Technical Paper

ISSN 1997-1400 Int. J. Pavement Res. Technol. 8(5):377-382

Asphalt Stiffness Reduction and Stress-Strain Calculation in Pavement Structures

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Abstract: Fatigue damage in asphalt pavements is primarily regulated by the stiffness of asphalt layer. Asphalt stiffness deteriorates with repetitive application of vehicular loads due to bending effect. As a result, both bending stress and strain levels increase with repetitions which ultimately leads to fatigue failure under non-control loading conditions. Therefore, the material degradation *i.e.* the reduction in stiffness parameter can be utilized to measure fatigue damage, irrespective of the cracks condition in pavement structures. This paper focuses on the asphalt stiffness reduction models as function of load repetitions that can take care of critical stress-strain variations in pavement structures. Stiffness reduction models, both in strain control and stress control modes have been developed using visco-elastic principles. The proposed stiffness models are used for stress-strain calculations under non-control loading conditions, and are validated with the results obtained from structural analysis program. FEM based ABAQUS software is used for 3-D nonlinear analysis of multilayered pavement structures. It is concluded that the proposed models can correlate the critical stress and strain variations of in-service pavements, and can be used for fatigue damage evaluation with load repetitions.

DOI: 10.6135/ijprt.org.tw/2015.8(5).377
Key words*: Asphalt pavement; Asphalt stiffness; Damage; Fatigue.*

Introduction

Asphalt pavement structures mostly fail due to repetitive application of vehicular loads. Due to repetitive loadings, the asphalt stiffness (E) deteriorates and ultimately it leads to fatigue failure. Therefore, the incremental fatigue damage with repetitions (n) becomes nonlinear, though normally a linear damage principle is adopted in various design practices. To estimate the nonlinear fatigue damage as function of n or to design a pavement section with reliability more than 50% (*i.e.* damage factor less than one at failure situation), it needs to evaluate the stiffness degradation at the intermediate condition of pavement structure [1]. Various researchers [2-5] had studied the stiffness degradation in different asphalt materials. Baburamani (1999) [6] had critically discussed the stiffness variation due to fatigue phenomenon. A general trend of asphalt stiffness (E) variation with load repetitions (n) is presented in Fig. 1.

The nonlinear fatigue propagation in asphalt pavements can be evaluated using field fatigue equation, and accounting the stiffness variation with load repetitions, even if no visible cracks exist [1,7]. A generic form of fatigue equation [8-13] is given in Eq. (1).

$$N = c_1\, \varepsilon^{-c_2} \times E^{-c_3} \tag{1}$$

where, N is fatigue life; ε is critical horizontal tensile strain at the bottom of asphalt layer; E is asphalt stiffness; and c_1, c_2, and c_3 are regression constants. Eq. (1) estimates N values for different ε and E values. For a given pavement section, the different N values can be calculated for different ε and E values as function of repetitions (n).

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Note: Submitted March 4, 2015; Revised Aug. 14, 2015; Accepted Aug. 15, 2015.

Initial fatigue life (N_0) can be obtained corresponding to initial strain (ε_0) and initial stiffness (E_0) of the pavement section. It may be mentioned that the effect of lowering layers (granular materials) on horizontal tensile strain (ε) at the bottom of asphalt layer is insignificant and therefore, the degradation in granular materials due to repetitions (n) may be neglected for fatigue evaluation.

Initially, the fatigue equation is developed under certain laboratory conditions and subsequently, it is calibrated using field data. Rajbongshi and Das (2009) [14] has presented a systematic procedure for calibration of laboratory equation. While developing the laboratory equation, normally the 3-point or 4-point bending test on beam samples is performed either in control strain or in control stress mode [3,6,10,15,16]. Cyclic loading is allowed at 8–10Hz frequency till failure. Traditionally, 50% reduction in asphalt stiffness is adopted as failure criterion [3,6,10,17] and the corresponding number of load repetitions is recorded as fatigue life. This is also depicted in Fig. 1. However, under field condition a pavement structure is subjected to neither in strain control nor in stress control mode. Both stress and strain increase with reduction in asphalt stiffness (E) due to load repetitions (n), even if for a

