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# Predicting strength behaviour of stabilized lateritic soil–ash matrix using regression model for hydraulically bound materials purposes

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**Abstract**

The multiple regression relationship models were applied on the strength properties of the treated lateritic soil to deduce models. These models were verified to be valid to be applied under different test conditions to determine the dependent variables of the compaction, Atterberg, California bearing ratio and unconfined compression tests. The results showed that the behaviour of the variables under laboratory condition was in tandem with the regression model results. This is a strong indication that the regression model can be applied in the field of soil stabilization in a statistical engineering application to predict consistent values to determine treatment variations.


*Keywords:* Regression model; Hydraulically bound material; Strength behaviour; Lateritic soil-ash matrix; Pozzolanic reaction; Cementitious material

## 1. Introduction

Strength behaviour of stabilized soil mixtures is an assessment and evaluation of the effect and response lateritic soils exhibit when mixed with materials with the intention to improve the geotechnical, geophysical, mechanical and geomorphic configuration and properties. The failure of various engineering structures both vertical and horizontal e.g. sub-grade and sub-base, of highway and airfield pavements, shallow foundational structures, submerged water retaining structures, etc. has necessitated the stabilization of soils for efficient performance [1–4]. The frequency with which the need for stabilization occurs has brought about the modelling of the performance of variables in a stabilization operation [5–10]. However, the need has arisen to generate models which experts can rely upon to monitor the design of admixture soil stabilization, improvement and optimization exercises [11–21]. Researchers have over the centuries of engineering advancement proposed methods and approaches to achieving a stabilized matrix of soil [22–25]. The use of cementitious materials e.g. ordinary Portland cement has been in operation for centuries [26–29]. This is a chemical method of soil stabilization where hydration, cation exchange; between the sodium and hydrogen ions in lateritic soil and calcium ion in cement and pozzolanic reaction and carbonation take place to bring about flocculation, hardening and densification of the stabilized mixture [30–35]. Supplementary cementing materials like fly ash, pozzolanas, slag, cement kiln dust, etc. and some biodegradable cementing materials

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**Nomenclature**

| | |
|---|---|
| Y | dependent variable of the linear multiple regression relationship |
| $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$ | independent variables of the linear multiple regression relationship |
| a, b, c, d and e | regression constants |
| $S_A$ | additive sample of ash |
| ε | random error |
| W | moisture content |
| $I_P$ | plasticity index |
| $w_P$ | plastic limit |
| $w_L$ | liquid limit |
| D | maximum dry density |
| Đ | deformation |
| NPBA | nanostructured palm bunch ash |
| CBR | California bearing ratio |
| UCS | unconfined compressive strength |
| P | applied vertical load |
| $\sum$ | summation of events of i = 1, 2, 3…,n occurrences |

like oyster shell dust, palm bunch ash, bagasse ash, etc. [36–37] have all been used by researchers to achieve a stabilized soil for construction purposes. Ash as an amorphous material though with a less density in the stabilization of soil has also been in use in recent times. Results have shown that ash materials react well with lateritic soil because of their pozzolanic properties and homogenous gradation when in contact with soil to bring about improved lateritic soil strength properties. Subsequently, the reduction in the ash materials to nanostructured materials to improve their reactive surface and bonding strength in the stabilization of lateritic soils for construction purposes is ongoing [17,38–41]. Results from this exercise showed improved strength and behaviour of stabilized soils [17]. Results have also shown that when solid, biodegradable waste materials are reduced to ashes, they become more useful to the geotechnical engineer in soil strength optimization [42–45]. In addition to construction materials' use in soil stabilization, physical or mechanical methods play an important role. For instance, compactive effort is a function of blows in soil stabilization and loads applied; axial and cell pressures applied depending on the type of machine and laboratory exercise being conducted. It has been shown that compactive effort brings about an improved densification of a stabilized mixture through the flocculation of the soil-admixture [46–48]. Loads applied during the laboratory examination also help in the determination of the strength beyond which a stabilized mixture can no longer be safe. The effort of this exercise was to bring these factors together, determine the variables in different laboratory investigations that the results we expect depend on and use the linear regression relationship to generate models that could be applied in different fields under different conditions while considering relevant and appropriate boundary conditions. The regression model was adopted in the prediction of strength behaviour of ash treated soil with particular emphasis on; (i) the formulation of the linear relationship between the variables in the strength properties' tests; compaction, Atterberg limits, California bearing ratio and unconfined compression tests by multiple regression method, (ii) the application of the laboratory results of lateritic soil stabilization obtained, design multiple regression models for the above laboratory test taking into account the field conditions and. (iii) the generation of sample results with the established models to show the behaviour of the models under the studied conditions.

## 2. Methodology

The reference soil sample was collected from Basic borrow pit at Olokoro in Umuahia south LGA of Abia State, Nigeria [16]. The soil sample was open air dried and prepared for preliminary and stabilization tests in accordance with BS 1377 [50], BS 1924 [51], and NGS [52]. The ordinary Portland cement used was Dangote cement bought at the Timber building materials Market, Umuahia that meets the requirements of cement materials in accordance with ASTM c150 [53]. The nanostructured ash admixture used to stabilize the lateritic soil was obtained by air drying, burning, completely pulverizing and UV–Vis Spectrophotometric characterization of palm bunch as nanostructured palm bunch as (NPBA) [16,54]. Finally the models were developed using the results of the stabilization tests and the regression relationship between variables of the strength behaviour of the stabilized lateritic soil according to Agunwamba [49].

## 3. Formulation of soil strength properties' regression models

The regression models formulation was conducted with the guidelines given by Agunwamba [49] and with strict adherence to the geotechnical principles of soil-admixture stabilization and improvement. The laboratory conditions were maintained at room temperature and various geophysical factors were assumed constant with respect to the test model under consideration. The stabilized soil strength behaviours were modelled with the linear multiple regression relationship with five data points as follows;

### 3.1. Compaction regression model

To achieve maximum dry density (D), the following independent variables were identified; moisture content (w), and varying percentage by weight proportion of additive sample ($S_A$), while cement content, compaction effort and other laboratory conditions remain constant. D is the dependent variable (Y) and w and $S_A$ are the independent variables $x_1$ and $x_2$ respectively hence,

$$Y_i = a + bx_{1(i)} + cx_{2(i)} + \varepsilon_i;\ i = 1, 2, 3..., n \tag{1}$$

where,

$a$, $b$ and $c$ are regression constants
$\varepsilon$ = random error which is assumed to be zero because of the precision with which the laboratory exercises were carried out.

