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Master Curves for Stiffness Asphalt Concrete

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Abstract: The present paper discusses a methodology and tries to construct the stiffness master curves for asphalt mixture. In asphalt pavements the stiffness modulus of asphalt concrete increases with decreasing temperature and increasing loading frequency. By shifting such stiffness modulus versus loading time relationship for various temperatures horizontally with respect to the curve chosen as reference, a complete modulus time behavior curve is chosen and consequently reference temperature can be assembled. The proposed model is based on physical observations. It is believed to give reasonable estimates for the mix stiffness at any arbitrary loading frequency. The technique to determine of the master curve is based on the principle of time temperature correspondence or thermo rheological simplicity. The experimental data are plotted against the log frequency or log loading timing and therefore, by choosing a reference temperature, the data of the other temperatures are shifted horizontally until they fit the curve for the reference temperature. Then the data acquired from other temperatures are shifted until they fix the extended reference curve. Master curve can be constructed by fitting a sigmoid function using non linear least square regression techniques. The shifting is done using an experimental approach by solving shift factors simultaneously with the parameters of the model. This is completed without the need to assume any functional form for the shift factor equation. Based on the material, the paper tries to show that a sigmoid model can be best described as the master curve of mix stiffness of asphalt concrete. This model can also explain the physical behavior of asphalt concrete.

***Key words**: Asphalt concrete; Master curve; Non-linear viscoelastic behavior; Stiffness.*

Introduction

The mechanistic-empirical design method is widely used in new pavement design or in evaluation of existing ones. Because material structural properties affect pavement response and performance, their specification is a fundamental of pavement design process. One of the key parameters for analytical methods is the evaluation of layer elastic modulus to be used in the elastic theory for pavement design. The resilient modulus, which is based on recoverable strains measured in laboratory under repeated load, is widely considered as a good estimator of the elastic modulus to be used with the elastic theory.

In order to fulfill design requirements, the resilient modulus becomes a goal during mix design phase and this evaluation is necessary in quality controls during pavement construction. In the design phase, resilient modulus can be determined using laboratory testing methods, while in quality control phase; non-destructive field trials such as FWD tests are recommended to evaluate elastic modulus of layers trough back-calculation.

Since asphalt concrete is a viscoelastic material, loading duration heavily affects resilient modulus. Because vehicle speed is strictly correlated to loading frequency, design values of resilient modulus should be selected commensurate with the design speed to be representative of in situ loading characteristics and layer stiffness.

Most of the mechanistic design methodologies for asphalt pavement are largely based on the assessment of the structural response i.e. the critical stresses/strains due to a certain load applied on the pavements. The critical strains, which are generally considered here, mean the horizontal flexural tensile strain at the bottom of the asphalt layer and the vertical compressive strain at the top of the sub-grade. Linear elastic multi-layer program like BISAR [1] or viscoelastic multi-layer programs such as Kenlayer [2] and VEROAD [3] are applied to calculate such stresses/strains. As such, in order to evaluate both the induced load and thermal stress and strain distribution in asphalt pavements, stiffness of bituminous mixture is a required parameter. Whether in laboratory or the field, stiffness is usually regarded as an indicator of the quality of mixtures and is required in the design to evaluate cumulative damage and also to assess age-hardening trends of bituminous mixtures [4]. The mix stiffness is generally estimated through the so-called master curves i.e. the relationship between the mix stiffness, loading time (or frequency), and temperature. In practice, this relationship is determined through indirect tensile test or the four-point bending test. This is performed by measuring the stiffness of asphalt concrete at different temperatures and frequencies. Typically, it is found that the stiffness modulus of asphalt mixes increases with decreasing temperature and increasing loading frequency. By shifting the stiffness modulus versus loading time for various temperatures horizontally with respect to the chosen reference curve, a complete modulus-time behavior curve can be assembled at a constant, arbitrary chosen reference temperature, T_{ref}. This study describes a methodology to construct the stiffness master curves for asphalt concrete. The model is based on physical observations and is believed to give 'reasonable' estimates for the mix stiffness at any arbitrary loading frequency.

Time-Temperature Superposition Principle

Mechanical properties of visco-elastic materials such as asphalt concrete are strongly influenced by temperature and loading time.