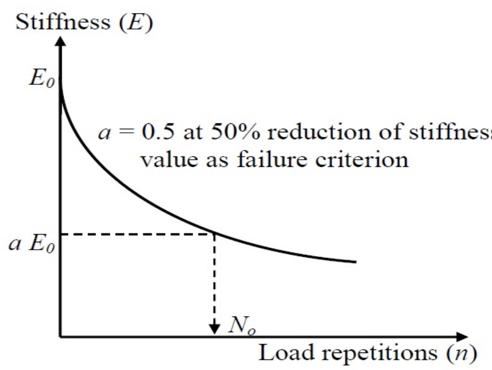

Fig. 1. Trend of Asphalt Stiffness Variation with Load Repetitions.

constant loading. Therefore, the stiffness variation under stress or strain control mode (*i.e.* laboratory condition) shall be capable of handling the stress-strain variations under non-control mode (field condition). This forms the scope of the present study. The objectives of the present study are - (i) to establish correlations for stress and strain variations with asphalt stiffness in pavement structures, (ii) to develop asphalt stiffness reduction model under control mode of loading, and (iii) to validate the stiffness model through numerical analysis of pavement structures. Stress and strain parameters mean the horizontal tensile stress and horizontal tensile strain at the bottom of asphalt layer.

This paper has five sections of which this is the first section. Next section discusses the pavement analysis and stress-strain variations with stiffness parameter. The development of asphalt stiffness reduction models has been explained in third section. The validation of the models is elaborated in the next section. Finally, the conclusions are drawn in last section. It also contains one annexure.

Analysis of Pavement Structure

This section attempts to correlate the stress-strain variations with asphalt stiffness of pavement structures under non-control mode. To this effect, a 3-layered 3-D asphalt pavement is modeled as nonlinear elastic with finite boundaries in ABAQUS environment. This is shown in Fig. 2. In FEM based ABAQUS analysis of the structure, the eight noded linear brick element with reduced integration is considered, including a rough interface between two layers. Each node is subjected to three degree of freedom *i.e.* displacement in X, Y, Z –directions. For analysis purpose, a pavement section of 10m×3.5m is taken with the boundary conditions of zero displacement in transverse (X) and longitudinal (Y) directions, and no displacement or rotation (fixed end) at the bottom of subgrade layer, as depicted in Fig. 2. A dual wheel load of 20kN each with center to centre distance of 30 cm and uniformly distributed tyre pressure of 0.7 MPa over its contact area has been adopted. The layers information used in the present analysis is given in Table 1.

The critical tensile strain (ε) and tensile stress (σ) values at the bottom of asphalt layer are obtained using ABAQUS analysis. For E = 2000 MPa, the ε and σ values are obtained as 0.000127 and 0.2449 MPa respectively, and recorded as initial strain (ε_0) and initial stress (σ_0) corresponding to initial stiffness (E_0) of 2000 MPa. In a similar way, the σ and ε parameters are determined for different E values in the range of 2000 to 800 MPa under same loading condition. Fig. 3 shows the variations of σ/σ_0 and $\varepsilon/\varepsilon_0$ with E/E_0 of the pavement section. It may be mentioned that this case, the $\sigma - \varepsilon$ variations are only due to the variation in asphalt stiffness (E) under non-control situation. The variations of σ/σ_0 under the strain control mode and $\varepsilon/\varepsilon_0$ under stress control mode are also depicted in Fig. 3. These cases, the stiffness parameter is expressed as $E = \sigma/\varepsilon_0$; where $\varepsilon = \varepsilon_0$ (strain control) and $E = \sigma_0/\varepsilon$; where $\sigma = \sigma_0$ (stress control). Fig. 3 shows a significant difference in $\sigma - \varepsilon$ variations between control and non-control modes. Fig. 4 presents a comparison of σ/σ_0 and $\varepsilon/\varepsilon_0$ parameters at the initial (*i.e.* $E = E_0$) and failure (*i.e.* E = 50% of E_0) conditions. As observed, the σ/σ_0 or $\varepsilon/\varepsilon_0$ value at failure is significantly different under control (*i.e.* laboratory) and non-control (*i.e.* field) loadings. This is one of primary reasons of experiencing large shift factor between laboratory and field fatigue equations. Moreover, both σ/σ_0 and $\varepsilon/\varepsilon_0$ values under the field condition are same *i.e.* nearly 1.4 at failure situation (E/E_0). That means, a pavement section fails at nearly 40% increase in stress or strain value corresponding to 50% reduction in stiffness value. Fig. 5 shows the variations of σ/σ_0 and $\varepsilon/\varepsilon_0$ parameters with different E/E_0 ratio under field situation. Using the least square method of curve fitting, the best fit equations are obtained as given in Eq. (2).

