Technical Paper

ISSN 1997-1400 Int. J. Pavement Res. Technol. 8(4):276-282

A New Failure Criterion for Asphalt Mixtures Under Fatigue Loading

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Abstract: In this paper a new criterion for determination of cycle number to failure of asphalt mixture at test in controlled strain mode is proposed. The proposed criterion is based on equality of total dissipated energy and total dissipated pseudo strain energy at failure. The proposed failure criterion is presented in three forms: in energy balance form, in terms of linear viscoelasticity theory and continuum damage mechanics. A new criterion applicability is shown by testing of hot mix asphalt mixture samples on four point bending equipment in controlled strain mode. Comparison of loading cycle numbers to failure of asphalt mixture obtained using the proposed and other often used criteria are carried out.

DOI: 10.6135/ijprt.org.tw/2015.8(4).276

***Key words**: A new failure criterion; Controlled strain mode; Hot mix asphalt; Fatigue; Test on four point bending equipment.*

Introduction

Fatigue cracking of asphalt mixture is one of the main types of flexible pavement failure [1-3]. Therefore, in flexible pavement design the evaluation of fatigue strength of asphalt mixture is important. Historically, evaluation of asphalt mixture fatigue strength in laboratory is carried out firstly [4-6], and then by coefficient multiplication that called "shift factor" is transferred to natural condition [7-9]. In other words, reliable laboratory evaluation method of asphalt mixture fatigue strength is base for flexible pavement strength determination. There are many laboratory determination methods of asphalt mixture fatigue strength. But, nowadays asphalt mixture test method on four point bending equipment is mainly used [2, 10-12]. In 1993 this test method was recommended by the American Strategic Highway Research Program (SHRP) for evaluation of asphalt mixture fatigue strength [13]. Subsequently it became a standard and frequently used method in the USA [14], Europe [15], Australia [16], New Zealand [12, 17], China [11], India [18, 19] and in many other countries around the world.

Important issue in laboratory testing of asphalt mixture on fatigue strength is exact determination of loading cycles number N_f, at which failure of testing sample of asphalt mixture happens. Currently, there are several criteria for determination of asphalt mixture fatigue strength N_f: reduction by 50 % of initial stiffness, reduction by 90 % of initial stiffness, ratio of dissipated energy change (RDEC), reaching maximum phase angle, complete failure of sample and etc. [20-24].

Asphalt mixture is viscoelastic material [2, 10]. Behavior of linear viscoelastic material under periodic loading (as in test on four point bending equipment) is characterized by two independent characteristics - complex modulus E^* and phase angle δ [25]. Each of the above listed criteria for asphalt mixture fatigue strength evaluation uses limit value of stiffness (absolute value of complex modulus) or phase angle. None of them does not consider both of these characteristics of asphalt mixture as viscoelastic material simultaneously.

In the present work a new criterion for asphalt mixture fatigue strength evaluation at the same time taking into account for stiffness E^* variation (decrease) and phase angle δ variation (increase) with growth of loading number at test in controlled strain mode is proposed.

Used Materials

Bitumen

In this paper bitumen of grade BND-90/130 has been used, meeting the requirements of Kazakhstan standard ST RK 1773. Performance grade of the bitumen is PG 64-40 [26]. Basic standard indicators for bitumen are shown in Table 1. Bitumen has been produced by Pavlodar processing plant from crude oil of Western Siberia (Russia) by direct oxidation.

Asphalt Mixture

Dense graded hot mix asphalt of type B under Kazakhstan standard ST RK 1225 has been prepared with the use of aggregate of fractions 5-10 mm (20%), 10-15 mm (13%), 15-20 mm (10%) from Novo-Alekseevsk rock pit (Almaty region), sand of fraction 0-5 mm (50%) from the plant “Asphaltconcrete-1” (Almaty city) and activated mineral powder (7%) from Kordai rock pit (Zhambyl region).

Bitumen content in hot mix asphalt is 4.8 % by weight of dry mineral material. Basic standard indicators for aggregate and hot mix asphalt are shown in Tables 2 and 3 respectively. Asphalt mixture grading curve for its mineral part is shown in Fig. 1.

Test Methods

Sample Preparation

Samples of hot mix asphalt in a form of rectangular beam with length of 380 mm, width of 50 mm and height of 50 mm were

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Note: Submitted February 4, 2015; Revised March 16, 2015; Accepted April 5, 2015.

