Modeling HMA Bulk Specific Gravities: A Theoretical and Experimental Investigation

Filippo G. Pratico¹, Antonino Moro², and Rachele Ammendola²

Abstract: Bulk specific gravity, Gₘₘₐₜ, is of outstanding importance in bituminous mixes. Life cycle costs, contract requirements, and Quality Control/Quality Assurance procedures are strongly dependent on the effective Gₘₘₐₜ obtained by suitable design and construction, and compliance with a number of boundary conditions. There are many ways to estimate Gₘₘₐₜ, and within-method variance is appreciable. The “transportability” of a Gₘₘₐₜ determined according to a given standard to another standard is therefore a critical issue. Our objective was to study relationships between Gₘₘₐₜ values determined using a given standard to Gₘₘₐₜ estimates calculated by applying other standards. We derived a model, and conducted experiments using five different methods. The model evaluates the elementary volumes that contribute to differences among the five considered methods. A tendency toward a common maximum value is assumed. The results are analyzed and interpreted, and the conceptual framework validated. Useful relationships between specific gravities determined by different methods are derived and analyzed. Power-asymptotic curve analysis yielded an algorithm well-grounded in logic to solve the issue of Gₘₘₐₜ transportability.

Key words: Bulk specific gravity; Hot mix asphalts; Osculatory volume.

Problem Statement

Bulk specific gravity, Gₘₘₐₜ, measures the specific gravity of a compacted Hot Mix Asphalt (HMA) sample. Gₘₘₐₜ is of outstanding importance in evaluation of bituminous mixes. Life cycle costs, contract requirements, and Quality Control/Quality Assurance procedures are strongly affected by the effective Gₘₘₐₜ values obtained by suitable design and construction, and application of appropriate boundary conditions.

Although specific gravity is important, there are many different ways to determine Gₘₘₐₜ, and many authors have attempted to find correlations among results obtained using the various procedures [1-8]. In this study, we construct a model to evaluate differences between procedures and seek information on the transportability of Gₘₘₐₜ values between such procedures.

The model is based on a conceptual framework in which variations between Gₘₘₐₜ values of a given sample originate from a difference in the osculatory external volume (V₀) of the sample and the mix bulk volume (Vₘₘₐₜ), termed the “roughness volume” R (Fig. 1). In the model, R corresponds to “extended” roughness (related to voids), Vₘₘₐₜ is the mix bulk volume, V₀ is an osculatory (external) volume, using the terms of the Dimensional Method, Rₕₜ is Bottom Surface Roughness, Rₜₙ refers to Lateral Surface Roughness, whereas Rₜₜₜₜ corresponds to Upper Surface Roughness (Fig. 1).

Thus,

\[ V_{mb} = V_0 - R \] (1)

where

\[ R = R_{bs} + R_{ls} + R_{un} \] (2)

Each of these three components (volumes), the Rᵢ values, can be split into three sub-components: Positive (P), Negative (N), and HyperNegative (HN):

\[ V_{mb} = V_0 - R_{bs} - R_{ls} - R_{un} = V_0 - [R_{bsp} + R_{bss} + R_{bsh}] \]

\[ - [R_{usp} + R_{usn} + R_{ush}] - [R_{lsp} + R_{lsh} + R_{lsh}]. \] (3)

Theoretically, the differences among measurement methods can be explained by the following equation:

\[ V_{mb} = V_0 - [R_{usp} + R_{usn} + R_{ush}] - [R_{bss} + R_{usn} + R_{lsh}] \]

\[ - [R_{ush} + R_{ush} + R_{lush}] \] (4)

It results in:

\[ G_{mb} = \frac{P_{mb}}{V_{mb}^w} = F(V_0, R) \] (5)

where Gₘₘₐₜ is the bulk specific gravity, Pₘₘₐₜ represents the (dry) mass of the bituminous mix, γₜ is the density of water at the same temperature, and F is a function. When R is small, the internal voids tend to be negligible, and then:

\[ \lim_{R \to 0} V_{mb} = V_0 \] (6)

\[ \lim_{R \to 0} G_{mb} = G_{mm} \] (7)

where Gₘₘₐₜ is the maximum theoretical specific gravity:

\[ G_{mm} = \frac{P_b + P_a}{V_b + V_a} \] (8)