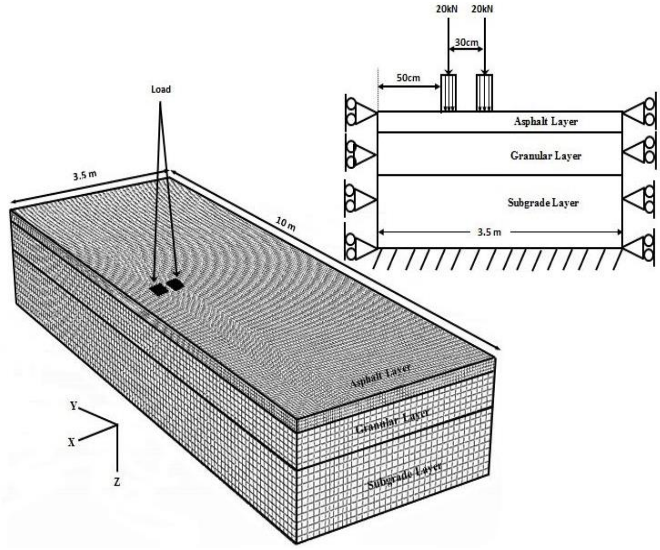

Fig. 2. Asphalt Pavement Section used in FEM Modeling.

Table 1. Data Used for Asphalt Pavement Analysis.

Layer	Thickness (cm)	E -value (MPa)	Poisson's Ratio
Asphalt	15	2000-800	0.30
Granular Base	30	350	0.35
Subgrade	100	60	0.35

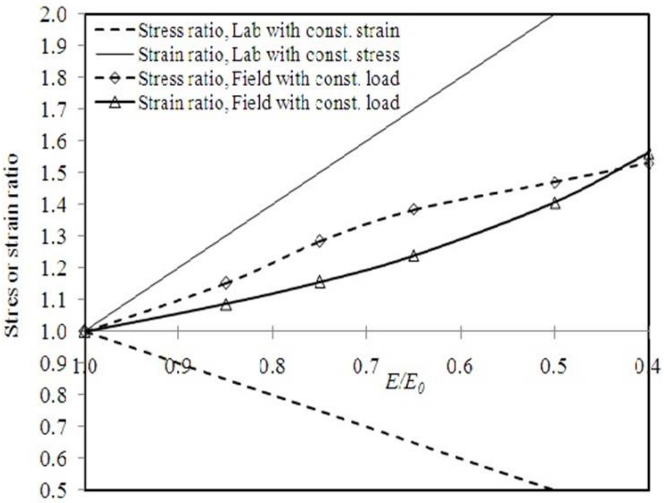

Fig. 3. Stress-strain Variations with Asphalt Stiffness Under Different Loading Modes.

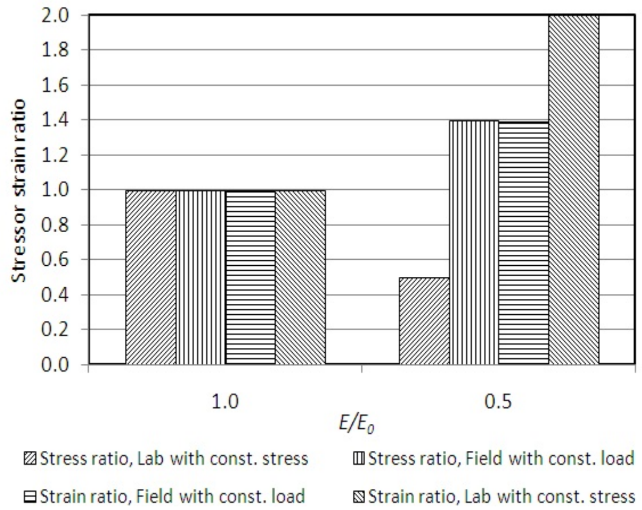

Fig. 4. Comparisons of Stress and Strain Ratio at Initial and Failure Conditions.