Eq. (1) is a linear multiple regression relationship for three variables in a compaction exercise required to achieve a dry density needed for the stabilization or allowable densification of the lateritic soil under varying proportions of additive material and moisture content. If the least square sum is minimized for the population of results, the following equations are obtained from Eq. (1).

$$\begin{aligned} \sum Y &= an + b\sum x_1 + c\sum x_2 \\ \sum Yx_1 &= a\sum x_1 + b\sum x_1^2 + c\sum x_1x_2 \\ \sum Yx_2 &= a\sum x_2 + b\sum x_1x_2 + c\sum x_2^2 \end{aligned} \tag{2}$$

Recall that,

$x_1$ = moisture content (w)
$x_2$ = additive sample varying proportion

By solving Eq. (2) simultaneously, $a$, $b$ and $c$ are determined for n number of data points. A matrix equation is formulated from the operation of the form;

$$AX = B \tag{3}$$

where,

$A$ = matrix of the independent variables
$B$ = matrix of dependent variables
$X$ = matrix of linear regression constants

Thus,

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{4}$$

To solve for the constants, Eq. (4) is solved with Crammer's rule.

Having determined $a$, $b$ and $c$, the linear multiple regression relationship involving maximum dry density ($D$); the dependent variable designated as $Y$ and the moisture content ($w$) designated as $x_1$ and additive sample percentage by weight proportion ($S_a$) designated as $x_2$ which are the independent variables may be described as;

$$Y_i = a + bx_{1(i)} + cx_{2(i)} + \varepsilon_i;\ i = 1, 2, 3..., n$$

This implies,

$$D_i = a + b\mathrm{w}_i + cS_{a(i)};\ i = 1, 2, 3..., n \tag{5}$$

### 3.2. Atterberg limits regression model

In this model, plasticity index ($I_P$) is the dependent variable while moisture content (w), plastic limit ($w_P$), liquid limit ($w_L$) and additive sample proportion ($S_a$) are the independent variables. The multiple regression relationship arising from the above is of the general form;

$$Y_i = a + bx_{1(i)} + cx_{2(i)} + dx_{3(i)} + ex_{4(i)} + \varepsilon_i;\ i = 1, 2, 3..., n \tag{6}$$

where,

$Y$ represents the dependent variable, plasticity index ($I_P$)
$x_1$ Represents moisture content (w)
$x_2$ Represents plastic limit ($w_P$)
$x_3$ Represents liquid limit ($w_L$)
$x_4$ Represents additive sample % by weight proportion ($S_a$)
$\varepsilon$ represents random error which is assumed to be negligible or zero
And $a$, $b$, $c$, $d$ and $e$ represent the regression constants.

Eq. (6) is the multiple regression relationship for five variables in an Atterberg limit model required to achieve the plasticity index ($I_P$) that meets the stabilized material requirements as stipulated in design standards for stabilized soil consistency. If the least square sum is also minimized for the population of results, the following equations are obtained from Eq. (6);

$$\begin{aligned} \sum Y &= an + b\sum x_1 + c\sum x_2 + d\sum x_3 + e\sum x_4 \\ \sum Yx_1 &= a\sum x_1 + b\sum x_1^2 + c\sum x_1x_2 + d\sum x_1x_3 + e\sum x_1x_4 \\ \sum Yx_2 &= a\sum x_2 + b\sum x_1x_2 + c\sum x_2^2 + d\sum x_2x_3 + e\sum x_2x_4 \\ \sum Yx_3 &= a\sum x_3 + b\sum x_1x_3 + c\sum x_2x_3 + d\sum x_3^2 + e\sum x_3x_4 \\ \sum Yx_4 &= a\sum x_4 + b\sum x_1x_4 + c\sum x_2x_4 + d\sum x_3x_4 + e\sum x_4^2 \end{aligned} \tag{7}$$

The regression constants are determined by solving Eq. (7) simultaneously. However, Eq. (7) can be translated to a matrix form; AX = B, thus;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \\ \sum Yx_3 \\ \sum Yx_4 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 & \sum x_3 & \sum x_4 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 & \sum x_1x_3 & \sum x_1x_4 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 & \sum x_2x_3 & \sum x_2x_4 \\ \sum x_3 & \sum x_1x_3 & \sum x_2x_3 & \sum x_3^2 & \sum x_3x_4 \\ \sum x_4 & \sum x_1x_4 & \sum x_2x_4 & \sum x_3x_4 & \sum x_4^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \tag{8}$$

By solving Eq. (8) by Gauss Reduction Method [49], $a$, $b$, $c$, $d$ and $e$ are determined. Having determined the regression constants $a$, $b$, $c$, $d$ and $e$, the linear multiple regression relationship involving the variables of Atterberg limit model may be described as;

$$I_{P(i)} = a + bw_i + cw_{P(i)} + dW_{L(i)} + eS_{a(i)};\ i = 1,2,3\ldots,n \tag{9}$$

From Eq. (9), it can be deduced that any change in the behaviour of the independent variables brings about a change in the plasticity index ($I_P$).

### 3.3. California bearing ration regression model

This model was used to study the regression relationship on the stabilized laterite–ash mixture between the California bearing ratio (CBR); the dependent variable and water content ($w$), maximum dry density ($D$) and additive sample percentage by weight proportion ($S_a$). These are the independent variables in this model. The linear multiple regression relationship arising from the above model is of the general form;

$$Y_i = a + bx_{1(i)} + cx_{2(i)} + dx_{3(i)} + \varepsilon_i;\ i = 1,2,3\ldots,n \tag{10}$$

where,

Y = CBR
$x_1$ = moisture content ($w$)
$x_2$ = maximum dry density ($D$)
$x_3$ = additive/admixture sample ($S_a$)
$\varepsilon$ = random error considered negligible or equal to zero
And $a$, $b$, $c$ and $d$ = regression constants in the linear relationship.

Eq. (10) is a linear multiple regression relationship for four variables in a CBR model required to achieve the sub-grade strength at the density obtained at optimum moisture content that meets the stabilized material requirement as stipulated in the design standards for stabilized sub-grade materials. If the least square sum is also minimized in this case for the population of the results, the following equations are obtained from Eq. (10);

$$\begin{aligned}
\sum Y &= an + b\sum x_1 + c\sum x_2 + d\sum x_3\\
\sum Yx_1 &= a\sum x_1 + b\sum x_1^2 + c\sum x_1x_2 + d\sum x_1x_3\\
\sum Yx_2 &= a\sum x_2 + b\sum x_1x_2 + c\sum x_2^2 + d\sum x_2x_3\\
\sum Yx_3 &= a\sum x_3 + b\sum x_1x_3 + c\sum x_2x_3 + d\sum x_3^2
\end{aligned} \tag{11}$$

Eq. (11) can be translated to a matrix of the general form, AX = B thus;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \\ \sum Yx_3 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 & \sum x_3 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 & \sum x_1x_3 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 & \sum x_2x_3 \\ \sum x_3 & \sum x_1x_3 & \sum x_2x_3 & \sum x_3^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \tag{12}$$