Table 1. Characteristics of Bitumen on Standard ST RK 1373.

Indicator	Measurement Unit	Test Method	Standard Requirements	Value
Penetration, 25°C, 100 gr., 5 s	0.1 mm	ASTM D 5	91-130	98
Penetration Index PI	-	ST RK 1373	-1.0… +1.0	-0.96
Ductility:	cm	ASTM D 113		
- 25°C			≥ 65	139
- 0°C			≥ 4.0	5.5
Softening Point	°C	ASTM D 36	≥ 43	45.3
Fraas Point	°C	DIN EN 12593	≤ -20	-24.6
Dynamic Viscosity, 60°C	Pa·s	DIN EN 12596	≥ 75	174.2
Kinematic Viscosity	mm^2/s	ASTM D 445	≥ 180	409.0

Table 2. Characteristics of Aggregate on Standard ST RK 1284.

Indicator	Measurement Unit	Test Method	Standard Requirements	Value	
				Fraction 5-10 mm	Fraction 10-20 mm
Average Density	g/cm^3	ASTM C 127	-	2.55	2.62
Elongated Particle Content	%	BS 812.105.2	≤ 25	13	9
Clay Particle Content	%	ST RK 1213	≤ 1.0	0.3	0.2
Bitumen Adhesion	-	ST RK 1218	-	satisfactory	satisfactory
Water Absorption	%	ASTM C 127	-	1.93	0.90

Table 3. Characteristics of Hot Mix Asphalt on Standard ST RK 1225.

Indicator	Measurement Unit	Test Method	Standard Requirements	Value
Average Density	g/cm^3	ST RK 1218	-	2.39
Water Saturation	%	ST RK 1218	1.5-4.0	2.3
Voids in Mineral Aggregate	%	BS EN 12697	≤ 19	14
Air Void Content in Hot Mix Asphalt	%	BS EN 12697	2.5-5.0	3.8
Voids Filled with Asphalt (Bitumen)	%	BS EN 12697	-	72.9
Compression Strength:	MPa	ST RK 1218		
- 0°C			≤ 13.0	7.3
- 20°C			≥ 2.5	3.4
- 50°C			≥ 1.3	7.3
Water Stability	-	ST RK 1218	≥ 0.85	0.92
Shear Stability	MPa	ST RK 1218	≥ 0.38	0.40
Crack Stability	MPa	ST RK 1218	4.0-6.5	4.1

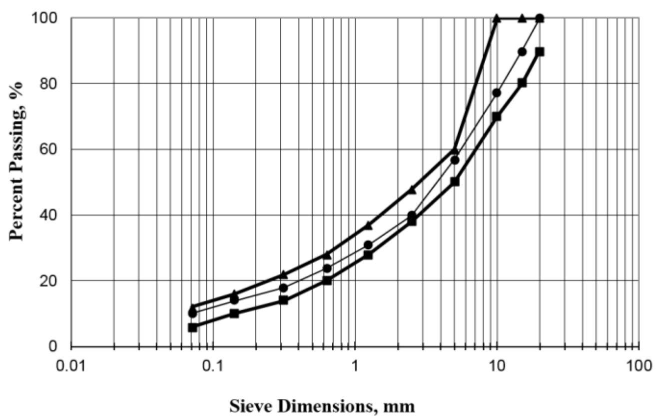

Fig. 1. Asphalt Mixture Grading Curve.

manufactured in the following way. First by, samples of hot mix asphalt were prepared in a form of square slab by roller compactor of Cooper company (United Kingdom, model CRT-RC2S) according to the standard EN 12697-33 [27]. Then samples in a form of beam were cut from hot mix asphalt slabs. Discrepancies in dimensions did not exceed 2 mm.

Test

Testing of hot mix asphalts samples in a form of rectangular beam was carried out according to the standard EN 12697-24 [15] on the device of Cooper company according to the scheme of four-point bending (4PB beam test, model CRT-SA4PT-BB) in controlled strain mode. Strain values were accepted as follows: ε = 200, 250, 300, 350 and 400 $\mu\varepsilon$. Frequency of loading and test temperature were accepted equal to 10 Hz and 10°C respectively. All samples were tested up to reach 10 % of its initial stiffness (complex modulus).