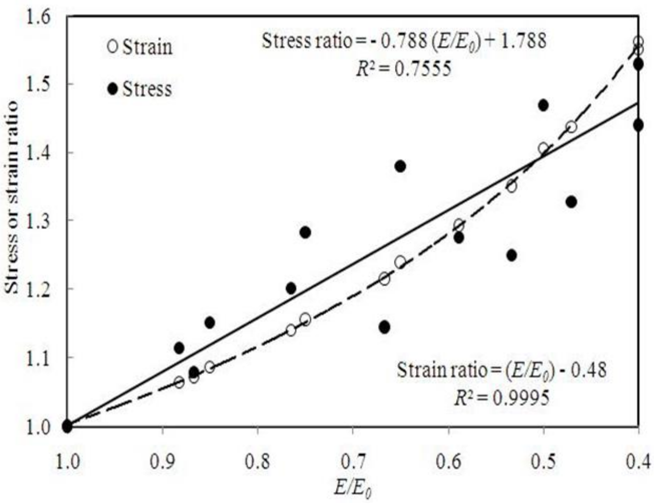

Fig. 5. Stress and Strain Variations with Asphalt Stiffness in Pavement Structure.

$$\frac{\varepsilon}{\varepsilon_0} = \left(\frac{E}{E_0}\right)^{-0.48}; \quad R^2 = 0.999$$
$$and, \tag{2}$$
$$\frac{\sigma}{\sigma_0} = 1.788 - 0.788\left(\frac{E}{E_0}\right); \quad R^2 = 0.755$$

Eq. (2) indicates the stress and strain variations due to asphalt stiffness reduction of a given pavement section under non-control situation. Such stiffness reduction in pavement structures happens due to load repetitions, which is neither under stress control nor under strain control condition. Moreover, the stiffness reduction as obtained from the laboratory is either in strain control (relaxation) or in stress control (creep) condition. Relaxation and creep conditions are widely used in visco-elastic analysis of asphalt materials. Using visco-elastic principles, the subsequent sections attempt to develop stiffness reduction models under control loading (laboratory) and to explore the use of such models for stress and strain calculations under non-control loading (field).

Development of Stiffness Reduction Models

Asphalt stiffness (E) parameter as function of load repetitions (n) can be obtained from laboratory tests under relaxation (strain control) or creep (stress control) condition. Under relaxation condition, the relaxation stiffness ($E(n)$) function may be expressed as given in Eq. (3).

$$E(n) = \frac{\sigma(n)}{\varepsilon_0}; \quad where,\ \varepsilon(n) = \varepsilon_0 \tag{3}$$

where, n is number of load repetitions. Taking Laplace transform of Eq. (3), it can be written as,

$$\bar{E}(s) = \frac{\bar{\sigma}(s)}{\varepsilon_0} = \frac{\bar{\sigma}(s)}{s\,\bar{\varepsilon}(s)}; \quad where,\ \bar{\varepsilon}(s) = \varepsilon_0 / s \tag{4}$$

where, $\bar{f}(s)$ is Laplace transform of $f(n)$; and S is Laplace transform variable. Similarly, under creep condition, the creep stiffness ($C(n)$) may be expressed as given in Eq. (5).

$$C(n) = \frac{\varepsilon(n)}{\sigma_0}; \quad where,\ \sigma(n) = \sigma_0 \tag{5}$$

Taking Laplace transform of Eq. (5), it can be written as,

$$\bar{C}(s) = \frac{\bar{\varepsilon}(s)}{\sigma_0} = \frac{\bar{\varepsilon}(s)}{s\,\bar{\sigma}(s)}; \quad where,\ \bar{\sigma}(s) = \sigma_0 / s \tag{6}$$

From Eqs. (4) and (6), one can write as,

$$\bar{C}(s) \times \bar{E}(s) = \frac{1}{s^2} \tag{7}$$

Thus, taking inverse Laplace of Eq. (7), the $E(n)$ and $C(n)$ functions can be correlated as given in Eq. (8).