By solving Eq. (12) by Gauss Reduction Method [49], $a$, $b$, $c$ and $d$ can be determined and having determined the regression constants from the solution of the global matrix Eq. (12), the linear regression relationship involving CBR, moisture content ($w$), dry density ($D$) and additive sample % by weight proportion with all the other factors of the laboratory test condition being constant, can be described as;

$$CBR_i = a + \mathrm{bw}_i + cD_i + dS_{a(i)};\ i = 1,2,3\ldots,n \tag{13}$$

### 3.4. Unconfined compression test model for the stabilized lateritic soil

Unconfined compression test is a specialized form of Triaxial test where all-round pressure (cell pressure) on the sample equals zero. The variables arising from this model are those factors responsible for deformation (Đ) of the test specimen which include vertical load ($P$) applied on the stabilized sample and additive sample % by weight proportion ($S_a$) used an admixture in the stabilization operation. While other factors of the test condition; cohesion, surface area of sample at failure, moisture content, spring stiffness and other UCS laboratory test conditions remain constant, the regression relationship for the four variables of this model was studied. The multiple regression relationship arising from the above is of the general form;

$$Y_i = a + bx_{1(i)} + cx_{2(i)} + \varepsilon_i;\ i = 1,2,3\ldots,n \tag{14}$$

where,

Y = deformation (Đ)
$x_1$ = vertical load applied (P)
$x_2$ = additive/admixture sample ($S_a$)
$\varepsilon$ = random error considered negligible or equal to zero owing to the precision and accuracy with which the lab exercises were conducted
And $a$, $b$ and $c$ = regression constants in the linear relationship

Eq. (14) is a multiple regression relationship for four variables of the unconfined compression test model required to achieve the deformation necessary for a stabilized soil matrix subjected to vertical loading to be considered to have met the minimum requirements for use as a consistent material according to sub-grade construction standards (cite) under the varying proportions of the additive ash materials. Also, if the least square sum is minimized for the population of results obtained, the following equations are deduced from Eq. (14).

$$\begin{aligned}
\sum Y &= an + b\sum x_1 + c\sum x_2\\
\sum Yx_1 &= a\sum x_1 + b\sum x_1^2 + c\sum x_1x_2\\
\sum Yx_2 &= a\sum x_2 + b\sum x_1x_2 + c\sum x_2^2
\end{aligned} \tag{15}$$

Eq. (15) can be translated to a matrix of the general form, AX = B thus;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{16}$$

By solving Eq. (16) by Gauss Reduction Method [49] the regression constants $a$, $b$ and $c$ can be determined for n number of data points.

Having determined the regression constants, the linear multiple regression relationship involving deformation, applied vertical load, the additive sample % by weight proportion and the curing time in an unconfined compression test model may be described as;

$$\mathrm{D}_i = a + b\,P_i + cS_{ai(i)}, i = 1, 2, 3..., n \tag{17}$$

## 4. Results and discussions

From the UV–Vis spectrophotometric characterization and preliminary tests conducted on the disturbed lateritic soil, Fig. 1 showed that the soil absorbance was 1.154 nm at a wavelength of 800 cm. Fig. 2 showed the gradation curve of the studied soil which was classified as an A-2-7 soil according AASHTO classification system as shown in Table 1; classification and index properties of the lateritic soil.

Fig. 3 shows the UV–Vis spectrophotometric characterization of the nanostructured palm bunch ashmaterial used for the stabilization of the lateritic soil. Its varying percentage by weight proportion ($S_a$) is one of the major independent variables of this regression model affecting all the strength properties of the stabilized lateritic soil. Table 2 showed the oxide composition of the nanostructured palm bunch ash. It can be deduced that the sum of components of the oxides responsible for pozzolinity in construction materials; $SiO_2$ (64.45%), $Al_2O_2$ (20.12%) and $Fe_2O_2$ (0.95) is equal to 85.52% > 70% which makes the nanostructured additive a highly pozzolanic material [55].

Table 3 shows the results of the compaction test of the stabilized lateritic soil conducted in accordance with BS 1377 [50] and BS 1924 [51]. The dependent variable from these results is the maximum dry density (D) which we called the Y and the independent variables are the moisture content (w) designated as $x_1$and additive ash sample ($S_a$) designated as $x_2$ in the multiple regression relationship model of these three variables.

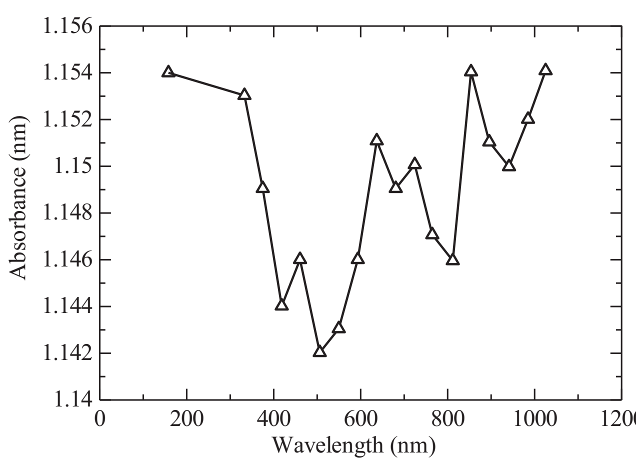


Fig. 1. Variation of absorbance against wavelength for the lateritic soil using UV/VIS spectrophotometer at 25 °C [16].

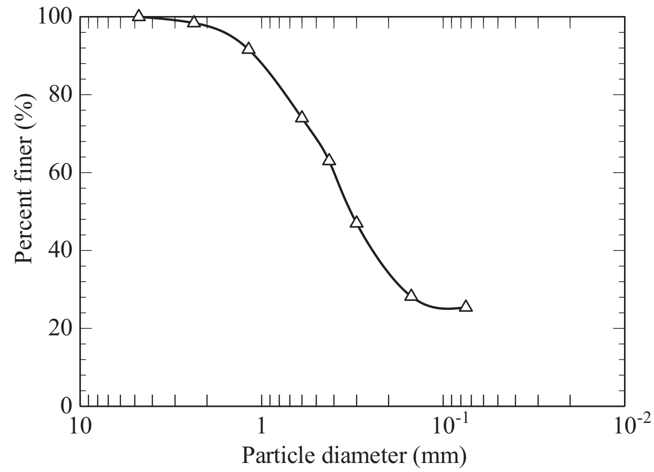


Fig. 2. Particle size distribution curve of the lateritic soil [16].