Development of Failure Criterion

Dissipated Pseudo Strain Energy

In works [21, 28] expressions for dissipated energy due to damage of viscoelastic material were obtained. These expressions are based on the concept of dissipated pseudo strain energy (DPSE) developed in researches of D.N. Little, Y.R. Kim and R.L. Lytton [29, 30].

In works [21, 28] it is shown that total dissipated pseudo strain energy of viscoelastic material in case of fatigue test can be divided into three components related to: 1) complex modulus E^* (stiffness) variation W_{R1}; 2) phase angle δ variation between successive loading cycles W_{R2}; and 3) phase angle δ variation within each separate loading cycle.

Further researches in the present work consider fatigue test of hot mix asphalt in controlled strain mode and assume that dissipated energy caused by phase angle variation within loading cycles is significantly less than dissipated energy, caused by variations of the complex modulus and phase angle between successive loading cycles. In the work [28] it is shown that at test in controlled strain mode the value of dissipated energy caused by phase angle variation within loading cycles does not exceed 5%. Then, for above mentioned case the next dependences are existing [21, 28]:

$$W_{R1} = \frac{1}{2}\varepsilon^2\left(E^*_{VE} - E^*_N\right) \tag{1}$$

$$W_{R2} = \pi E^*_{VE}\,\varepsilon^2 \sin\left(\delta_N - \delta_{VE}\right) \tag{2}$$

where,

ε - applied constant strain;

E^*_{VE} - complex modulus of intact viscoelastic material;

δ_{VE} - phase angle of intact viscoelastic material;

E^*_N - complex modulus of viscoelastic material in cycle N;

δ_N - phase angle of viscoelastic material in cycle N.

The total of dissipated pseudo strain energy in cycle N is determined as the sum of Eqs. (1) and (2):

$$W_R = W_{R1} + W_{R2}$$

$$W_R = \varepsilon^2\left[\frac{1}{2}\left(E^*_{VE} - E^*_N\right) + \pi E^*_{VE}\sin\left(\delta_N - \delta_{VE}\right)\right] \tag{3}$$

A New Failure Criterion Formulation

Total dissipated energy of viscoelastic material at cyclic loading in each cycle is calculated according to [25]:

$$W = \pi\sigma\varepsilon\sin\delta_N \tag{4}$$

where

σ - stress;

ε - strain;

δ_N - phase angle.

Product $\sigma\varepsilon$ for controlled strain mode is replaced with equivalent product $E^*_N\,\delta^2$ and put it in the Eq. (4):

$$W = \pi E^*_N\,\varepsilon^2\sin\delta_N \tag{5}$$

At test in constant strain mode with increase of loading number the complex modulus decreases, and phase angle increases. As results of numerous experiments have showed the value of total dissipated energy decreases with increase of loading number. In initial cycles material deformation conditions are close to deformation of pure viscoelastic material, because damage in the tested material is absent or very little. With increase of cycles number amount of cumulative damage increases, and ability of the material to deform as viscoelastic material decreases. And in some cycle material completely loses its ability to deform as viscoelastic material, and allocated amount of energy characterizes condition of its final failure.

Now, let us formulate a new failure criterion: “Failure of viscoelastic material at test in cyclic controlled strain mode occurs in such cycle, in which the value of dissipated pseudo strain energy is equal to the value of total dissipated energy”. Otherwise, failure of viscoelastic material occurs when the following condition is true:

$$W = W_R \tag{6}$$

We will substitute Eqs. (3) and (5) in Eq. (6):

$$\pi E^*_N\,\varepsilon^2\sin\delta_N = \varepsilon^2\left[\frac{1}{2}\left(E^*_{VE} - E^*_N\right) + \pi E^*_{VE}\sin\left(\delta_N - \delta_{VE}\right)\right] \tag{7}$$

We will divide right and left parts of Eq. (7) into product $\pi\varepsilon^2$:

$$E^*_N\sin\delta_N = \frac{1}{2\pi}\left(E^*_{VE} - E_N\right) + E^*_{VE}\sin\left(\delta_N - \delta_{VE}\right) \tag{8}$$

The left part of the Eq. (8), i.e. product $E^*_N\sin\delta_N$ represents the loss modulus of viscoelastic material $E^{//}_N$ [25].