$$C(n) = L^{-1}\left[\frac{1}{s^2 \times \bar{E}(s)}\right]$$
$$Or, \tag{8}$$
$$E(n) = L^{-1}\left[\frac{1}{s^2 \times \bar{C}(s)}\right]$$

Eq. (8) represents the relationship between $E(n)$ and $C(n)$. $C(n)$ can be obtained for any given $E(n)$ and vice versa. Further, from Eqs. (40) and (6) the stress and strain functions can be written as given in Eq. (9).

$$\bar{\sigma}(s) = \bar{E}(s) \times s\,\bar{\varepsilon}(s)$$
$$\text{and,} \tag{9}$$
$$\bar{\varepsilon}(s) = \bar{C}(s) \times s\,\bar{\sigma}(s)$$

From Eq. (9) and using the convolution integral theorem [18], the stress and strain as function of n can be expressed as given in Eq. (10).

$$\sigma(n)=\int_0^n E(n')\,\frac{\partial\varepsilon(n-n')}{\partial n'}\,dn'=\int_0^n \frac{\partial E(n-n')}{\partial n'}\,\varepsilon(n')\,dn'$$
$$Or, \tag{10}$$
$$\varepsilon(n)=\int_0^n \frac{\partial C(n-n')}{\partial n'}\,\sigma(n')\,dn'=\int_0^n C(n')\,\frac{\partial\sigma(n-n')}{\partial n'}\,dn'$$

where, *n'* is an integrating variable.

Eq. (10) represents the stress-strain relationship in functional form under non-control mode. While using Eq. (10), it needs to know the $E(n)$ or $C(n)$ function under laboratory condition. At this juncture $E(n)$ and $C(n)$and based on the literatures [3,6,10,19], a simple form of exponential stiffness reduction may be considered as given in Eq. (11).

$$E(n)=\lambda\, e^{-n/\rho} \tag{11}$$

where, λ and $1/\rho$ are the model parameters (constants). Eq. (11) represents the Maxwell form of relaxation modulus under relaxation condition. Using two boundary conditions, viz. $E(n = 0) = E_0$ and $E(n = N_0) = aE_0$ at failure in Eq. (11), one can write $\lambda = E_0$ and $-1/\rho = \ln(a)/N_0$; where, a indicates the stiffness ratio at failure condition. In the present work, the 50% reduction in stiffness value is considered as failure criterion, *i.e.* $E(n = N_0)/E_0 = a = 0.5$ for example case. Thus, the Eq. (11) can be re-written as given in Eq. (12).

$$E(n)=E_0\, e^{\ln(a)\times n/N_0} \tag{12}$$

From Eqs. (2) and (12) the strain and stress functions can be written as given in Eq. (13).

$$\varepsilon(n)=\varepsilon_0\, e^{-0.48\ln(a)\times n/N_0}$$
$$and, \tag{13}$$
$$\sigma(n)=\sigma_0\left\{1.788-0.788e^{\ln(a)\times n/N_0}\right\}$$

$\sigma(n)$ and $\varepsilon(n)$ in Eq. (13) are developed based on the analysis of pavement structures (Eq. (2)). Further, using the visco-elastic principles (refer to Eq. (10)), the $\sigma(n)$ for any given $\varepsilon(n)$ can be derived as given in Eq. (14).

$$\begin{aligned}\sigma(n)&=\int_0^n E_0\, e^{\ln(a)\times n'/N_0}\times[-0.48\ln(a)/N_0]\,\varepsilon_0\, e^{-0.48\ln(a)\times(n-n')/N_0}\,dn'\\&=0.3243E_0\varepsilon_0\times e^{0.48\ln(2)\times n/N_0}\left\{1-e^{-1.48\ln(2)\times n/N_0}\right\}+Integ.const.;\\&\quad for\, a=0.5\end{aligned} \tag{14}$$

Putting, $\sigma(n = 0) = \sigma_0$ in Eq. (14), the integrating constant can be obtained as σ_0. Thus, the Eq. (14) can be re-written as,

$$\sigma(n)=0.3243E_0\varepsilon_0\, e^{0.48\ln(2)\times n/N_0}\left\{1-e^{-1.48\ln(2)\times n/N_0}\right\}+\sigma_0;\quad for\ a=0.5 \tag{15}$$