In this equation Pₖ and Vₖ are the mass and volume of asphalt binder, respectively, whereas Pₐ and Vₐ refer to aggregates. Thus,
Eqs. (6) and (7) suggest pursuit of the $G_{mb}$ transportability objective by consideration of the tendency toward restriction of the variability zone as $G_{mb}$ increases. The approach of Eqs. (4-8) can be integrated with an analysis focused on solid versus fluid volumes (Fig. 2 shows the relevant schematic for the case saturated surface dry (SSD) versus paraffin-coated (FIN)):

$$V_{mbSSD} = \alpha_1 + \sum_{i=3}^{K} \alpha_i$$  \hspace{1cm} (9)

$$V_{mbFIN} = \sum_{i=1}^{\xi} \alpha_i$$  \hspace{1cm} (10)

where all $\alpha_i$ values are volumes and $\alpha_i = \alpha'_i$ represents the solid volume (aggregate + bitumen), whereas each $\alpha_i, \forall i \neq 1$, corresponds to a fluid (such as air, water, or paraffin) volume. For mixes that are not too “open”, $K$ may be assumed to be 4 and $\xi$ to be 5; $\alpha_0 + \alpha_5$ represent paraffin.

In this case, and using Eqs. (9) and (10) above, the following equations are derived:

$$V_{mbSSD} = \alpha_1 + \alpha_5 + \alpha_4$$  \hspace{1cm} (11)

$$V_{mbFIN} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$$  \hspace{1cm} (12)

$$V_{mbFIN} = V_{mbSSD} + \alpha_2 + \alpha_5$$  \hspace{1cm} (13)

$$v_{mbFIN} > v_{mbSSD}$$  \hspace{1cm} (14)

From Eqs. (5) and (14) it follows that:

$$G_{mbSSD} < G_{mbFIN}$$  \hspace{1cm} (15)

Moreover, if $\gamma_a = \alpha_5 \gamma_a$ and $\gamma_w = \alpha_5 \gamma_w$, where $\gamma_a$ represents the density of water, the following equations are obtained:

$$\frac{1}{G_{mbFIN}} = \frac{1}{G_{mbSSD}} + \frac{\alpha_2 + \alpha_5}{P_{mb}}$$  \hspace{1cm} (16)

$$\frac{100}{G_{mbFIN}} = \frac{100}{G_{mbSSD}} + \frac{\alpha_2 + \alpha_5}{P_{mb}}$$ \hspace{1cm} (17)
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\[ \Delta V_{SSD-FIN} = \frac{\alpha_2 - \alpha_1}{P_{mb}} \times 100 = \frac{G_{mb,SSD} - G_{mb,FIN}}{G_{mb,FIN}G_{mb,SSD}} \]  \hspace{1cm} (18)

where \( \Delta V_{SSD-FIN} \) is the percentage difference between the “specific” volumes of SSD and FIN. Similarly it is possible to define \( \Delta V_{FIN-COR}, \Delta V_{COR-FILM}, \) and \( \Delta V_{FILM-DIM} \) where COR, FILM, and DIM are three different methods explained below.

**Experimental Plan**

The mixes tested are described below, and the procedures used are summarized in the following section.

**Mixes**

A total of 205 mixes were tested and five methods used for \( G_{mb} \) determination.

DGFCs (Dense-Graded Friction Courses) and PEMs (Porous European Mixes) were obtained (Fig. 3 and Table 1). For each set, a subset of specimens was used in composition analyses for asphalt binder content (CNR [Consiglio Nazionale delle Ricerche] n.38/73, EN [European Standard] 12697-1:2006), aggregate grading (CNR n.43/53, EN 12697-2:2008), and aggregate apparent specific gravity (CNR n. 63/78, EN 12697-5:2008) (Fig. 3 and Table 1). Fig. 3 shows aggregate gradations and Superpave requirements. NMAS is the Nominal Maximum Aggregate Size. Note that DGFCs generally conform to Superpave requirements (some Through Restricted Zone (TRZ), and some Below Restricted Zone (BRZ)). On the contrary, as is well known, PEM gradations are of the BRZ type and are outside control points.