The above proposed failure criterion can be formulated in the following manner, according to the Eq. (8): Failure of viscoelastic material at test in cyclic controlled strain mode occurs in such cycle, in which the loss modulus $E^{//}$ reaches its critical value $E^{//}_{Ncr}$. Otherwise, failure occurs when the following condition is true:

$$E^{//}_N = E^{//}_{Ncr} \tag{9}$$

If we divide right and left parts of the Eq. (8) into $E^{*/}_N\sin\delta_N$, we will obtain:

$$1 = \frac{\frac{1}{2\pi}\left(E^*_{VE} - E^*_N\right) + E^*_{VE}\sin\left(\delta_N - \delta_{VE}\right)}{E^*_N\sin\delta_N} \tag{10}$$

The analysis of this expression shows that in initial loading cycles, when $E^*_N \approx E^*_{VE}$ and $\delta_N \approx \delta_{VE}$, its right part is close to zero, and at failure is equal to unity. Therefore we call the right part of the Eq. (10) as damage of viscoelastic material and denote by the letter D_N:

$$D_N = \frac{\frac{1}{2\pi}\left(E^*_{VE} - E^*_N\right) + E^*_{VE}\sin\left(\delta_N - \delta_{VE}\right)}{E^*_N\sin\delta_N} \tag{11}$$

Comparing Eqs. (7), (8), and (11), one can noted that the proposed failure criterion of viscoelastic material in the Eq. (7) is presented in energy balance form, in the Eq. (8) it is written in terms

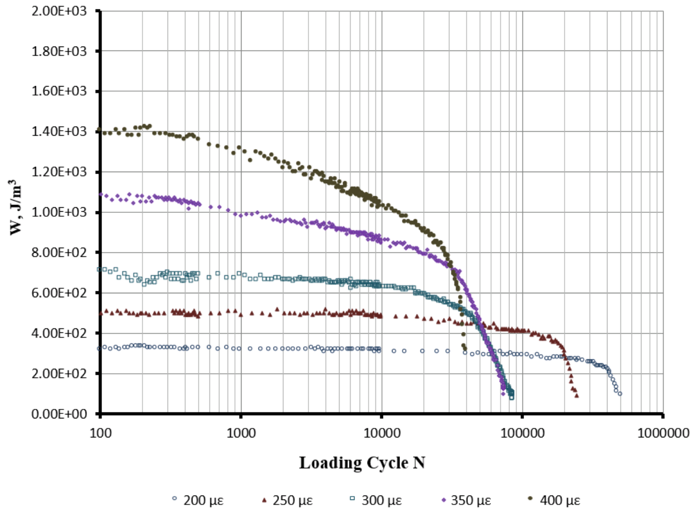

Fig. 2. Total Dissipated Energy versus Loading Cycle Number.

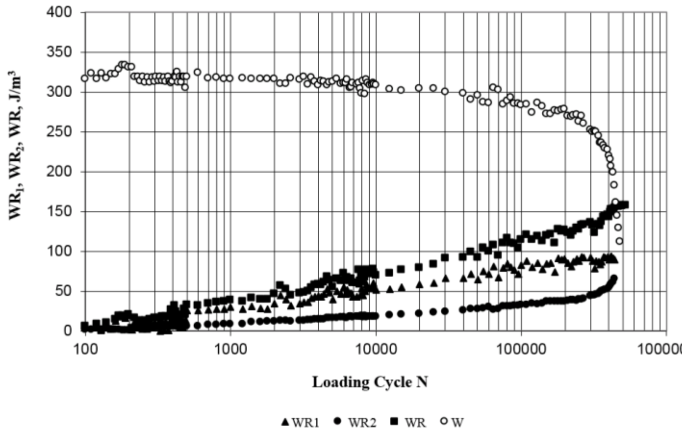

Fig. 3. Dissipated Pseudo Strain Energies W_{R1}, W_{R2} and W_R versus Loading Cycle Number at Constant Strain 200 με.

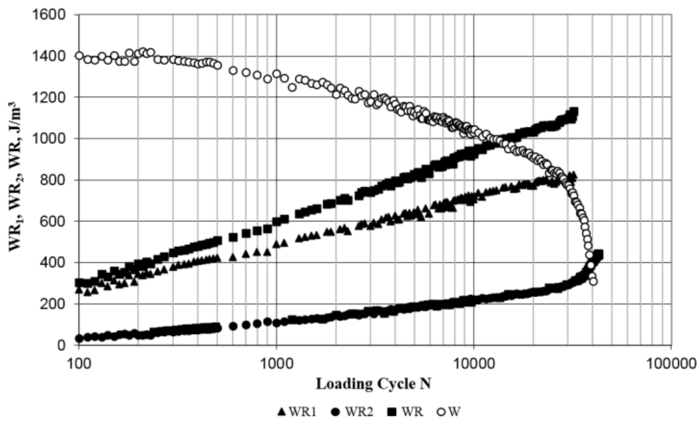

Fig. 4. Dissipated Pseudo Energies W_{R1}, W_{R2} and W_R versus Loading Cycle Number at Constant Strain 400 με.

of linear viscoelasticity theory, and in the Eq. (11) it is presented in terms of continuum damage mechanics.