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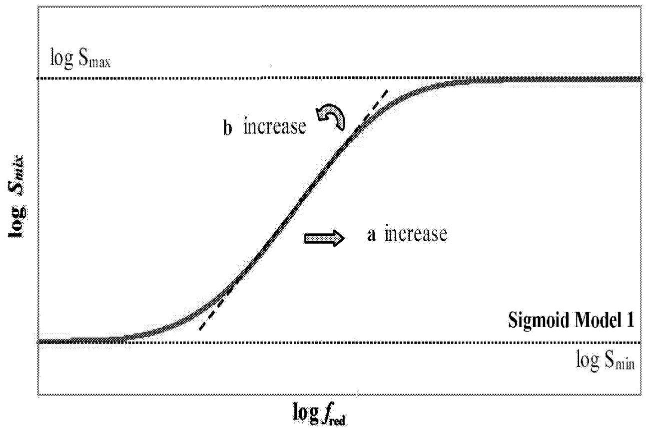

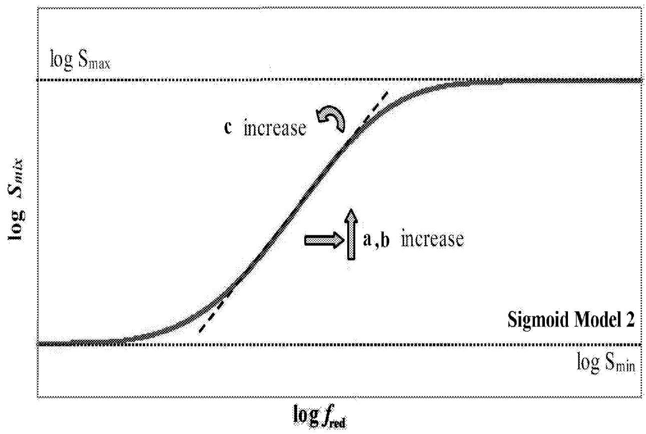

Fig. 1. Parameters in the Sigmoid Model.

For an asphalt mix in linear visco-elastic phase, stiffness variations, obtained from modifying temperature at fixed loading frequency, can be replicated by changing loading duration at the same test temperature; this behavior is defined as termo-rehological simplicity and it allows the application of time-temperature superposition principle. Experimental data which collected at various test temperatures and drawn in stiffness versus frequency graph, lie on different isothermal curves.

The data collected at different temperature levels can be "*shifted*" relatively to the time of loading (or frequency), so that the various curves can be aligned to form a single master curve. Further, this master curve can be constructed using an arbitrary selected reference temperatures (T_{ref}) to which all data are shifted. At reference temperature, the shift factor is equal to one. The technique of determining the master curve is based on the corresponding time-temperature principle, or thermo-rheological simplicity, which uses the equivalence between frequency and temperature for the stiffness modulus of bituminous mixes as:

$$Logf_{red} - Logf = Loga_t \quad \Rightarrow \quad f_{red} = a_T f \tag{1}$$

Where f_{red} is the frequency where the master curve be read (*Hz*), f is loading frequency (*Hz*) and a_T is the shifting factor. The shifting factor can be determined in different ways and the most commonly used formula is an Arrhenius type equation [5, 6]:

$$Loga_T = \frac{E}{2.303R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right) \tag{2}$$

Where T is the experimental temperature (°*K*), T_{ref} indicates the reference temperature (°*K*), E is activation energy (*J/mol*), and R is ideal gas constant, 8.314 (*J/mol°K*). In some other literatures, different values have been reported for the constant E; Francken and Clauwaert [5] put E = 209,087, Lytton et al. [7] reported E = 250,062 and according to Jacobs [6] E = 147,050.

To calculate the shift factor, Williams-Landel-Ferry (WLF) gave following equation [8]:

$$Loga_T = -\frac{C_1(T - T_{ref})}{C_2 + (T - T_{ref})} \tag{3}$$

Where C_1 and C_2 are empirical constants, T is the experimental temperature (°*K*), T_{ref} indicates the reference temperature (°*K*). According to Sayegh [9] C_1= 9.5 and C_2=95. It has also been defined by Lytton et al. [7] as C_1=19 and C_2= 92.