![Fig. 3. Gradations.](image)

**Table 1. Composition of the Mixes Tested (Averages).**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Density (g/cm³)</th>
<th>NMAS (mm)</th>
<th>P200 (%)</th>
<th>S (%)</th>
<th>Compaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGFC</td>
<td>5.5</td>
<td>726</td>
<td>10</td>
<td>4.4</td>
<td>31.1</td>
</tr>
<tr>
<td>PEM</td>
<td>5.1</td>
<td>829</td>
<td>19</td>
<td>5.2</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Note: % b = asphalt binder content referred to aggregate weight; \( \gamma_g \) = aggregate apparent density; \( P_{200} \) = filler content; S(%) = Sand content; NMAS = Nominal Maximum Aggregate Size

**Procedures**

Five different methods were considered:
- the dimensional method (DIM);
- four water displacement methods, being i) Saturated Surface Dry (SSD); ii) Paraffin (FIN); iii) Paraffin (FILM); and, iv), the Vacuum Sealing Method, VSD (COR).

In Table 2, algorithms and standards for each indicator are detailed.

The DIM is based on height and diameter/width measurements. In water displacement methods, specimen volume is calculated by weighing in a water bath and outside of the water bath. In the SSD approach the specimen volume is calculated by subtracting the mass of the specimen in water from the mass of a SSD sample. The FIN method determines volume according to the water displacement principle but uses melted paraffin wax for the external sealing (we emphasize: not the filling) of a sample’s internal air voids.

In the FILM method, a thin film is used to wrap the specimen and weights are obtained in and out of water. In the COR method,
specimen volume is determined as in the FILM approach but after shrink-wrapping the sample in a high-quality plastic bag, in a vacuum chamber.

**Correlations**

To pursue our objectives, we considered many possible relationships. Particular attention was paid to the reasons for differences among bulk specific gravity values obtained using different standards. Once the model was formalized, analysis was focused on difference dependence on mix characteristics. The following parameters were taken into account: NMAS = Nominal Maximum Aggregate Size; \( P_{200} \) = filler percentage (%); \( S(\%) \) = sand percentage (%); and \( h(\%) \) = asphalt content.

The following types of fitting curves were considered for each association of standards \( i \) and \( j \):

\[
G_{\text{mby}} = a_y G_{\text{mb}} + b_y \quad \text{(linear)} \tag{19}
\]

\[
G_{\text{mby}} = G_{\text{mb}} + \frac{1}{G_{\text{mb}}} \quad \text{(power, asymptotic)} \tag{20}
\]

\[G_{\text{mby}} = G_{\text{mb}} + a(G_{\text{mm}} - G_{\text{mb}}) \quad \text{(linear, asymptotic)} \tag{21}\]

In these expressions, \( j \) represents the \( j \)-th standard used to measure \( G_{\text{mby}} \) (e.g., FILM, SSD, VSD, or FIN), whereas \( i \) represents the standard used to measure \( G_{\text{mb}} \) and \( a \) is a coefficient.

It was necessary to split Eq. (20) into two subsets, because of different curve behaviors. Thus, curves approached to the line of equality either from above or below, depending on the standards being compared. The subset equations were:

\[G_{\text{mby}} = G_{\text{mb}} + \frac{1}{G_{\text{mb}}} \quad \text{(G_{\text{mby}} approaches to G_{\text{mb}} from above)}
\]

\[G_{\text{mby}} = G_{\text{mb}} - \frac{1}{G_{\text{mb}}} \quad \text{(G_{\text{mby}} approaches to G_{\text{mb}} from below)}
\]

In addition, the parameter \( \beta \) in Eq. (20) was expressed through the following equations:

\[\beta_y = b_y^\text{a} (\%So) + b_y^\text{b} (\%NMAS) + b_y^\text{c} (\%S) + b_y^\text{d} (P_{200}) \quad \text{(a)}
\]

\[\beta_y = b_y^* (\%NMAS) \quad \text{(b)}
\]

\[\beta_y = b_y^* (P_{200} + \%S) \quad \text{(c)}
\]

\[\beta_y = b_y^* (P_{200} + \%S) \quad \text{(d)}
\]

\[\beta_y = b_y^* (\%NMAS) + b_y (P_{200} + \%S) + b_y^* (\%NMAS) \quad \text{(e)}
\]

The corresponding equations are referred to below as Eqs. (20(a)) to (20(e)).