From the compaction model formulation, Eq. (4); the global matrix equation for the regression exercise is shown below;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

From the laboratory results in Table 3, the variables of Eq. (4) can be determined as follows;

$$\sum Y = \sum D = 10.86; \sum x_1 = \sum w = 73.08; \sum x_2 = \sum S_A = 45; \sum x_1^2 = 892.88; \sum x_1x_2 = 551.28; \sum x_2^2 = 495; \sum Yx_1 = 132.14; \sum Yx_2 = 80.97; n = 6 \tag{18}$$

Substitute the values of Eq. (18) into Eq. (4), it becomes;

$$\begin{bmatrix} 10.86 \\ 132.14 \\ 80.97 \end{bmatrix} = \begin{bmatrix} 6 & 73.08 & 45 \\ 73.08 & 892.88 & 551.28 \\ 45 & 551.28 & 495 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{19}$$

Solving for a, b and c in Eq. (19) by Gauss Reduction Method [49],

$$a = 2.4;\; b = -0.046;\; \text{and } c = -0.0021 \tag{20}$$

Substituting the values of a, b and c from Eq. (20) in Eq. (5),

$$D_i = 2.4 - 0.046w_i - 0.0021S_{a(i)}; \mathrm{i} = 1, 2, 3..., n \tag{21}$$

Eq. (21) is the compaction linear regression model equation of a stabilized lateritic soil for maximum dry density, moisture content and the additive ash material in varying

Table 1
Classification Properties of the Lateritic Soil [49]

| Property/Unit | | Quantity |
|---|---|---|
| % Passing BS No. 200 sieve | | 25.40 |
| Natural Moisture Content, (%) | | 10 |
| Liquid Limit, (%) | | 47 |
| Plastic Limit, (%) | | 25 |
| Plasticity Index, (%) | | 22 |
| Specific Gravity | | 2.67 |
| AASHTO classification | | A-2-7 |
| USCS | | CH |
| Optimum Moisture Content, (%) | | 13 |
| Maximum Dry Density ($g/cm^3$) | | 1.84 |
| California bearing ratio, (%) | | 14 |
| Unconfined Compressive Strength, ($kN/m^2$) | 28 days | 230.77 |
| | 14 days | 219.11 |
| | 7 days | 194.26 |
| Color | | Reddish Brown |

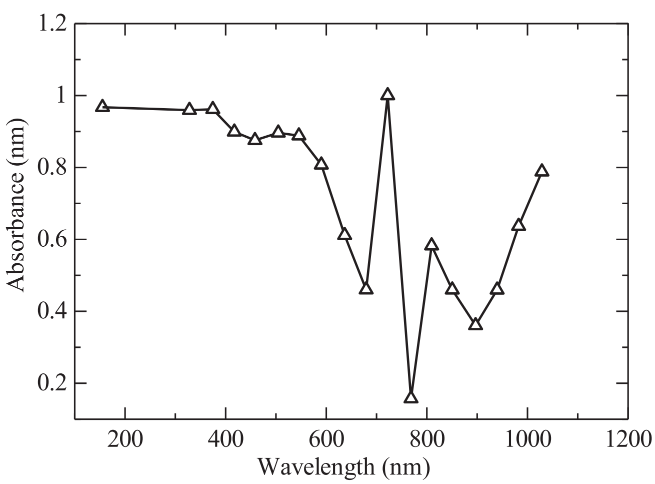


Fig. 3. Variation of absorbance against wavelength for the nanostructured palm bunch ash particles using UV/VIS spectrophotometer at 25 °C [16].

percentages by weight of the dry sample. Table 4 shows various model effects of the independent variables on the dependent variable. This shows the efficiency of the operation to monitor the design of admixtures for soil compaction to achieve the proper densification at varying moisture contents and proportions of additive sample. The moisture content of the stabilized matrix can be altered by application of the model equation to generate an estimated maximum dry density mathematically. The reduction in the density values, an indication of the addition in varying proportions of a material of lesser density and of amorphous property agrees with the lab results [16]. Finally, it can be deduced that the model equation can be used under different conditions in a soil compaction operation [16,30,36].

Fig. 4 shows the graphical behaviour of the density in a compaction model with the addition of the proportions of ash additive (nanostructured palm bunch ash). The density–additive relationship with $R^2$ of 0.981 has a logarithmic relationship of $Y = -40.5\ln(x) + 27.76$ where $Y$ is the dependent variable; density and x is the independent variable; ash additive added in percentages by weight of the dry sample.

Table 5 shows the results of the Atterberg limits test of the stabilized lateritic soil conducted in accordance with BS 1377 [50] and BS 1924 [51]. The dependent variable from these results is the plasticity index ($I_P$) which we called the Y and the independent variables are the moisture content ($w$) designated$x_1$; the plastic limit ($w_P$) designated $x_2$; liquid limit ($w_L$) designated $x_3$ and additive ash sample ($S_a$) designated$x_4$ in the multiple regression relationship model of these five variables.

Table 2
Chemical properties of nanosized palm bunch ash [16].

| Constituents | CaO | MnO | MgO | ZnO | PbO | CuO | CdO | $Fe_2O_2$ | $Al_2O_2$ | $SiO_2$ | $Na_2O$ | $P_2O_5$ | $K_2O$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| % wt in NPBA | 12.7 | 0.13 | 0.01 | 0.78 | 0.07 | Trace | Trace | 0.95 | 20.12 | 64.45 | 0.71 | 0.64 | 0.14 |

Table 3
Effect of NPBA on the compaction of the lateritic soil.

| NPBA Proportion ($S_a$) (%) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| MDD (D) ($g/cm^3$) | 1.84 | 1.72 | 1.93 | 1.84 | 1.76 | 1.77 |
| OMC (w) (%) | 13.00 | 11.84 | 11.10 | 11.74 | 12.50 | 12.90 |

Table 4
Compaction regression model results.

| $w$ (%) | $D_1;S_a = 0$ (g/cm$^3$) | $D_2;S_a = 3$ (g/cm$^3$) | $D_3;S_a = 6$ (g/cm$^3$) | $D_4;S_a = 9$ (g/cm$^3$) | $D_5;S_a = 12$ (g/cm$^3$) | $D_6;S_a = 15$ (g/cm$^3$) |
|---|---|---|---|---|---|---|
| 10 | 1.94 | 1.9337 | 1.9274 | 1.9211 | 1.9148 | 1.9085 |
| 11 | 1.894 | 1.8877 | 1.8814 | 1.8751 | 1.8688 | 1.8625 |
| 12 | 1.848 | 1.8417 | 1.8354 | 1.8291 | 1.8228 | 1.8165 |
| 13 | 1.802 | 1.7957 | 1.7894 | 1.7831 | 1.7768 | 1.7705 |
| 14 | 1.756 | 1.7497 | 1.7434 | 1.7371 | 1.7308 | 1.7245 |
| 15 | 1.71 | 1.7037 | 1.6974 | 1.6911 | 1.6848 | 1.6785 |
| 16 | 1.664 | 1.6577 | 1.6514 | 1.6451 | 1.6388 | 1.6325 |
| 17 | 1.618 | 1.6117 | 1.6054 | 1.5991 | 1.5928 | 1.5865 |

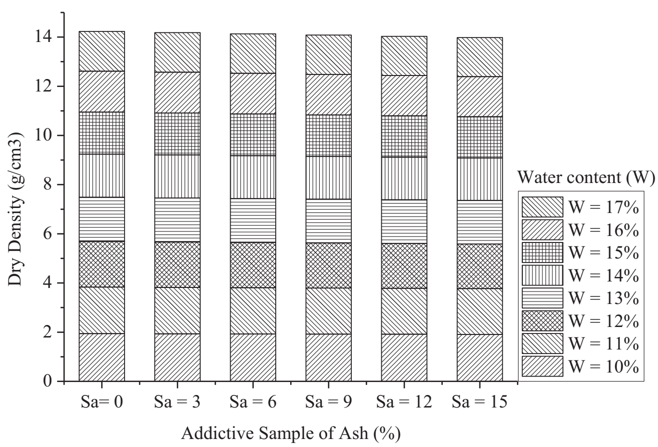


Fig. 4. Compaction variables relationship model.