Master Curve Using Sigmoid Model

The experimental (stiffness) data are plotted against log frequency or log loading time. By choosing a reference temperature, the data from other temperatures are shifted horizontally until they fit the curve for the reference temperature (the shift can be obtained by interpolation or extrapolation). Thereafter, the data obtained at the other temperatures are shifted until they fix the extended reference curve. It is quite common to use the generalized power law to define the frequency dependant behavior of bituminous materials at low and moderate temperatures. If higher temperatures data is included, polynomial fitting functions are also used [10]. Meanwhile, extrapolation of polynomial fits can result in some problems, which will be highlighted later. It is worth to mention that if the data intend to include wide range of frequencies, testing at temperatures higher and lower than the reference temperature will be needed.

In the present study, two sigmoid models are being presented that show how the master curve could be constructed by fitting a sigmoid function and using non-linear least square regression techniques. The shifting is done by using an experimental approach as well as by solving shift factors simultaneously with the parameters of the model. Also there is no need to assume any functional form for the shift factor equation. These two models are described as follow:

Model 1

$$LogS_{mix} = LogS_{\min} + (LogS_{\max} - LogS_{\min})\left(1 - \frac{1}{e^{\left(\frac{10 + Log\, f_{red}}{a}\right)^b}}\right) \tag{4}$$

Model 2

$$LogS_{mix} = LogS_{\min} + (LogS_{\max} - LogS_{\min})\left(\frac{a}{1 + b\, e^{-c\, Log\, f_{red}}}\right) \tag{5}$$

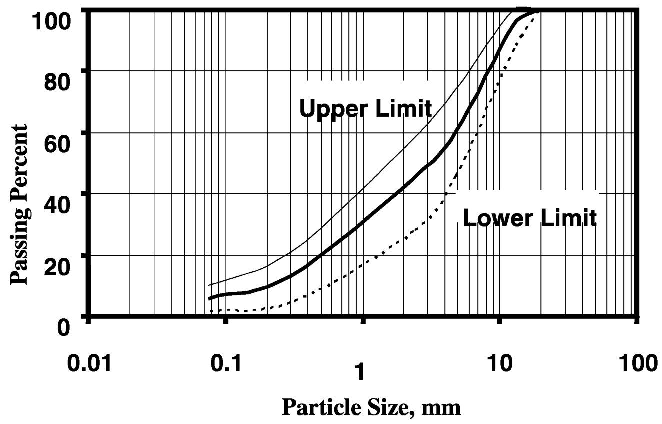

Fig. 2. Aggregate Gradation.

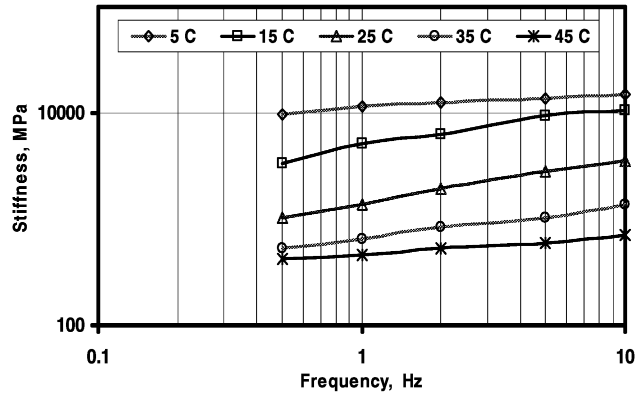

Fig. 3. Mix Stiffness at Different Frequencies and Temperatures.

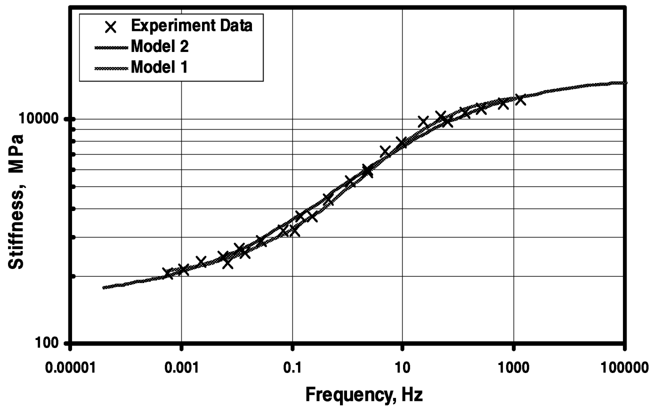

Fig. 4. Comparing Master Curve of Both Models with Experimental Data (Arrhenius Equation).