**Results**

Figs. 4 to 8 and Tables 3 to 5 summarize the results. Four main classes of fitting curves were analyzed: linear (Eq. (19)), power-asymptotic (Eq. (20)), power-asymptotic with composition dependence (Eqs. (20(a)) to (20(e))), and linear asymptotic (Eq. (21)).

Eqs. (20) to (21) take into consideration the conceptual framework of Eq. (7), the predicted tendency of all \( G_{\text{mb}} \) values to reach a common asymptotic number \( G_{\text{mm}} \).

The data of Fig. 4 (DGFCs + PEMs) are organized as functions of \( G_{\text{mby}} \). Fig. 5 shows plots in which \( G_{\text{mbFILM}} \) is the independent variable. In Fig. 6 the independent variable is \( G_{\text{mbCOR}} \), whereas Fig. 7 refers to \( G_{\text{mby}} \) values and Fig. 8 to \( G_{\text{mby}} \) data.

Fig. 4 shows the characteristics of convergence from above for all mixes and all specific gravities displayed on the y-axis. Similarly, Fig. 5 shows that when the independent variable is \( G_{\text{mby}} \), then \( G_{\text{mbCOR}} \) approaches to the line of equality from below (dotted line). Similar behavior can be seen in Figs. 6 to 8.

Table 3 contains relevant statistics and (linear) correlations. When the principal statistics are ranked for all mixes (DGFC, PEM):

\[G_{\text{mby}} > G_{\text{mbFILM}} > G_{\text{mbCOR}} > G_{\text{mby}} > G_{\text{mbFILM}} \quad \text{(22)}
\]

In particular, for DGFCs:

\[G_{\text{mby}} = 0.97 G_{\text{mbFILM}} = 0.95 G_{\text{mbCOR}} = 0.93 G_{\text{mbFIN}} = 0.92 G_{\text{mbSSD}}
\]
Similarly, for PEMs:

\[ G_{\text{DIM}}^{\text{DIM}} = 0.97G_{\text{Film}}^{\text{Film}} = 0.92G_{\text{COR}}^{\text{COR}} \quad \text{and for} \quad \text{DGFCs and PEMs:} \]

\[ G_{\text{DIM}}^{\text{DIM}} = 0.96G_{\text{Film}}^{\text{Film}} = 0.92G_{\text{COR}}^{\text{COR}}. \]

The skewness (S) values of bulk specific gravities are always positive for PEMs (more data in the right tail) and often negative for DGFCs (more data in the left tail).

It is important to observe that the dimensional method always shows the lowest skewness values.

As far as the kurtosis, k, of the probability distribution is concerned, most variance usually arises because of infrequent but extreme variations (distributions are usually leptokurtic, k>0). The coefficient of variation (CV) ranges from 2.6 to 5.0%. Note that the DIM shows the maximum value, whereas the VSD method yields low values for both DGFCs and PEMs.

In Table 3, R-square values (linear, Eq. (19)) are listed, for all five standards and three mix classes (DGFC, PEM, and DGFC + PEM).

In practice, the SSD and FIN methods were quite unreliable for

| Table 3. Main Statistics and R-square Values (Two Mix Types x Five Standards). |
|---|---|---|---|---|---|---|---|---|---|
| \( \bar{x} \) | \( \sigma \) | \( S_\text{s} \) | \( k \) | CV = (\( \sigma / \bar{x} \)) (%) |
| | | | | | | | | | |
| **DIM** | 2.041 | 1.840 | 1.851 | 0.076 | 0.081 | 0.093 | -1.280 | 0.706 | 0.838 | 3.720 | 1.67 | 1.112 | 3.7 | 4.4 | 5.0 |
| **Film** | 2.100 | 1.906 | 1.920 | 0.069 | 0.075 | 0.090 | -0.482 | 1.425 | 1.269 | 0.949 | 3.88 | 1.787 | 3.3 | 3.9 | 4.7 |
| **COR** | 2.135 | 1.995 | 2.007 | 0.064 | 0.068 | 0.080 | -0.013 | 1.570 | 1.328 | 1.415 | 4.84 | 2.373 | 3.0 | 3.4 | 4.0 |
| **FIN** | 2.194 | 0.060 | 0.060 | 0.134 | 0.016 | 0.017 | -0.309 | 0.480 | 0.468 | 0.446 | 0.347 | 0.347 | 2.8 | 3.0 | 3.2 |
| **SSD** | 2.220 | 0.057 | 0.057 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 0.464 | 2.6 | 2.6 | 2.6 |