From the Atterberg model formulation, Eq. (8); the global matrix equation for the regression exercise is shown below;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \\ \sum Yx_3 \\ \sum Yx_4 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 & \sum x_3 & \sum x_4 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 & \sum x_1x_3 & \sum x_1x_4 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 & \sum x_2x_3 & \sum x_2x_4 \\ \sum x_3 & \sum x_1x_3 & \sum x_2x_3 & \sum x_3^2 & \sum x_3x_4 \\ \sum x_4 & \sum x_1x_4 & \sum x_2x_4 & \sum x_3x_4 & \sum x_4^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

From the laboratory results in Table 4, the variables of Eq. (8) can be determined as follows;

$$\begin{aligned} &\sum Y = \sum I_P = 87.13; \sum x_1 = \sum \mathrm{w} = 73.08; \\ &\sum x_2 = \sum w_P = 99.99; \sum x_3 = \sum w_L = 165.43; \\ &\sum x_4 = \sum S_A = 45; \sum x_1^2 = 723.88; \\ &\sum x_1x_2 = 1216.16; \sum x_1x_3 = 2223.24; \\ &\sum x_1x_4 = 551.28; \sum x_2x_3 = 3448.44; \\ &\sum x_2x_4 = 592.44; \sum x_3x_4 = 1075.62; \\ &\sum x_2^2 = 1826.31; \sum x_3^2 = 6528.63; \sum x_4^2 = 495; \\ &\sum Yx_1 = 1054.08; \sum Yx_2 = 1622.13; \\ &\sum Yx_3 = 3080.19; \sum Yx_4 = 483.18; n = 6 \end{aligned} \quad (22)$$

Substituting the values of the variables in Eq. (8);

$$\begin{bmatrix} 87.13 \\ 1054.08 \\ 1622.13 \\ 3080.19 \\ 483.18 \end{bmatrix} = \begin{bmatrix} 6 & 73.08 & 99.99 & 165.43 & 45 \\ 73.08 & 723.88 & 1216.16 & 2223.24 & 551.28 \\ 99.99 & 1216.16 & 1826.31 & 3448.44 & 592.44 \\ 165.43 & 2223.24 & 3448.44 & 6528.63 & 1075.62 \\ 45 & 551.28 & 592.44 & 1075.62 & 495 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \quad (23)$$

Eq. (23) was solved by Gauss Reduction Method to determine the values of a, b, c, d and e as follows; $a = -0.57$; $b = 0.023$; $c = 0.944$; $d = 0.00108$; $e = -0.13$.

Substituting for the values of a, b, c, d and e in Eq. (9);

$$\begin{aligned} I_{P(i)} &= -0.57 + 0.023w_i + 0.944w_{P(i)} + 0.00108w_{L(i)} \\ &\quad - 0.13S_{a(i)}; \mathrm{i} = 1, 2, 3..., \mathrm{n} \end{aligned} \quad (24)$$

Eq. (24) is the regression relationship model of the Atterberg limits of the stabilized soil-nanostructured ash material. The five variables of this model were Plasticity index, moisture content, plastic limit, liquid limit and

Table 5
Effect of NPBA on the consistency limits of the lateritic soil.

| NWPA Proportion, (%) by weight ($S_a$) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| OMC ($w$) (%) | 13.00 | 11.84 | 11.10 | 11.74 | 12.50 | 12.90 |
| Liquid limit, $w_L$ (%) | 47.00 | 38.36 | 35.91 | 29.60 | 21.69 | 14.56 |
| Plastic limit, $w_P$ (%) | 25.15 | 19.97 | 18.21 | 14.99 | 12.23 | 9.44 |
| Plasticity index, $I_P$ (%) | 21.85 | 18.39 | 17.70 | 14.61 | 9.46 | 5.12 |

percentage by weight proportion additive sample. Table 6 shows the behaviour of the model variables in a consistency test operation. '$w$' is the laboratory values of the moisture content while $w^*$ is the applied values of moisture content used to examine the behaviour of the dependent variable $I_P$ shown as $I_P{}^*$ in Table 6. The results show that the treated soil-admixture matrix with moisture and admixture generated model results that are in tandem with laboratory observations [16,30,36].

Figs. 5 and 6 show the graphical behaviour of the of plasticity index as a dependent variable with water content and ash additive. From Fig. 5, it can be deduced that the Ip reduced considerably with $R^2$ of 0.968 having a polynomial relationship with the ash additive as $Y = -0.014x^2 - 0.330x + 17.69$ where $x$ is the ash additive in varying percentage proportions by weight of the dry sample in the Atterberg limits regression model. Fig. 6 gave rise from the manipulation of the water content of the model which eventually produced a polynomial relationship as $Y = -0.019x^2 - 0.245x + 17.79$ with $R^2$ of 0.969. This relationship agrees perfectly as the previous curve. This is indication that the model can be manipulated with figures to achieve different results under different Atterberg conditions. The reduced $I_P$ and $I_P{}^*$ is an indication that the ash additive reduced the adsorbed water to form a flocculated matrix of during the lab operation [16].

Table 7 shows the results of the California bearing ratio test of the stabilized lateritic soil conducted in accordance with BS 1377 [50] and BS 1924 [51] under laboratory conditions and soaked for 28 days to relate field conditions. The dependent variable from these results is the CBR which we called the Y and the independent variables are the moisture content (w) designated as $x_1$, the dry density (D) designated as $x_2$ and the additive ash sample ($S_a$) designated as $x_3$ in the multiple regression relationship model of these four variables.