$\sigma(n)$ in Eq. (15) utilizes the relaxation stiffness ($E(n)$) parameter as expressed in Eq. (12). Similarly, $\varepsilon(n)$ can be derived utilizing the creep stiffness ($C(n)$) parameter (refer to Eq. (10)). To this effect, from Eqs. (8) and (12) the $C(n)$ function can be obtained as given in Eq. (16).

$$C(n)=\frac{1}{\lambda}+\frac{n}{\lambda\rho}=\frac{1}{E_0}-\frac{1}{E_0}\frac{\ln(a)}{N_0}n \tag{16}$$

Eq. (16) represents the Kelvin form of creep modulus under creep condition. It may be mentioned that $1/C(n = 0) = E(n = 0)$, and $1/C(n) \neq E(n)$; for $n > 0$. This is illustrated further in a later section. Thus, in a similar way the $\varepsilon(n)$ for any given $\sigma(n)$ can be derived (refer to Eq. (10)) as given in Eq. (17).

$$\begin{aligned}\varepsilon(n)&=\int_0^n\left\{1/E_0-1/E_0\times\ln(a)(n-n')/N_0\right\}\times\sigma_0\left\{-0.788\ln(a)/N_0\times e^{\ln(a)\times n'/N_0}\right\}dn'\\&=1.576\sigma_0/E_0\left\{1-e^{-\ln(2)\times n/N_0}-\ln(2)/2\times n/N_0\times e^{-\ln(2)\times n/N_0}\right\}+\varepsilon_0;\quad for\ a=0.5\end{aligned} \tag{17}$$

Eq. (17) provides the $\varepsilon(n)$ prediction based on $C(n)$ as given in Eq. (16) and, Eq. (15) provides the $\sigma(n)$ prediction based on $E(n)$ as given in Eq. (12). The validation of $E(n)$ and $C(n)$ models are discussed in the next section.

Validation of Stiffness Models

To validate the relaxation stiffness and creep stiffness models, the following information are used.

Another 3-layered pavement section is modeled in ABAQUS environment. The layers information is given in Table 2 and all other information used as mentioned in second section. Thus, from the analysis of pavement section with initial asphalt stiffness of E_0 = 1700 MPa the initial stress and initial strain values are obtained as σ_0 = 0.282 MPa and ε_0 = 0.000138 respectively. Subsequently, using Eq. (1) the fatigue life of the pavement structure is calculated as N_0 = 400×10^6 axles. The parameters of Eq. (1) *i.e.* c_1, c_2 and c_3 are taken as 2.21×10^{-4}, 3.89, and 0.854 respectively, as per IRC (2012) [9].

Using the relaxation stiffness model (Eq. (12)), the different E values can be obtained for different load repetitions (n). Accordingly, for each E-value the σ and ε values can be calculated from the pavement analysis program. Calculated σ and ε values are given in Table 3. Also, for different n values the σ and ε values are predicted using Eqs. (15) and (17) respectively. Predicted σ and ε values are also presented in the Table 3. From Table 3, it is seen that σ–ε values calculated through pavement analysis program are close to the predicted σ–ε values from Eqs. (15) and (17). The comparisons on the calculated and predicted σ and ε parameters are presented in Figs. 6 and 7 respectively. As observed in Figs. 6 and 7, and in Table 3 it may be concluded that developed E(n) and C(n) models can correlate the stress-strain behaviors under non-control loading. These models contain two elements (λ and 1/ρ) that represent the stiffness reduction behavior in asphalt materials under control modes. $E(n)$ and $1/C(n)$ functions are evaluated and presented in Fig. 8 for the present example case. It is seen that $E(n)\neq1/C(n)$; for

Table 2. Data Used for Validation of Stiffness Models.

Layer	Thickness (cm)	E-value (MPa)	Poisson's ratio
Asphalt	15	1700	0.30
Granular Base	35	400	0.35
Subgrade	100	50	0.35

Table 3. Comparisons of Result for Validation of Stiffness Models.