Where S_{mix}, S_{min}, and S_{max} are mix stiffness, minimum, and maximum mix stiffness, respectively; f_{red} is the reduced frequency whereas a, b, and c are the shape parameters that are related to the curvature of the S-shaped function and the horizontal distance from the turning point to the origin, respectively (Fig. 1).

Physical observation is justified by fitting the data to a sigmoid model. The upper part of this model approaches asymptotically to the maximum stiffness of the mix, which is dependent on the limiting binder stiffness of the mix. At high temperatures, the aggregate skeleton plays a more dominant role than the viscous binder. The modulus starts to reach a limiting equilibrium value, which depends on the gradation of the aggregates [10].

Table 1. Variables in Indirect Tensile Modulus Test.

Frequencies	0.5, 1, 2, 5, and 10*Hz*
Temperatures	5, 15, 25, 35, and 45°C
Load Amplitudes	10 % of indirect tensile strength
Strain Amplitudes	Less than 100$\mu m/m$
Loading Wave	Haversine wave
Preloading Pulse	5 pulses
Stiffness Measurement	after 20 pulses

Experiment to Construct the Stiffness Master Curve

The proposed models have been applied to construct the master curve for the asphalt concrete. The testing has been carried out through UTM-25 machine at the Pavement Laboratory of Amirkabir University of Technology in Tehran.

Materials

The materials applied in the proposed models include 60/70-penetration grade bitumen, aggregates with gradation characterized by 19.5*mm* as shown in Fig. 2 (in accordance with the Pavement Guidelines of Iran), and limestone mineral filler.

Sample Preparation and Mix Composition

Specimens have been prepared by using a Marshall Compactor. Based on the standard specified for heavily trafficked roads (ASTM D1559), compaction blows were 75 blows per face. Mixing and compaction temperatures were designated at 160 and 140ºC, respectively. Specimens for the resilient modulus test were cut smoothly from both ends, with 35*mm* in height and 100*mm* in diameter.

In addition, the optimum binder content for the mixture was found to be 5.5%, the filler content 5 %, and void in total mix (VTM) 4.8 %.

Mix Stiffness at Different Temperatures and Loading Frequencies

The mix stiffness at different temperatures and frequencies has been determined through the indirect tensile modulus test (ASTM D4123) by using Universal Testing Machine (UTM). In the above test, load amplitude should be about 10 to 15 % of tensile strength in order to maintain the deformation in elastic range. The proper test parameters are highlighted in Table 1 and results of the mix stiffness at different temperatures and frequencies are shown in Fig. 3.

Application of the Sigmoid Models

Fitting experimental data to the sigmoidal model is a non linear optimization problem; in order to solve this problem, the Generalized Reduced Gradient (GRG) algorithm has been used, which is a nonlinear extension of the simplex method for linear programming. The GRG algorithm reduces the original nonlinear

Table 2. The Parameters of Sigmoid Model.

	E (J/mol)	*a*	*b*	*c*	$S_{max,}$ *(MPa)*	S_{min}, *(MPa)*	Sum Square Errors
Model 1	216,885.25	10.563	7.236	---	15,900	385	2,720
Model 2	216,885.25	1.14	0.92	0.695	13,890	253	4,098

Table 3. The Parameters of Sigmoid Model.

	C_1	C_2	*a*	*b*	*c*	$S_{max,}$ *(MPa)*	S_{min}, *(MPa)*	Sum Square Errors
Model 1	29.547	224.56	10.486	7.562	----	16,250	372	3,144
Model 2	29.547	224.56	1.119	1.125	0.685	18,430	286	4,389

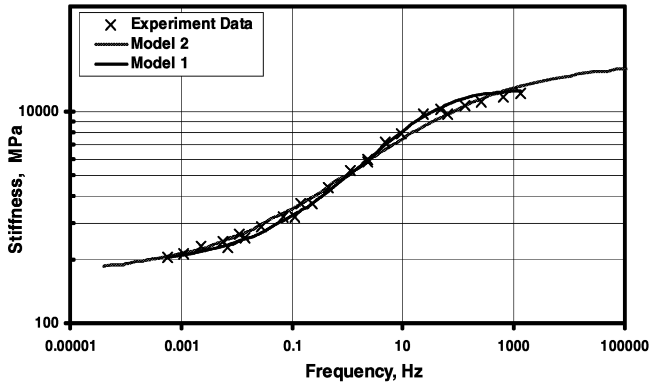

Fig. 5. Comparing Master Curve of Both Models with Experimental Data (WLF Equation).