From the CBR model formulation, Eq. (12); the global matrix equation for the regression exercise is shown below;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \\ \sum Yx_3 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 & \sum x_3 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 & \sum x_1x_3 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 & \sum x_2x_3 \\ \sum x_3 & \sum x_1x_3 & \sum x_2x_3 & \sum x_3^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

From the laboratory results in Table 5, the variables of Eq. (12) can be determined as follows;

Table 6
Behaviour of the variables of the Atterberg limits regression model.

| | | | | | | |
|---|---|---|---|---|---|---|
| $W$ (%) | 13 | 11.84 | 11.1 | 11.74 | 12.5 | 12.9 |
| $w^*$ (%) | 10 | 11 | 12 | 13 | 14 | 15 |
| $w_P$ (%) | 25.15 | 19.97 | 18.21 | 14.99 | 12.23 | 9.44 |
| $w_L$(%) | 47.00 | 38.36 | 35.91 | 29.6 | 21.69 | 14.56 |
| $S_A$ (%) | 0 | 3 | 6 | 9 | 12 | 15 |
| $I_P$ (%) | 23.52 | 18.16 | 16.14 | 12.71 | 9.73 | 6.71 |
| $I_P{}^*$ (%) | 23.45 | 18.18 | 16.16 | 12.74 | 9.77 | 6.75 |

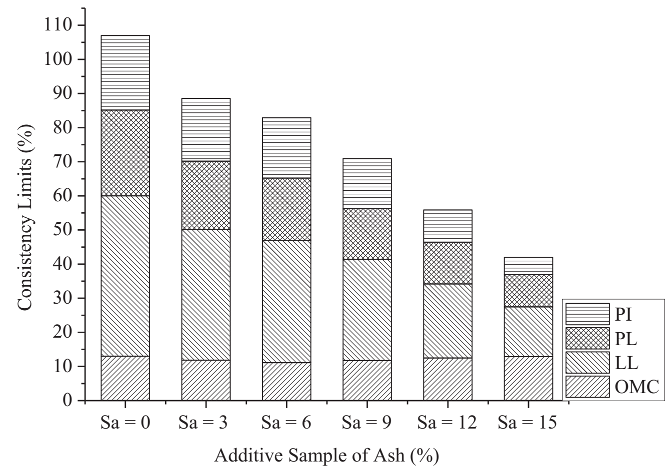


Fig. 5. Consistency behaviour of treated soils model 1.

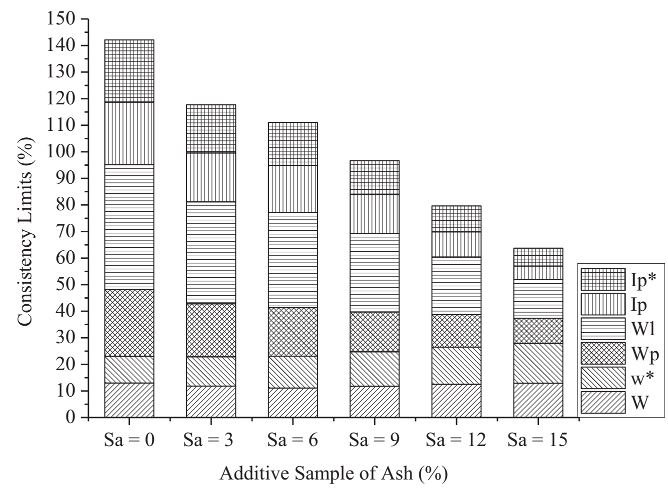


Fig. 6. Consistency limits variables relationship of model 2.

$$\begin{aligned} &\sum Y = \sum CBR = 111; \sum x_1 = \sum \text{w} = 73.08; \\ &\sum x_2 = \sum D = 10.86; \sum x_3 = \sum S_A = 45; \\ &\sum x_1^2 = 723.88; \sum x_1x_2 = 132.14; \sum x_1x_3 = 551.28; \\ &\sum x_2x_3 = 80.97; \sum x_2^2 = 19.685; \sum x_3^2 = 495; \\ &\sum Yx_1 = 1343.7; \sum Yx_2 = 202.25; \sum Yx_3 = 918; \text{n} = 6 \end{aligned} \tag{25}$$

Substitute the variables of Eq. (25) into Eq. (12);

$$\begin{bmatrix} 111 \\ 1343.7 \\ 202.25 \\ 918 \end{bmatrix} = \begin{bmatrix} 6 & 73.08 & 10.86 & 45 \\ 73.08 & 723.88 & 132.14 & 551.28 \\ 10.86 & 132.14 & 19.685 & 80.97 \\ 45 & 551.28 & 80.97 & 495 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \tag{26}$$

Solving Eq. (25) with Gauss Reduction Method [49], the regression constants a, b, c and d were determined as follows;

a = −94.79; b = 0.015; c = 59.49 and d = 0.724.

Table 7
Effect of NPBA on CBR of the Lateritic Soil.

| NPBA Proportion by weight ($S_a$), (%) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| Dry Density, (D)(g/cm$^3$) | 1.84 | 1.72 | 1.93 | 1.84 | 1.76 | 1.77 |
| Moisture content (w) (%) | 13 | 11.84 | 11.10 | 11.74 | 12.50 | 12.90 |
| CBR (%) | 14 | 10 | 21 | 30 | 16 | 20 |

Table 8
Behaviour of the variables of the CBR regression model.

| | | | | | | |
|---|---|---|---|---|---|---|
| $W$ (%) | 13 | 11.84 | 11.1 | 11.74 | 12.5 | 12.9 |
| $w^*$ (%) | 10 | 11 | 12 | 13 | 14 | 15 |
| $D$ (g/cm$^3$) | 1.84 | 1.72 | 1.93 | 1.84 | 1.76 | 1.77 |
| $S_a$ (%) | 0 | 3 | 6 | 9 | 12 | 15 |
| CBR (%) | 14.865 | 9.880 | 24.541 | 21.362 | 18.788 | 21.563 |
| CBR$^*$ (%) | 14.82 | 9.87 | 24.554 | 21.381 | 18.81 | 21.6 |

Substituting the values of a, b, c and d in Eq. (13),

$$CBR_i = -94.79 + 0.015w_i + 59.49D_i + 0.724S_{a(i)}; i = 1, 2, 3 \ldots, n \tag{27}$$

Eq. (27) is the regression relationship model of the CBR of the stabilized soil-nanostructured ash material matrix. The four variables of this model were CBR, moisture content, dry density and percentage by weight proportion of additive sample. Table 8 shows the CBR results generated from the regression model relationship for a treated soil-admixture matrix. The generated model is in tandem with the laboratory results and shows it can be used to monitor the dependent variable; CBR behaviour of a treated soil-admixture matrix under different moisture contents and admixture proportions [16,30,36].

Figs. 7, 8, and 9 are the graphical behaviour of the CBR regression model with the independent variables. Fig. 7 shows the combined behaviour of the variables which also shows that the various proportions of the ash additive reacted differently with the CBR. In Fig. 8, it can be deduced that the optimum CBR was achieved at 6% ash additive while the optimum minimum was achieved at 3% by weight additive of the ash which agrees with the laboratory results [16]. This regression curve has a polynomial relationship of $Y = -0.195x^2 + 3.085x + 10.39$ with $R^2$ of 0.513 where $Y$ is the dependent variable; CBR and $x$ is the in dependent variable; ash additive in varying propor-

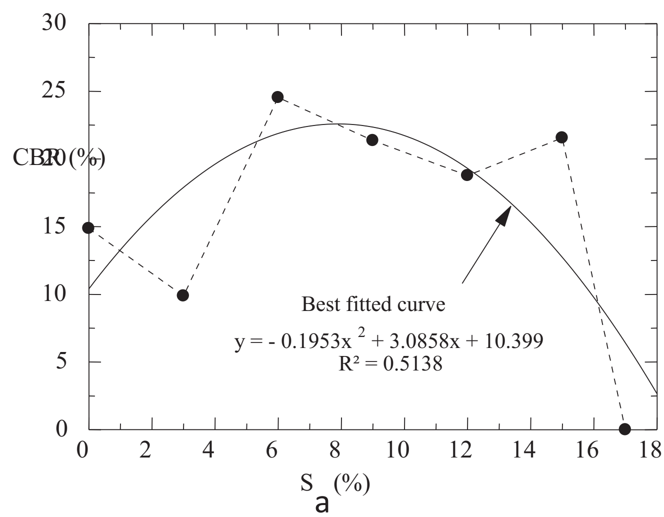


Fig. 8. CBR/$S_a$ relationship model.

