]NC-Not calculated.

spacing decreases with increasing $k$ / $w$. This is because sufficient shear stresses to fracture the pavement can be generated from a smaller length of interface when the shear stiffness is high. Furthermore, the crack spacing plateaus with increasing $k/w$, after which a further increase in $k$ / $w$ does not change crack spacing. This is because the size of the zone where spring resistance is in place, i.e. region where $R_x(x) = ku_x < R_x^f$, becomes small beyond a certain magnitude of $k$ / $w$. Then, shear stresses outside this zone dominate the onset of fracture and are constant and take the value $R_f$. Timm et al.'s estimates for the bounds of average crack spacing cannot be determined when $k$ / $w$ = 1 and 10 MPa because the term within the square root in Eq. (5) becomes negative. The incorporation of fracture properties in the modelling or the unreasonably small shear stiffness assigned may have taken the outcome beyond the range admissible to the analytical model used by Timm et al.

### *Sensitivity to $w_c$, ResidualLlimit*

Results from the numerical modelling are presented in Table 2. This is the most critical parameter identified in this paper, as this has been ignored as a criterion in previous studies such as by Timm et al. [8]. The average crack spacing obtained when $w_c$ = 10 μm was so small that adequate potential crack pathways could not be provided, similar to the sensitivity analysis on $S$ with $S$ = 100 kPa. It is apparent from Table 2 that increase of $w_c$ has caused an increase in crack spacing. This is to be expected, as an increase of $w_c$ causes the fracture energy to increase, causing an increase in the resistance to fracture. Thus it is apparent that $w_c$ does have an effect on crack patterns obtained, and ignoring it when modelling analytically or numerically may lead to erroneous results. This parameter can be measured experimentally through methods such as digital image correlation and/or through extraction of local fracture model and properties using inverse analysis (Aragao and Kim [30], Shen and Paulino [31]). Thus, in general, it is unadvisable to depend on models which rely solely on a criterion as normal stresses reaching tensile strength to assume fracture. Adequate strain energy must be present to be released to accommodate the energy required to create a fractured surface.

## Numerical Mdelling of a Pvement Sction at the Minnesota Road

### Description of Field Test

It is important to compare observations made in the field with results from the numerical model. A statistical analysis of crack spacing observed at a testing facility was carried out by Timm [32]. The asphalt thickness of these pavement sections was 0.15 m. The average crack spacings obtained in three cells mapped individually were 12 m, 8 m and 13 m, with standard deviations of 4.88 m, 4.27 m, and 8.23 m respectively. Some variability in crack distribution is to be expected because of the presence of flaws and weak regions in the asphalt. Such spatial random variability can be accommodated in numerical modelling as detailed in Amarasiri and Kodikara [33]. The granular base was found to be wet and frozen, which accounts for its high stiffness. The maximum $\Delta T$ at the time cracks appeared was found to be -15°C and $T_2 - T_1$ was measured to be 8°C.

### Material Properties Used in Modelling the Field Test

Most of the material properties are as reported in Table 1. The stiffness of the base the pavement rests on has to be replaced by a Winkler shear stiffness at the interface. The base in the numerical model can then be made very stiff, because its pliability has been lumped at the interface. Timm et al. suggest the equation:

$$\frac{k}{w} = \frac{4\pi E_b}{\lambda(1-\nu_b{}^2)(4+3\log 3)}\left(1+\frac{(4+3\log 3)(1-\nu_b)\lambda}{8\pi h_b}\right) \tag{10}$$

to determine an equivalent $k/w$. When the values reported by Timm et al. [8] were substituted into the above equation, $k$ / $w$ =

7.25 GPa/m was obtained and was used in the numerical modelling. It should be noted here that the base, pavement and base-pavement interface can be readily modelled numerically using the current methodology. However, Timm et al.'s approximation given by Eq. (10), and a stiff base are used in order to replicate the geometry used in the analytical model. Two values of $w_c$ were investigated, i.e. 100 µm and 10 µm.