Repetitions (n)	E -value (MPa)	ε - Calculated	ε - Predicted	σ (MPa)- Calculated	σ (MPa)- Predicted
0.0	1700	0.000138	0.000138	0.282	0.282
72×10^6	1500	0.000147	0.000152	0.314	0.299
155×10^6	1300	0.000157	0.000167	0.339	0.316
306×10^6	1000	0.000178	0.000194	0.360	0.346
435×10^6	800	0.000198	0.000215	0.375	0.370
601×10^6	600	0.000228	0.000239	0.448	0.401

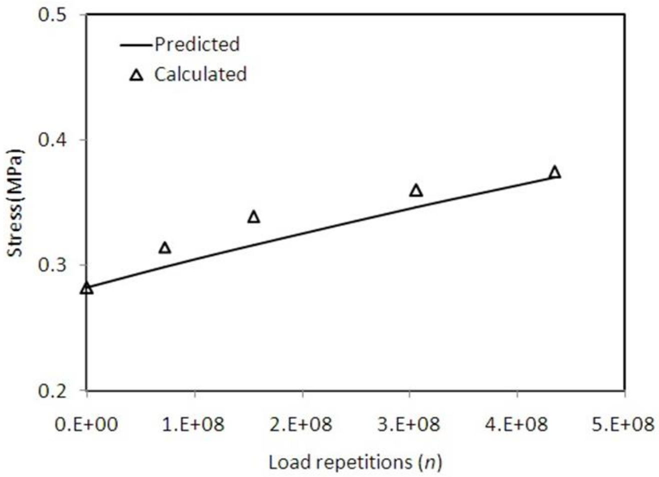

Fig. 6. Comparison of Observed and Predicted Stress Parameters.

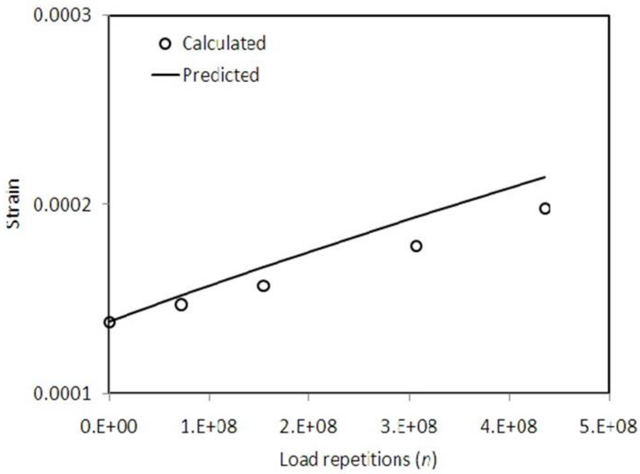

Fig. 7. Comparison of Observed and Predicted Strain Parameters.

$n > 0$, *i.e.* $E(n) \neq \sigma_0/\varepsilon(n)$ under laboratory condition (stress control). Thus it implies that the stiffness variation under strain control and under stress conditions is different. $E(n)$ or $C(n)$ for asphalt mix either at constant strain or constant stress amplitude can be obtained using 3-point or 4-point laboratory beam test [3, 6, 10, 15, 20]. The expressions of $E(n)$ and $C(n)$ with 3-point and 4-point bending test are given in *Annexure I*.

Conclusions

This paper presents the theoretical stiffness reduction models in asphalt materials. Convolution integral theorem and Laplace transform are used for models development. The following conclusions are drawn from the present work.

a) Stress and strain variations with asphalt stiffness reduction have been studied using 3-D nonlinear analysis of multilayered pavement structures. A very good correlation has been established between the strain and stiffness parameters, with R^2=0.999. However, in case of stress variation with stiffness parameter, a relatively poor correlation is observed (R^2 = 0.755).

b) A stiffness reduction model ($E(n)$) under strain control mode has been developed using linear visco-elastic principles. It is established that the proposed $E(n)$ model is good enough to predict the stress variation for any given strain variation under non-control loading conditions. Therefore, using the strain based fatigue equation and to evaluate the fatigue life as function load repetitions (n) the proposed $E(n)$ model can be used.

c) The relaxation stiffness ($E(n)$) under strain control mode can be obtained for known creep stiffness ($C(n)$) under stress control mode. Accordingly, the $E(n)$ function can be used for fatigue performance evaluation. In other words, the $C(n)$ function can be used with the stress based fatigue equation for fatigue damage evaluation.