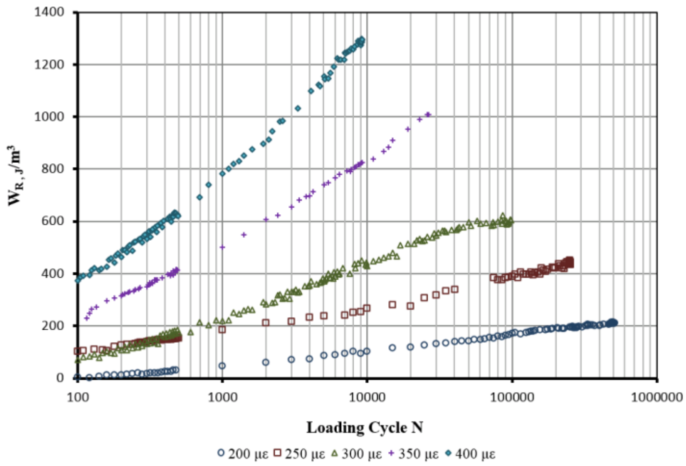

Fig. 5. Total Dissipated Pseudo Strain Energy at Different Constant Strains versus Loading Cycle Number.

Application of the Proposed Failure Criterion

Determination of Fatigue Durability

In calculations carried out on Eqs. (1), (2), (3), (8), and (11), the complex modulus E^*_{VE} and phase angle δ_{VE} values, corresponding to intact, viscoelastic condition of asphalt mixture were determined by test results of asphalt mixture beams at constant strain ε = 50 με. By results of these tests it was accepted, that E^*_{VE} = 6097.6 MPa and $\delta_{VE} = 24.54^{o}$.

In Fig. 2 diagrams of total dissipated energy per asphalt mixture unit volume versus loading cycle number, based on the Eq. (4) using the experimental data are shown. It is seen that with increasing of loading cycle number the total dissipated energy decreases due to combined effect of asphalt mixture stiffness and phase angle. In Figs. 3 and 4 diagrams of total dissipated energy W, dissipated pseudo strain energies, related to the complex modulus variation W_{R1} and phase angle variation W_{R1}, total dissipated pseudo strain energy W_R at constant strains 200 με and 400 με versus loading cycle number are presented. Fig. 5 shows diagrams of total dissipated pseudo strain energy variation versus loading cycle number. As you can see, total dissipated pseudo strain energy W_R in semi-logarithmic coordinate increases almost on linear dependence with increasing of loading cycle number. Dissipated pseudo strain energy value related to phase angle increase W_{R1} is approximately two times larger than dissipated pseudo strain energy related to stiffness decrease W_{R2}. In these figures it is possible to determine loading cycle number corresponding to failure of asphalt mixture samples. According to the proposed criterion (Eq. (6)) they are found as cycle numbers corresponding to crossings of total dissipated energy W and total dissipated pseudo strain energy W_R lines. Cycle numbers to failure of asphalt mixture samples determined by above indicated way at different constant strain are shown in Table 4.

As it was shown in the previous section, cycle number to failure of asphalt mixture can be calculated also by the condition (Eq. (8)). In Fig. 6 the upper curve shows the loss modulus $E^*_N \cdot \sin\delta_N$ variation and the bottom line corresponds to the expression in the right-hand side of the Eq. (8) at constant strain ε = 350 με. The

Table 4. Cycles Number to Failure of Asphalt Mixture Samples.