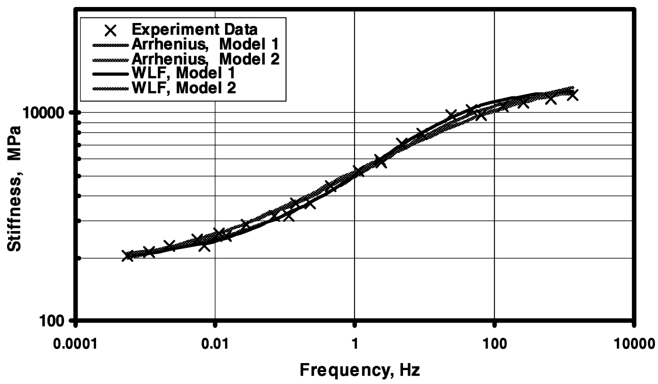

Fig. 6. Comparison between Arrhenius and WLF Equations Sigmoid Models.

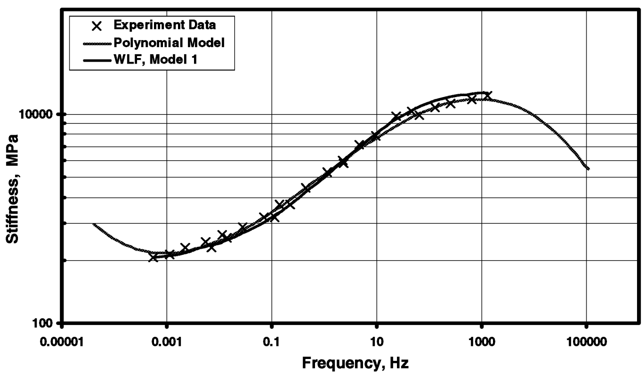

Fig. 7. Comparison of Stiffness Master Curve by Using Polynomial Model and Sigmoid Model.

problem to a less complicate problem which can be solved by using gradient method.

For the proposed models, a reference temperature of 20°C was chosen. By fitting experimental data to the sigmoid models, all model parameters and the constants of the Arrhenius or the WLF equations can be obtained. This can be performed by minimizing the sum of the square of the errors as well as using the Solver Function in the Curve Expert (1.3) software [11].

Fitting the Experimental Data by Using the Arrhenius Equation

The shift factor is defined by the Arrhenius equation, as shown in Eq. (6):

$$a_T = \exp\frac{E}{R}(\frac{1}{T} - \frac{1}{T_{ref}}) \tag{6}$$

Table 2 shows the sigmoid models parameters and the activation energy *E,* which are obtained by minimizing the sum of the square of the errors of the experimental and model values. Fig. 4 shows the goodness of the fit of the above models by using the Arrhenius equation versus the experimentally determined mix stiffness.

Fitting the Experimental Data by using WLF Equation

As stated above in Eq. (3), WLF equation, the shift factor is defined as:

$$Log\ a_T = -\frac{C_1(T - T_{ref})}{C_2 + (T - T_{ref})} \tag{3}$$

The sigmoid models parameters and the empirical parameters of the WLF equation can be obtained at the same time by minimizing the sum of the square of the errors of values obtained from experiment and model. The parameters obtained are shown in Table 3. Fig. 5 shows the good fit between the experiment data and the sigmoid model (using the WLF equation for the shift factor) for mix stiffness.

Fig. 6 compares the two equations given by Arrhenius and WLF. Considering the good-fit of both sigmoid models to the experimental data by using the Arrhenius and the WLF equations for the shift factor, it may therefore be concluded that either of the two equations can be used to estimate the shift factor. A comparative analysis of Figs. 3, 4, and 5 shows that Model 1 fits better to the data

Table 4. The Parameters of Polynomial Model.