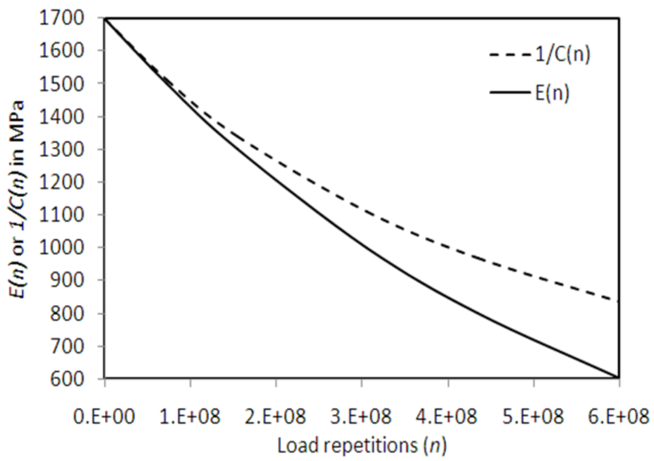

Fig. 8. Comparison of Stiffness Reduction in Strain Control and Stress Control Modes.

d) $E(n) \neq 1/C(n)$ and, that is why the number of load repetitions obtained corresponding to 50% reduction in stiffness values are not same under constant strain and constant stress amplitude fatigue tests.

e) Proposed $E(n)$ and $C(n)$ models are not specific to any fatigue equation. These models contain two parameters namely, λ and $1/\rho$. However, more parametric stiffness models may be studied for better prediction of stress-strain parameters.

Annexure I

Expression for relaxation stiffness ($E(n)$) and creep stiffness ($C(n)$) with 3-point and 4-point bending tests:

3-point Bending Test

Let δ_0 be the maximum deflection corresponding to load P_0 applied at the middle of a simply supported asphalt beam with size $b \times h \times l$. Then, applying pure bending theory the corresponding strain (ε_0) and stress (σ_0) parameters can be expressed as given in Eq. (18).

$$\varepsilon_0 = \left(\frac{6h}{l^2}\right)\delta_0 \quad and \quad \sigma_0 = \left(\frac{3l}{2bh^2}\right)P_0 \tag{18}$$

Accordingly, the relaxation stiffness ($E(n)$) and creep stiffness ($C(n)$) in functional form can be evaluates as given in Eqs. (19) and (20) respectively.

$$E(n) = \frac{\sigma(n)}{\varepsilon_0} = \left(\frac{3l}{2bh^2\varepsilon_0}\right)P(n); \quad where,\ \varepsilon(n) = \varepsilon_0 = \left(\frac{6h}{l^2}\right)\delta_0 \tag{19}$$

$$C(n) = \frac{\varepsilon(n)}{\sigma_0} = \left(\frac{6h}{l^2\sigma_0}\right)\delta(n); \quad where,\ \sigma(n) = \sigma_0 = \left(\frac{3l}{2bh^2}\right)P_0 \tag{20}$$

4-point Bending Test

Let δ_0 be the maximum deflection corresponding to load P_0 applied at two locations ($P_0/2$ each at equal span of $l/3$) of a simply supported beam with size $b \times h \times l$. Using the pure bending theory, the corresponding strain (ε_0) and stress (σ_0) parameters can be expressed as given in Eq. (21).

$$\varepsilon_0 = \left(\frac{108h}{23l^2}\right)\delta_0 \quad and \quad \sigma_0 = \left(\frac{l}{bh^2}\right)P_0 \tag{21}$$

Accordingly, the relaxation stiffness ($E(n)$) and creep stiffness ($C(n)$) in functional form can be evaluated as given in Eqs. (22) and (23) respectively.

$$E(n) = \frac{\sigma(n)}{\varepsilon_0} = \left(\frac{l}{bh^2\varepsilon_0}\right)P(n); \quad where,\ \varepsilon(n) = \varepsilon_0 = \left(\frac{108h}{23l^2}\right)\delta_0 \tag{22}$$

$$C(n) = \frac{\varepsilon(n)}{\sigma_0} = \left(\frac{108h}{23l^2\sigma_0}\right)\delta(n); \quad where,\ \sigma(n) = \sigma_0 = \left(\frac{l}{bh^2}\right)P_0 \tag{23}$$

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