| R² | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **DIM** | 0.861 | 0.859 | 0.891 | 0.864 | 0.819 | 0.851 | 0.818 | 0.818 | 0.739 |
| **Film** | 0.861 | 0.859 | 0.891 | 0.864 | 0.819 | 0.851 | 0.818 | 0.818 | 0.739 |
| **COR** | 0.864 | 0.819 | 0.851 | 0.833 | 0.890 | 0.916 | 0.817 | 0.817 | 0.757 |
| **FIN** | 0.818 | 0.817 | 0.817 | 0.931 | 0.931 | 0.931 | 0.931 | 0.931 | 0.947 |
| **SSD** | 0.739 | 0.757 | 0.757 | 0.947 | 0.947 | 0.947 | 0.947 | 0.947 | 0.891 |
Table 4. Main Results of Regression Analyses.

<table>
<thead>
<tr>
<th>Standard Couple (y / x)</th>
<th>Coefficients</th>
<th>$R^2$</th>
<th>$R^2_{ad}$</th>
<th>Standard Couple (y / x)</th>
<th>Coefficients</th>
<th>$R^2$</th>
<th>$R^2_{ad}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear, Eq. (19)</td>
<td></td>
<td></td>
<td></td>
<td>Power-Asymmetric, Eq. (20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FILM-DIM</td>
<td>$a = 0.861; b = 0.322$</td>
<td>0.891</td>
<td>0.890</td>
<td>FILM-DIM</td>
<td>$\beta_y = 4.489 (+)$</td>
<td>0.899</td>
<td>0.898</td>
</tr>
<tr>
<td>COR - DIM</td>
<td>$a = 0.741; b = 0.631$</td>
<td>0.851</td>
<td>0.851</td>
<td>COR - DIM</td>
<td>$\beta_y = 3.079 (+)$</td>
<td>0.860</td>
<td>0.859</td>
</tr>
<tr>
<td>FIN - DIM</td>
<td>$a = 0.611; b = 0.923$</td>
<td>0.818</td>
<td>0.795</td>
<td>FIN - DIM</td>
<td>$\beta_y = 2.875 (+)$</td>
<td>0.833</td>
<td>0.812</td>
</tr>
<tr>
<td>SSD - DIM</td>
<td>$a = 0.521; b = 1.134$</td>
<td>0.739</td>
<td>0.701</td>
<td>SSD - DIM</td>
<td>$\beta_y = 2.612 (+)$</td>
<td>0.755</td>
<td>0.720</td>
</tr>
<tr>
<td>DIM - FILM</td>
<td>$a = 1.034; b = -0.131$</td>
<td>0.891</td>
<td>0.890</td>
<td>DIM - FILM</td>
<td>$\beta_y = 4.158 (-)$</td>
<td>0.846</td>
<td>0.845</td>
</tr>
<tr>
<td>COR - FILM</td>
<td>$a = 0.841; b = -0.392$</td>
<td>0.916</td>
<td>0.916</td>
<td>COR - FILM</td>
<td>$\beta_y = 3.773 (+)$</td>
<td>0.918</td>
<td>0.918</td>
</tr>
<tr>
<td>FIN - FILM</td>
<td>$a = 0.800; b = -0.509$</td>
<td>0.817</td>
<td>0.802</td>
<td>FIN - FILM</td>
<td>$\beta_y = 3.259 (-)$</td>
<td>0.822</td>
<td>0.807</td>
</tr>
<tr>
<td>SSD - FILM</td>
<td>$a = 0.700; b = 0.745$</td>
<td>0.757</td>
<td>0.735</td>
<td>SSD - FILM</td>
<td>$\beta_y = 2.928 (+)$</td>
<td>0.763</td>
<td>0.741</td>
</tr>
<tr>
<td>DIM - COR</td>
<td>$a = 1.149; b = -0.450$</td>
<td>0.851</td>
<td>0.851</td>
<td>DIM - COR</td>
<td>$\beta_y = 2.719 (-)$</td>
<td>0.853</td>
<td>0.852</td>
</tr>
<tr>
<td>FILM - COR</td>
<td>$a = 1.089; b = -0.226$</td>