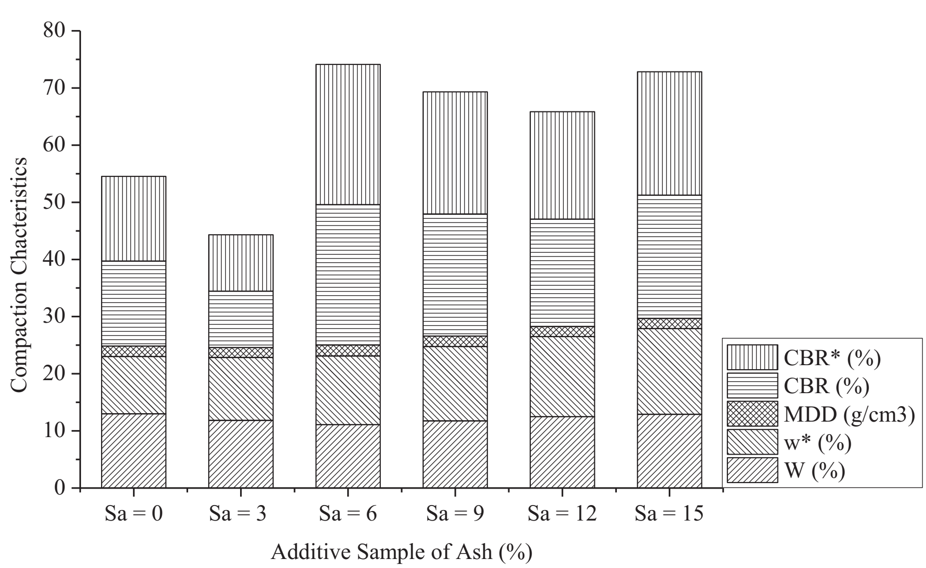


Fig. 7. California bearing ratio variables relationship model.

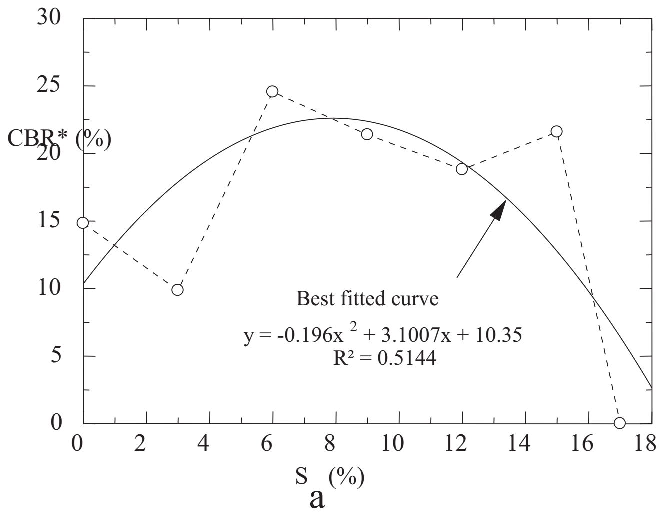


Fig. 9. CBR*/$S_a$ relationship model.

tions by weight of dry sample. From Fig. 9, another set of values of CBR* were generated and a model with the polynomial relationship $y = -0.196x^2 + 3.100x + 10.35$ with $R^2$ of 0.514 was observed from a manipulated operation with the values of the moisture content ($w$) which agrees with the original regression model result of Fig. 8.

Table 9 shows the results of the unconfined compressive strength test of the stabilized lateritic soil with a capacity of 50 kg, specimen size of 38 mm diameter by 76 mm long [50–51]. The dependent variable from these results is the deformation (Đ) which we called the Y and the independent variables are the applied axial load (P) designated as $x_1$and the additive ash sample ($S_a$) designated as $x_2$ in the linear multiple regression relationship model of these three variables.

From the UCS model formulation, Eq. (16); the global matrix equation for the regression exercise is shown below;

$$\begin{bmatrix} \sum Y \\ \sum Yx_1 \\ \sum Yx_2 \end{bmatrix} = \begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1x_2 \\ \sum x_2 & \sum x_1x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

From the laboratory results in Table 6, the variables of Eq. (16) can be determined as follows;

$$\begin{aligned} &\sum Y = \sum \text{Đ} = 1780.37; \sum x_1 = \sum \mathrm{P} = 2.171; \\ &\sum x_2 = \sum S_a = 45; \sum x_1^2 = 1.0677; \\ &\sum x_1x_2 = 18.504; \sum x_2^2 = 495; \sum Yx_1 = 681.11; \\ &\sum Yx_2 = 15176.34; n = 6 \end{aligned} \quad (28)$$

Substitute the variables of Eq. (28) into Eq. (16);

$$\begin{bmatrix} 1780.37 \\ 681.11 \\ 15176.34 \end{bmatrix} = \begin{bmatrix} 6 & 2.171 & 45 \\ 2.171 & 1.0677 & 18.504 \\ 45 & 18.504 & 495 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (29)$$

Solving Eq. (29) with Crammer's rule, the regression constants $a$, $b$ and $c$ were determined as follows; $a = 198.47$; $b = 44.62$; and $c = 10.948$.

Substituting the values of $a$, $b$ and $c$ in Eq. (17),

$$\mathrm{D}_i = 198.47 + 44.62P_i + 10.948\, S_{a(i)}; i = 1, 2, 3..., n \quad (30)$$

Eq. (30) is the linear multiple regression relationship model for the three variables; deformation, applied axial load and varying proportions of the admixture as material in percentage by weight of dry sample in an unconfined

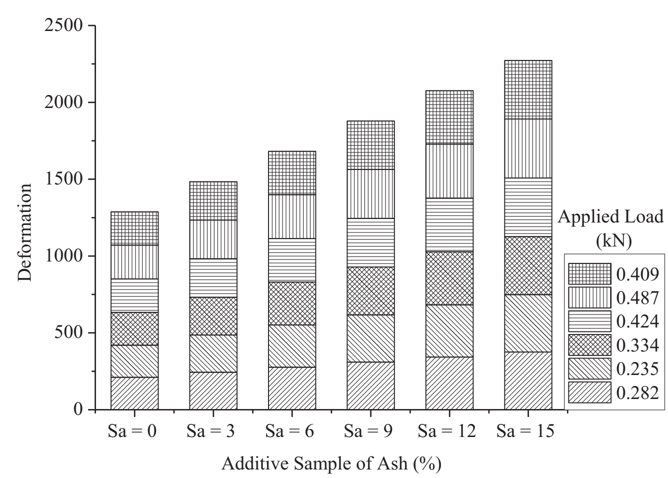


Fig. 10. Deformation of treated soils at various value of applied load.