Test Constant Strain ε, με	200	250	300	350	400
Cycles Number to Failure N_f	453 000	197 250	42 500	32 720	14 200

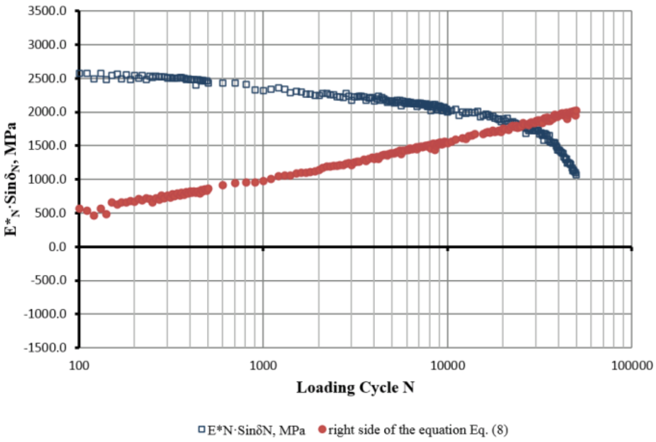

Fig. 6. Determination of Cycles Number to Failure According to Condition (8) at Constant Strain 350 με.

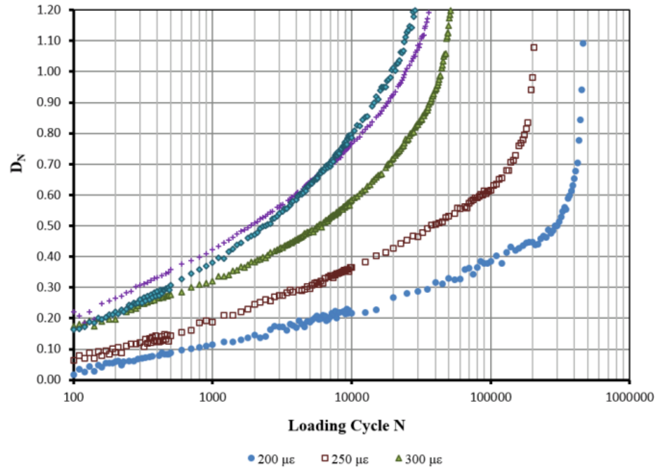

Fig. 7. Damage of Hot Mix Asphalt versus Loading Cycle Number at Different Constant Strains.

intersection point corresponds to loading cycles number to failure N_f.

In our opinion, one of the most logically appropriate attempts to explain materials failure is the use of continuum damage mechanics base. Rabotnov and Kachanov introduced the damage concept, which theoretically varies from 0 to 1 [31]. Damage equal to 0 corresponds to initial undamaged material condition. Material condition, characterized as its complete failure, corresponds to damage amount that is equal to 1.

In Fig. 7 diagrams of asphalt mixture damage at different constant strain of test built by the Eq. (11) using the experimental data are shown. It is seen that in semi-logarithmic coordinates damage nonlinearly depends on loading cycle number. Damage nonlinearity increases with loading cycle number and test constant strain value.

It should be noted that as Eqs. (8) and (11) obtained from the condition (Eq. (6)) purely analytical way, values of loading cycles number to failure of asphalt mixture obtained on these three expressions are the same and equal to the values presented in Table 4.

Comparison of the Proposed Criterion with Others

The interest is comparison of loading cycles number to failure of asphalt mixture determined according to the proposed criterion and by often used others. As already noted, in the case of asphalt mixture test on fatigue strength at controlled strain mode commonly used criterion for determining failure point is reduction by 50 % of initial stiffness. This criterion is included in standards of the USA [14] and Europe [15]. Another frequently used criterion is the peak value of the following parameters combination (normalized stiffness):

$$\frac{N \cdot E_N^*}{E_{50}^* \cdot 50} \qquad (12)$$

where N - loading cycles number;

E_N^* - complex modulus in loading cycle N;

E_{50}^* - initial complex modulus defined in 50th loading cycle;

50 - loading cycle number at which the initial modulus is defined.

This criterion is also included in the current USA standard [32] and is often used by researchers in tests of bitumen binders [33] and asphalt mixtures [20, 21, 29, 34].

In Fig. 8 diagrams of asphalt mixture normalized stiffness variation depending on loading cycle number at constant test strains 200 με and 250 με are shown. It is clear that in both diagrams after reaching peak value of normalized stiffness of asphalt mixture it decreases sharply. It is accepted that this peak value corresponds to failure point N_f.