<td>0.916</td>
<td>0.916</td>
<td>FILM - COR</td>
<td>$\beta_y = 3.530 (-)$</td>
<td>0.915</td>
<td>0.915</td>
</tr>
<tr>
<td>COR - COR</td>
<td>$a = 0.884; b = -0.291$</td>
<td>0.931</td>
<td>0.925</td>
<td>COR - COR</td>
<td>$\beta_y = 4.196 (+)$</td>
<td>0.932</td>
<td>0.926</td>
</tr>
<tr>
<td>SSD - COR</td>
<td>$a = 0.813; b = 0.468$</td>
<td>0.947</td>
<td>0.942</td>
<td>SSD - COR</td>
<td>$\beta_y = 3.585 (+)$</td>
<td>0.950</td>
<td>0.945</td>
</tr>
<tr>
<td>DIM - FIN</td>
<td>$a = 1.339; b = -0.864$</td>
<td>0.818</td>
<td>0.795</td>
<td>DIM - FIN</td>
<td>$\beta_y = 2.647 (-)$</td>
<td>0.824</td>
<td>0.802</td>
</tr>
<tr>
<td>FILM - FIN</td>
<td>$a = 1.021; b = -0.135$</td>
<td>0.817</td>
<td>0.802</td>
<td>FILM - FIN</td>
<td>$\beta_y = 3.092 (-)$</td>
<td>0.819</td>
<td>0.804</td>
</tr>
<tr>
<td>COR - FIN</td>
<td>$a = 1.054; b = -0.158$</td>
<td>0.931</td>
<td>0.925</td>
<td>COR - FIN</td>
<td>$\beta_y = 4.103 (-)$</td>
<td>0.931</td>
<td>0.925</td>
</tr>
<tr>
<td>SSD - FIN</td>
<td>$a = 0.876; b = 0.294$</td>
<td>0.891</td>
<td>0.881</td>
<td>SSD - FIN</td>
<td>$\beta_y = 4.829 (+)$</td>
<td>0.892</td>
<td>0.882</td>
</tr>
<tr>
<td>DIM - SSD</td>
<td>$a = 1.417; b = -1.072$</td>
<td>0.739</td>
<td>0.701</td>
<td>DIM - SSD</td>
<td>$\beta_y = 2.369 (-)$</td>
<td>0.743</td>
<td>0.706</td>
</tr>
<tr>
<td>FILM - SSD</td>
<td>$a = 1.081; b = -0.293$</td>
<td>0.757</td>
<td>0.735</td>
<td>FILM - SSD</td>
<td>$\beta_y = 2.738 (-)$</td>
<td>0.760</td>
<td>0.738</td>
</tr>
<tr>
<td>COR - SSD</td>
<td>$a = 1.165; b = -0.429$</td>
<td>0.947</td>
<td>0.942</td>
<td>COR - SSD</td>
<td>$\beta_y = 3.453 (-)$</td>
<td>0.948</td>
<td>0.943</td>
</tr>
<tr>
<td>FIN - SSD</td>
<td>$a = 1.107; b = -0.059$</td>
<td>0.891</td>
<td>0.881</td>
<td>FIN - SSD</td>
<td>$\beta_y = 4.798 (-)$</td>
<td>0.891</td>
<td>0.881</td>
</tr>
</tbody>
</table>

Eq. (20(a))

\[
\begin{align*}
\beta_y^{\text{FILM-DIM}} &= -0.09148; b_1^{\text{FILM-DIM}} = 0.21265; b_2^{\text{FILM-DIM}} = 0.08089; b_3^{\text{FILM-DIM}} = 0.06371 \\
\beta_y^{\text{COR - DIM}} &= -0.03012; b_1^{\text{COR - DIM}} = 0.14038; b_2^{\text{COR - DIM}} = 0.06378; b_3^{\text{COR - DIM}} = 0.02312 \\
\beta_y^{\text{FIN - DIM}} &= 0.54846; b_1^{\text{FIN - DIM}} = 0.15759; b_2^{\text{FIN - DIM}} = 0.04521; b_3^{\text{FIN - DIM}} = -0.0759 \\
\beta_y^{\text{SSD - DIM}} &= 0.50692; b_1^{\text{SSD - DIM}} = 0.13579; b_2^{\text{SSD - DIM}} = -0.03267; b_3^{\text{SSD - DIM}} = -0.1196 \\
\end{align*}
\]