Table 9
Effect of NPBA on the UCS of the Lateritic Soil with cross sectional area at failure = 0.00122 m$^2$.

| NPBA Proportion by weight ($S_a$) (%) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| Applied load, (P) (kN) | 0.282 | 0.235 | 0.334 | 0.424 | 0.487 | 0.409 |
| 28 days curing (Đ) (kN/m$^2$) | 230.77 | 192.97 | 274.06 | 347.85 | 399.46 | 335.26 |

Table 10
Behaviour of the variables of the UCS regression model.

| P (kN) | 0.282 | 0.235 | 0.334 | 0.424 | 0.487 | 0.409 |
|---|---|---|---|---|---|---|
| Đ$_1$; $S_a$=0 (kN/m$^2$) | 211.05 | 209.0 | 213.37 | 217.39 | 220.2 | 216.72 |
| Đ$_2$; $S_a$ =3 (kN/m$^2$) | 243.87 | 241.78 | 246.19 | 250.21 | 252.03 | 249.54 |
| Đ$_3$; $S_a$ =6 (kN/m$^2$) | 276.74 | 274.64 | 279.06 | 283.08 | 285.89 | 282.41 |
| Đ$_4$; $S_a$ =9 (kN/m$^2$) | 309.58 | 307.49 | 311.9 | 315.92 | 318.73 | 315.25 |
| Đ$_5$; $S_a$ =12(kN/m$^2$) | 342.43 | 340.34 | 344.75 | 348.77 | 351.58 | 348.1 |
| Đ$_6$; $S_a$ =15 (kN/m$^2$) | 375.27 | 373.18 | 377.59 | 381.61 | 384.42 | 380.94 |

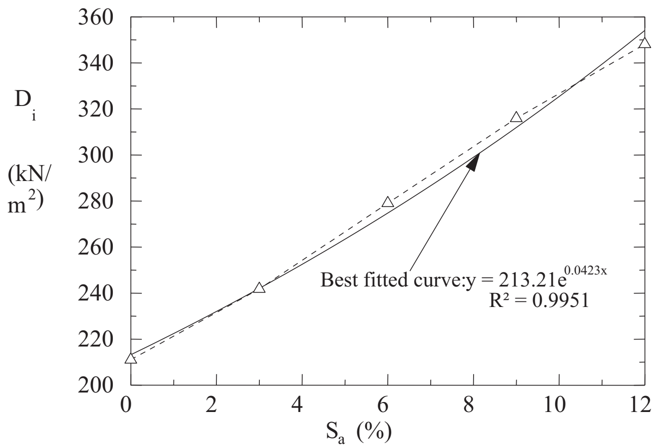


Fig. 11a. Deformation/$S_A$ relationship model.

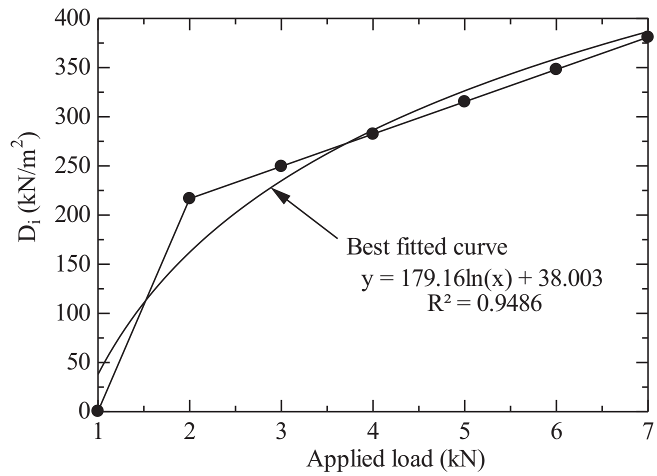


Fig. 11c. UCS Regression model relationship.

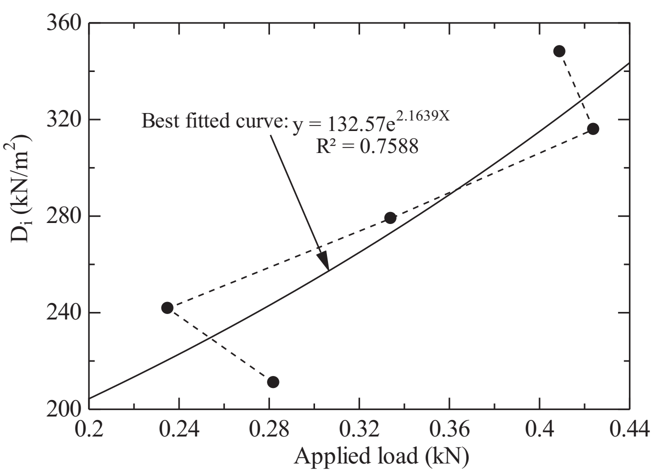


Fig. 11b. Deformation/Applied load relationship model.

compression test. Table 10 and Fig. 4show the UCS regression model relationship results. The results show that the generated model can be used under varying applied loads to monitor the deformation of a treated soil-admixtire matrix in an unconfined compression test operation. This is because the results are in tandem with the laboratory results [16,30].

Fig. 10 shows the regression model relationship between deformation (D) and ash additive ($S_a$) with an exponential curve equation $y = 213.2e^{0.042x}$ with $R^2$ of 0.995; a near perfect correlation. This exponential increment in D is only possible in a laboratory and field condition where the ash additive increment donates more cation for the cation exchange reaction within the diffused double layer of the stabilized matrix. This is also as result of the rearrangement of the crystalline arrangement of the lateritic soil [16]. Figs. 11a–c show the relationship between the deformation and applied axial load. The curve has an exponential relationship $y = 132.5e^{2.163x}$ with $R^2$ of 0.758.

## 5. Concluding remarks

It can be concluded as follows;

The strength properties of the soil-admixture treated matrix have been modelled with the multiple regression relationship between the independent variables and the dependent variables of the various test operations; compaction, Atterberg, California bearing ratio and unconfined compression tests to generate relationship models between covariants and also between variants. These models were discovered to be in tandem with the laboratory results conducted earlier in terms of behaviour and variables reactions and the resultant effect on the dependent variables. With the regression model relationships, the strength properties of the treated soils can be deduced under different conditions to determine design results to be applied in the field.

## 6. Conflict of interests

There are no conflicts of interests recorded in this article.

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