### Results from Modelling of Field Test

Once the tensile strength is fixed in a cohesive softening law, the fracture energy is solely dependent on $w_c$. Since the literature review suggested a large range of possible fracture energies of asphalt concrete at low temperatures, it was thought more appropriate to investigate which value of $w_c$ would yield reasonable results. When $w_c$ was assigned a value of 100 µm and 10 µm, an average crack spacing of 13.1 m and 11.1 m respectively were obtained in the modelling of the field experiment, while the average crack spacing in the three cells in the field test is about 11.0 m. A $w_c$ of 100 µm corresponds to a fracture energy of about 98 N/m, which is somewhat lower than the range obtained in the literature review. The incorporation of more accurate material properties, particularly the fracture properties of asphalt concrete, and creep analyses may lead to a better fit to field data being generated. Furthermore, the shear stiffness of the base-pavement interface needs to be accurately determined, as the sensitivity analyses showed that it has a significant impact on the modelling outcome. The shear stiffness and strength of the base could possibly be obtained by laboratory experimentation. Similarly, the necessary data was obtained from laboratory experiments for numerically modelling the interface between soil and its base by Amarasiri and Kodikara [7]. As the material properties change significantly during cooling, the incorporation of such changes in the analyses could lead to much better replication of field behaviour. Many previous numerical models of thermal cracking have demonstrated that the exact matching of field data is difficult to attain, eg. Marasteanu [1].

## Conclusions and Recommendations

It is possible using the distinct element method and cohesive cracks to model thermal cracking of pavements. Though a Winkler model was used to account for the shear stresses generated at the base on the shrinking slab and the pliability of the base, it is possible to use the same methodology with experimentally determinable properties for interfaces, base and sub-base materials. A parametric study was carried out to determine the sensitivity of the generated cracks to three key material properties i.e. the tensile strength of the pavement, the shear stiffness of the pavement-base interface, and the fracture energy of the pavement. The following could be concluded:

- The spacing of cracks increased with increasing tensile strength.
- Within a range of shear stiffnesses of the pavement-base interface there was decreasing crack spacing with increasing shear stiffness.
- When the shear stiffness was increased beyond this range, no effect on crack spacing was observed.
- An increase of crack spacing was observed with increasing fracture energy. This is because increasing fracture energy of pavement materials signifies an increase in fracture resistance. The results obtained by numerical modelling were consistent with an analytical model available in literature.
- The thermal cracking during a cooling event observed at an experimental pavement section could be approximately replicated using this methodology.

The methodology proposed in this paper and the following recommendations could make the numerical modelling closer to representative of the physical behaviour of asphalt pavements.

- Vehicular loading could be included in the analyses.
- The creep properties of asphalt are well documented (eg. [1, 20, 34] ) and could be included in the numerical analyses.
- The assumption has been made that the stresses in the pavement were negligible at the start of the cooling event in this work and many other previous studies (eg: [8]). This may not be necessarily so, and is worthy of further study, perhaps by field experimentation.
- The variation of pavement temperature with time can be numerically modelled to take into account the expected thermal behaviour of surrounding materials. UDEC has the capacity to carry out both creep and thermal flow analyses, and is highly suited for such numerical modelling.
- The exact cohesive law can be experimentally determined as done by Amarasiri et al. [27], Amarasiri and Kodikara [26] and Bolander and Sukumar [35] for respectively clay, an artificial soft rock, and concrete. It may well turn out that a bi-linear softening law as shown in Fig. 2 is more appropriate for asphalt concrete as is certainly the case for Portland cement concrete [25]. This has to be investigated by further laboratory fracture experimentation.
- Since it is well documented that many pertinent asphalt concrete properties change significantly with cooling, such changes have to be accommodated in the model. Amarasiri and Kodikara [7] discuss the implementation of such changes in a numerical model of desiccation cracking. Since the tensile strength is also evolving with cooling, it is necessary to accommodate the softening law changing while a cohesive crack is semi-open. Amarasiri and Kodikara [7] address this issue, and the same methodology could be used for future numerical models of thermal cracking of pavements. The presence of the programing language FISH in UDEC facilitates the incorporation of the above features efficiently in a numerical model of thermal cracking of asphalt pavements.

Modelling including the above features are being planned by the authors as further experimental data becomes available.

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