Eq. (20(b))

\[
\begin{align*}
\beta_y^{\text{FILM-DIM}} &= 0.93159 \\
\beta_y^{\text{COR - DIM}} &= 0.60672 \\
\beta_y^{\text{FIN - DIM}} &= 1.00953 \\
\beta_y^{\text{SSD - DIM}} &= 0.91844 \\
\end{align*}
\]

Eq. (20(c))

\[
\begin{align*}
\beta_y^{\text{FILM-DIM}} &= 0.25679 \\
\beta_y^{\text{COR - DIM}} &= 0.17141 \\
\beta_y^{\text{FIN - DIM}} &= 0.28745 \\
\beta_y^{\text{SSD - DIM}} &= 0.26103 \\
\end{align*}
\]

Eq. (20(d))

\[
\begin{align*}
\beta_y^{\text{FILM-DIM}} &= -0.38166 \\
\beta_y^{\text{COR - DIM}} &= -0.25624 \\
\beta_y^{\text{FIN - DIM}} &= -0.08751 \\
\beta_y^{\text{SSD - DIM}} &= -0.07966 \\
\end{align*}
\]

Eq. (20(e))

\[
\begin{align*}
\beta_y &= 5.80379; \beta_1 = -0.02377; \beta_2 = -0.05318 \\
\beta_y &= 4.65518; \beta_1 = -0.01742; \beta_2 = -0.07179 \\
\beta_y &= 0.62975; \beta_1 = -0.02969; \beta_2 = 0.32597 \\
\beta_y &= 0.62187; \beta_1 = -0.01454; \beta_2 = 0.24808 \\
\end{align*}
\]

The collection of PEM data and a lower number of R-square values are therefore reported in the Table 3.

Differences between R-square values do not seem to clearly depend on mix type. As far as DGFCs are concerned, FILM, FIN, and VSD
Table 5. Differences $\Delta V_{ij}$

<table>
<thead>
<tr>
<th></th>
<th>$\Delta V_{SSD-FIN}$</th>
<th>$\Delta V_{FIN-COR}$</th>
<th>$\Delta V_{COR-FILM}$</th>
<th>$\Delta V_{FILM-DIM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGFC</td>
<td>Min -0.487</td>
<td>0.415</td>
<td>0.466</td>
<td>0.101</td>
</tr>
<tr>
<td></td>
<td>Max 1.124</td>
<td>1.677</td>
<td>2.580</td>
<td>2.386</td>
</tr>
<tr>
<td></td>
<td>Mean 0.460</td>
<td>0.858</td>
<td>1.168</td>
<td>1.017</td>
</tr>
<tr>
<td>PEM</td>
<td>Min 0.804</td>
<td>0.057</td>
<td>0.537</td>
<td>0.913</td>
</tr>
<tr>
<td></td>
<td>Max 4.345</td>
<td>5.397</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean 2.377</td>
<td>1.913</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DGFC</td>
<td>Min -0.487</td>
<td>0.415</td>
<td>0.466</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>Max 1.124</td>
<td>1.677</td>
<td>4.345</td>
<td>5.397</td>
</tr>
<tr>
<td></td>
<td>Mean 0.460</td>
<td>0.858</td>
<td>2.287</td>
<td>1.863</td>
</tr>
</tbody>
</table>

data were better correlated with results from the other approaches than were SSD and dimensional data. Overall, the FILM and VSD methods were confirmed to be more closely related than were the other methods.

Table 4 summarizes the results obtained using different fitting curves. ‘+(+)’ stands for Eq. (20) used with a positive sign, and ‘-(−)’ the opposite. Note that, beside each R-square value, an adjusted R-square value is reported:

$$R^2_{adj} = 1 - \left(1 - R^2\right) \frac{n-1}{n-K-1}$$ (23)

where $n$ is the number of observations and $K$ is the number of independent variables.