Loading cycles numbers to failure of asphalt mixture determined using the proposed criterion (N_{fWR}), initial stiffness reduction by 50 % criterion (N_{f50}) and peak value of normalized stiffness criterion (N_{fE}) are shown in Fig. 9. It is possible to notice some differences in values of N_f determined using different criteria. The values of these differences are given in Table 5. It is seen that the values of N_f determined using the proposed criterion at small constant test strains (200 and 250 με) are higher than those determined by initial stiffness reduction by 50 % criterion, and at large test strains (300, 350 and 400 με), on the contrary, are less. This difference increases from 4.9 % at strain of 200 με to 27.2 % at strain 400 με. Differences also take place between values of N_f determined using peak value of normalized stiffness criterion and initial stiffness reduction by 50% criterion. This difference has the maximum value at the most test constant strain 400 με and equal to 61.5 %. Here it is important to note that at low strain values, which are close to strain values in asphalt mixture pavement layers during trucks passage, all three of the considered criteria give similar results. But in practice usually correlation dependence equation is used, coefficients of which are calculated using tests results including at large strains.

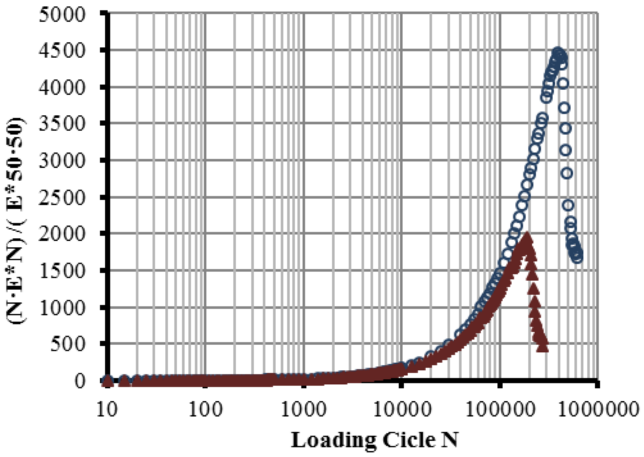

Fig. 8. Normalized Stiffness of Hot Mix Asphalt Versus Loading Cycle Number at Constant Strains 200 µε and 250 µε.

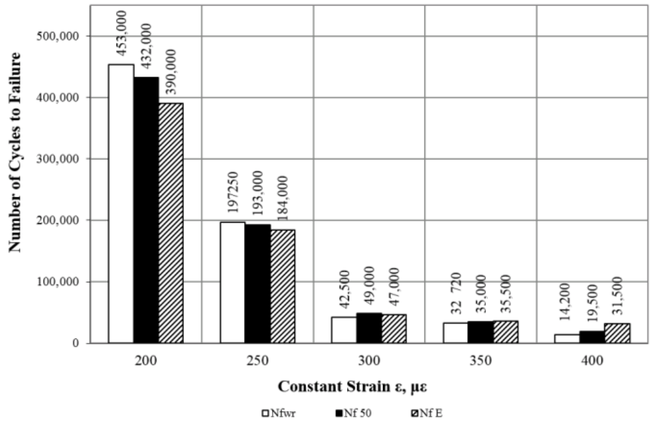

Fig. 9. Number of Cycles to Failure of Hot Mix Asphalt Determined Using Different Criteria.

Table 5. Differences between Failure Points of Asphalt Mixture at Different Values of Test Constant Strain.

Difference between Failure Points	Difference value, %				
	200 µε	250 µε	300 µε	350 µε	400 µε
Difference between N_{fWR} and N_{f50}	+4,9	+2,2	-13,3	-6,5	-27,2
Difference between N_{fE} and N_{f50}	-9,7	-4,7	-4,1	+1,4	+61,5

This possibly leads to significant differences in values of N_f determined by different criteria. Therefore, it is necessary to choose the most reasonable of them.

Conclusion

In this paper a new criterion for determination fatigue strength of asphalt mixture is proposed. According to this criterion, under cyclic loading in controlled strain mode the asphalt mixture failure occurs in such cycle, in which total dissipated energy is equal to total dissipated pseudo strain energy.

Proposed failure criterion is presented in three forms: in energy balance form, in terms of linear viscoelasticity theory and in terms of failure mechanics.

Results of hot mix asphalt samples test using bitumen of grade 90-130 on four point bending equipment at temperature of 10°C and loading frequency of 10 Hz at different constant strain showed applicability of the proposed criterion. Comparison of fatigue strength determination results on the proposed criterion, by initial stiffness reduction by 50% criterion and criterion of normalized stiffness peak value showed that for the tested hot mix asphalt cycles numbers to failure at strains close to real in pavement structure are similar.

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