C_1	C_2	a_0	a_1	a_2	a_3	Sum Square Errors
29.547	224.56	3.42	0.37	-0.00158	-0.0139	2,622

than the Model 2. Again, the sum of the square of the errors in Tables 2 and 3 is smaller for Model 1 when compare to Model 2. It is concluded that Model 1 is better than Model 2 for the construction of the master curve.

Comparing the Polynomial and the Sigmoid Models

The polynomial and the sigmoid models (Model 1) are compared by using the WLF equation to estimate the shift factor. For the polynomial model, a third degree polynomial is assumed to describe the relationship between the mix stiffness and the reduced frequency:

$$Log(S_{mix}) = a_0 + a_1(\log f_{red}) + a_2(\log f_{red})^2 + a_3(\log f_{red})^3 \quad (7)$$

Where, a_i is the regression coefficient. Where, a_i is the regression coefficient and other parameters have already been discussed in the previous equations. Table 4 shows the polynomial model parameters and the WLF constants. Fig. 7 shows the master curves obtained for the mix stiffness at 20°C by using the polynomial and the sigmoid models.

According to Fig. 7, both models are seem to fit quite well to the range of experimental data, but once as they move outside that range, the polynomial model may not be correct. As such, it is known that the stiffness of asphalt concrete increases with the increase of frequency until a threshold value (S_{max}) and it does not decrease after reaching a maximum value as suggested by the polynomial model. Furthermore, the stiffness decreases with the decrease of frequency until a threshold value (S_{min}) and it does not increase after reaching a minimum value as suggested by the same model. In other words, the polynomial model does not depict the observed behavior of mix stiffness outside the range of the data. However, if there is a need to increase the frequency range to have the polynomial model to be valid, more tests are needed at temperatures higher and lower than the reference temperature.

Construction the Master Curve Using Experiments at Three Temperatures

This part shows how to construct a master curve at a reference temperature of 20°C by using the data obtained from three temperatures of 5, 25, and 45°C. Fig. 8 shows good conformity between the master curves constructed from data obtained from 3 and 6 temperatures by using the sigmoid Model 1. At least two further tests are necessary to be able to catch the upper part of the master curve (obtained from a low temperature data) and the lower part of the master curve (obtained from a high temperature data). First test should be performed at a rather high temperature and the second test at a rather low temperature. The third test may be executed at a medium temperature.

Conclusion

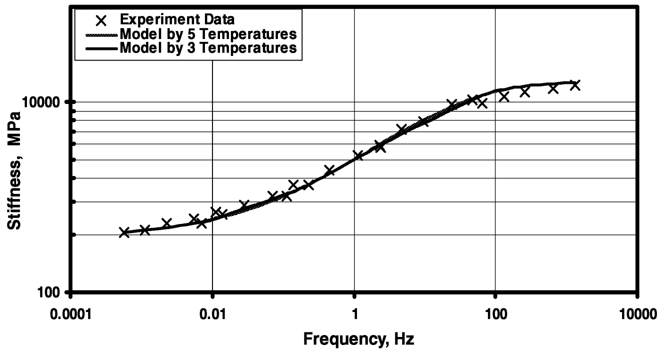

Fig. 8. Comparing Master Curve by Using Data Obtained from 3 and 6 Temperatures.

Based on the material presented above, the following conclusions are drawn:

- Sigmoid models can best describe as the master curve of mix stiffness of asphalt concrete. These models can also explain the physical behavior of asphalt concrete.
- The parameters of the equations for the shift factor need not be assumed, as they can be obtained together with the parameters of the Sigmoid Model by using proper software.
- The Arrhenius and the Williams-Landel-Ferry equations provide quite comparable results for the shift factor.
- The Polynomial Model may encounter some problems if it is intended to estimate values of the mix stiffness outside the range of the data.
- The Sigmoid Model can be described adequately based on the results obtained from tests at three temperatures alone. In sum, two other tests are needed to catch the lower and the upper part of the curve: one such test can be performed at a rather high temperature and the other at a rather low temperature. However, a third test may be executed at a medium temperature.

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