Finally, Table 5 summarizes results derived using Eq. (18). The average values of specific volumes are positive, although some negative values were obtained when comparing SSD and FIN.

The following observations may be made:

1. Eqs. (19) to (21) yield, in general, similar R-square values.
2. In particular, the power-asymptotic law yields the best R-square values ($R^2_{adj} = 0.71$ to 0.95, $\beta \approx 2.4$ to 4.8). These values are generally higher than those obtained using the linear-asymptotic curve (Eq. (21)) or the linear expression (Eq. (19)).
3. When the coefficient $\beta$ in the power-asymptotic equation is expressed as a function of mix characteristics (Eqs. (20(a)) to (20(c))), no appreciable optimization is obtained. Interesting results are seen when NMAS is analyzed ($R^2_{adj} = 0.67$ to 0.84, Eq. (20(c))).
4. In linear regression, the highest R-square value is always referable to a relationship between the SSD and VSD methods ($R^2_{adj} = 0.94$, DGFCs), whereas the lowest R-square value is seen when SSD and Dimensional (DGFCs) are compared. The VSD, FILM, and FIN methods are well-correlated with all remaining methods. In particular, the method explains, on average, 91% of VSD variance, 87% that of FIN, 85% that of FILM, 84% that of SSD, and 83% that of DIM.
5. The linear equation (Eq. (19)) shows quite good performance but this does not necessarily consider any tendency toward the equality line.
6. The power-asymptotic equations (Eq. (20)) show very good “performance” (in terms of $R^2_{adj}$) but there is a small drawback in that a few standards showed changes in functional signs. It is very interesting to observe that transportability seems to be quite immediate and coefficients are contained within a small range (2.4 to 4.8). Note that this range becomes even smaller when SSD and DIM models are not considered (the range is then 3.4 to 4.3).
7. The power-asymptotic equations in which composition parameters are included (Eqs. (20(a)) to (20(c))) seem not to offer significant optimization from either a conceptual viewpoint or when quantitative-fitting was analyzed.
8. The linear-asymptotic curves (Eq. (21)) are both significant and interesting. Moreover, these curves do not have the drawback of a sign change (all $a$ values have proper signs). They confirm the conceptual framework outlined above, but $G_{ms}$ calculation appears to be more a conceptual than a practical necessity.
9. Another question to be considered deals with the robustness and logical admissibility of the three classes of fitting curves. In fact, both linear curves (Eq. (19)) and linear asymptotic curves (Eq. (21)) are associated with the possibility that two different fitting curves $G_{ms}$ and $G_{mb}$ both expressed as functions of a third ($G_{mah}$), may intersect on the graph (for example, Figs. 4, 7, and 8). This means that the relationship between any two methods $j$ and $k$ changes (from $>$ to $<$ or vice versa), perhaps because of statistical defects. On the contrary, when $\beta_2 = \beta_3$, there is no possibility that two curves of the power-asymptotic type (Eq. (20)) can intersect.
10. Regarding the four differences $\Delta V_{ij}$ (Eq. (18)), Table 5 shows that the lowest difference usually relates to the SSD-FIN comparison, whereas the highest difference is seen in the comparison of COR and FILM.

Further, although the averages are always positive (and agree with Eqs. (15) and (22)), significant deviations can be easily detected when SSD and FIN methods are compared.

**Concluding Remarks**

Transportability of bulk specific gravities is a theoretical and practical necessity and, in this paper, constituted our main objective. The following conclusions were reached:

1. Even if there is a need for further research, the formalized model, in which the higher the specific gravity the lower the differences between different measurement methods, appears to be basically validated.
2. The discretization of volumes used in the model offers a simple and intuitive tool for interpreting differences among methods.
3. The introduction of composition parameters (such as NMAS) does not yield any significant improvement in the regressions.
4. Paraffin, vacuum-sealing, and paraffin procedures (even if not applicable to PEMs) emerge as possible key methods.
5. The vacuum-sealing method is the best descriptor of the other methods.
6. Linear relationship evaluation is efficient but power-asymptotic curves are more robust and preferable from a theoretical viewpoint. In particular, power-asymptotic curves yield a well-grounded logical algorithm permitting an approach to the issue of $G_{mb}$ transportability.
References


